STABLITY, CONVERGENCE OF BALLOON PARTICLE SWARM OPTIMIZER AND ITS APPLICATION ON VECHILE MODELLING

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Abstract: Particle Swarm Optimizer, PSO, exhibits good performance for optimization problem, although, PSO can not guarantee convergence of a global minimum, even a local minimum. However, there are some adjustable parameters and restrictive conditions which can affect performance of the algorithm. In this paper, a new adaptive PSO algorithm—Balloon PSO (BPSO) is proposed. The sufficient conditions for asymptotic stability of acceleration factor and inertia weight are deduced. Furthermore it is proved that BPSO is a global research algorithm. Simulation results of power spectral density (PSD) of vehicle vibratory signal estimation show the good performance of BPSO.

Keywords: Particle Swarm Optimizer (PSO), Balloon PSO (BPSO), asymptotic stability, global convergence

1. INTRODUCTION

Particle Swarm Optimization (PSO) algorithm is a population based parallel optimization technique (Kennedy and Eberhart, 1995), which exhibits good performance for optimization problems. There are some adjustable parameters, such as inertia weight, acceleration factor, scaled factor, and so on, which greatly influence the convergence performance and stability of the algorithm. Some papers (Clerc and Kennedy, 2002, Bergh, 2002, Jie Chen, *et al*., 2003) about the stability analysis have been published with many improved methods proposed.

At time k+1, the update equations of the ith particle in the dth dimension search space of standard PSO algorithm are defined as following:

$$v_{id}^{k+1} = w \cdot v_{id}^{k} + c_1 \cdot rand_{pd} \cdot (p_{gd}^{k} - x_{id}^{k}) + c_2 \cdot rand_{id} \cdot (p_{id}^{k} - x_{id}^{k}) \quad (1)$$

$$x_{id}^{k+1} = x_{id}^{k} + \eta \cdot v_{id}^{k+1} \quad (2)$$

Satisfying $|v_{id}| \le V_{\max}$. As an upper bound of velocity vector in every epoch, $V_{\max}$ can be presented as the Lipschitz condition of particles dynamic systems.

$$\left| x_{id}^{k+1} - x_{id}^{k} \right| \le V_{\max} \quad (3)$$

Where w is the inertia weight, x_{id}^{k} is the current position of the particle; v_{id}^{k} is the velocity vector, p_{id}^{k} is the personal best position of the particle; p_{gd}^{k} is the swarm best position among all particles; c_1, c_2 are acceleration factors, respetively; $rand_{id}$ and $rand_{gd}$ are random number in the range [0,1]. In the search space, particles "fly" to the target guided by the swarm information p_{gd}^{k} and its own information p_{id}^{k}.

In this paper, standard PSO algorithm is analyzed as a discrete dynamic system. Sufficient conditions for asymptotic stability are deduced. On the basis of the analysis, a new adaptive PSO algorithm——Balloon PSO (BPSO) is proposed and proved to be a global research algorithm.

2. STABILITY ANALYSIS OF PSO

Adjustable parameters of PSO are always tuned empirically. The state equations of particles are simplified and analyzed as a constant coefficient dynamic system (Clerc and Kennedy, 2002). In this section, the sufficient asymptotic stability conditions without Lipschitz condition constrains for acceleration factor φ and inertia weight w are deduced. First, a Lemma is introduced (Yang Xiao, 2002).

Lemma 1: Given time varying discrete dynamic system as below:

$$x(n+1) = A(n)x(n)$$

Sufficient condition for asymptotic stability of the discrete-time system is that: there exists *M*>2 and $k \geq M$ satisfying:

$$\rho(T(k,k-M)) < 1$$

Where $T(k,k-M) = \prod_{i=1}^{M} A(k-i)$ is the transfer matrix, ρ is the spectral radius and

$$\rho(T(k,k-M)) = \max\{|\lambda_i|;\quad i \in n\} < 1 \qquad \square$$

Based on the Lemma 1, theorem 1 can be deduced.

Theorem 1: The sufficient condition for asymptotic stability of PSO algorithm is that: the acceleration factor φ^{k+1} and inertia weight w satisfying the following conditions:

$$\eta \cdot w \in (-1,1) \tag{4}$$

$$\{\eta \cdot \varphi^{k+1} \in U \mid U \subset (\varphi_{\min}^{k+1}, \varphi_{\max}^{k+1})\}$$
$$\varphi_{\min}^{k+1} = \frac{-\eta\varphi^k(1+\eta w)}{1+\eta w-\eta\varphi^k}$$
$$\varphi_{\max}^{k+1} = \frac{2(1+(\eta w)^2)-\eta\varphi^k(1+\eta w)}{1+\eta w-\eta\varphi^k} \tag{5}$$

Proof: The standard PSO algorithm can be expressed as

$$\begin{bmatrix} x_{id}^{k+1} \\ x_{id}^{k} \end{bmatrix} = \begin{bmatrix} (1+\eta\cdot w-\eta\cdot\varphi^{k+1}) & -\eta\cdot w \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_{id}^{k} \\ x_{id}^{k-1} \end{bmatrix} + \begin{bmatrix} \varphi^{k+1} \\ 0 \end{bmatrix} p \tag{6}$$

Where acceleration factor $\varphi^{k+1} = \varphi_1^{k+1} + \varphi_2^{k+1}$, $\varphi_i^{k+1} = c_i \times rand_i$, and input vector is $p = (\varphi_1^{k+1}\cdot p_{id}^k + \varphi_2^{k+1}\cdot p_{gd}^k)/\varphi^{k+1}$. The transfer matrix of Eq.(6) is:

$$T(k,k-2) = \begin{bmatrix} 1+\eta\cdot w-\eta\cdot\varphi^k & -\eta\cdot w \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1+\eta\cdot w-\eta\cdot\varphi^{k+1} & -\eta\cdot w \\ 1 & 0 \end{bmatrix}$$

From Lemma 1, if the condition $\rho(T(k,k-2)) = \max|\lambda_i| < 1$ is satisfied, the system described by Eq(6) is asymptotic stable.

$$|\lambda E - T(k,k-2)| = \begin{vmatrix} \lambda - L^k L^{k+1} + \eta\cdot w & \eta\cdot w\cdot L^k \\ -L^k & \lambda+\eta\cdot w \end{vmatrix} \tag{7}$$

Where $L^k = (1+\eta\cdot w-\eta\cdot\varphi^k)$. The characteristic equation of Eq.(7) is described as:

$$D(\lambda) = \lambda^2 + \lambda\cdot[2\eta w - L^k(1+\eta w) + L^k\cdot\eta\cdot\varphi^{k+1}] + (\eta w)^2 \tag{8}$$

Based on Jury criterion, the sufficient condition of $\rho(\cdot) < 1$ is:

$$\begin{cases} (\eta w)^2 < 1 \\ \left|2\eta w - L^k(1+\eta w) + L^k\cdot\eta\cdot\varphi^{k+1}\right| < 1+(\eta w)^2 \end{cases} \tag{9}$$

By calculation, $w \in (-1/\eta, 1/\eta)$ satisfying Eq.(9), moreover:

$$\frac{-\eta\varphi^k(1+\eta w)}{1+\eta w-\eta\varphi^k} \leq \eta\cdot\varphi^{k+1} \leq \frac{2(1+(\eta w)^2)-\eta\varphi^k(1+\eta w)}{1+\eta w-\eta\varphi^k}$$

$\square$

At time *k*+1, the acceleration factor $\eta\cdot\varphi^{k+1}$ and inertia weight w should meet the Eq.(4), Eq.(5) to ensure asymptotic stability. If the range of $\eta\cdot\varphi^{k+1}$ satisfys $\eta\cdot\varphi^{k+1} \in \{U \mid [\varphi_{\min}^{k+1}, \varphi_{\max}^{k+1}]\}$, particle systems are asymptotic stable; Otherwise, swarm system will disperse.

3. BALLOON PSO AND ITS GLOBAL CONVERGENCE PROOF

3.1 A kind of adaptive PSO —— Balloon Particle Swarm Optimizer, BPSO

Same as GA and other evolutionary computing techniques, PSO is facing a dilemma between rapid searching and premature convergence. According to the analysis result of theorem 1, a kind of adaptive PSO (BPSO) is proposed in this paper. At every epoch, according to the current value of acceleration factor $\eta\cdot\varphi^k$, $\eta\cdot\varphi^{k+1}$ can be adjusted dynamically, so as to control the convergence or disperse of the swarm. The swarm, which is looked as a balloon expands and shrinks continuously, can search the resolution space repeatedly.

The logical flow of BPSO is:
(1). Create and initialize a PSO swarm.
(2). Evaluate each particle in the swarm
(3). If the available time has expired, or reach the termination, return the best solution, if not, go to (4)
(4). If the consecutive failure times exceed $CF_{\max}$ or $DV_{swarm} < DV_{\min}$, update particles based on Eq.(4), Eq.(5), to expand the swarm, else if $DV_{swarm} > DV_{\max}$, shrink the swarm
(5) Update particles by Eq.(4), Eq. (5), to shrink the swarm. Go to (2)

In the above flow, the term $CF_{\max}$ is the upper bound of consecutive failure times, where failure means the current fitness is worse than swarm best fitness before. $DV_{swarm}= \sum Deviation(Swarm)$ is defined as the sum of swarm variance, and $DV_{\max}$ denotes the variance of searching space.

3.2 Global convergence proof of BPSO

Solis and Wets (Solis and Wets, 1981) provide some conditions and results for global convergence of random search algorithms as follows.

(***H1***) $f(D(x,\xi)) \le f(x)$ and if $\xi \in S^n$, $f(D(x,\xi)) \le f(\xi)$.

Where ξ^k is generated from sample space (R^n, B, μ_k) ; $x^{k+1} = D(x^k, \xi^k)$, $D: S^n \times R^n \to S^n$; S^n is a subset of R^n; B is the σ -algebra of subset of R^n; μ_k is the probability measure.

(***H2***) For any (Borel) subset A of S^n with $\upsilon(A) > 0$, there exists $\prod_{k=0}^{\infty}[1-\mu_k(A)] = 0$, where υ is a nonnegative measure defined on B , generally is Lebesgure measure.

Lemma 2 (Global Search): Suppose that f is a measurable function, S^n is a measurable subset of R^n, (***H1***) and (***H2***) are satisfied. Let $\{x^k\}_{k=0}^{\infty}$ be a sequence generated by the algorithm. Then

$$\lim_{k\to\infty} P[x^k \in R_{\varepsilon,M}] = 1$$

where $P[x^k \in R_{\varepsilon,M}]$ is the probability that at step k, the point x^k generated by the algorithm in the optimality region. □

It has been proved that PSO doesn't satisfy (***H2***) and Lemma 2 (Bergh, 2002). In this paper, convergence properties of BPSO is studied. Without Lipschitz constrain condition, Eq.(1), and Eq.(2) of standard PSO can be represented as Eq.(10) and Eq.(11)

$$v_{id}^{k+1} = w \cdot v_{id}^k + \varphi^{k+1} \cdot (p - x_{id}^{k+1}) \tag{10}$$

$$x_{id}^{k+1} = x_{id}^k + \eta \cdot v_{id}^{k+1} \tag{11}$$

where p is a hypercube whose vertex is p_{gd}^k and p_{id}^k.

From the above, we have theorem 2.

Theorem 2: BPSO is a global search algorithm.

Proof: D function of BPSO is defined below (Bergh, 2002):

$$D(p_{gd}^k, x_{id}^k) = \begin{cases} p_{gd}^k & f(p_{gd}^k) \le f(x_{id}^k) \\ x_{id}^k & f(p_{gd}^k) > f(x_{id}^k) \end{cases} \tag{12}$$

It is clear that Eq.(12) satisfys (***H1***). At time k, the support M_i^k of the i th particle is defined as:

$$M_i^{k+1} = x_{id}^k + \eta \cdot w \cdot v_{id}^k + \eta \cdot \varphi^{k+1} \cdot (p - x_{id}^k)$$

where M_i^{k+1} is a hyper sphere whose center is x_{id}^k and radius is $\rho_i^{k+1} = \eta \cdot w \cdot v_{id}^k + \eta \cdot \varphi^k \cdot (p - x_{id}^k)$. To BPSO, Searching is a process of "Shrink-Expand-Shrink" course. Moreover, according to Lemma 1, there is no restriction of the φ^{k+1} value, especially, if the swarm is a expand status. So the range of ρ_i^{k+1} has no limitation in theory. If only the following condition is met:

$$\eta \cdot \varphi^{k+1} \ge \frac{\left\| S_{\max}^n, S_{\min}^n \right\|_2 - \eta \cdot w \cdot v_{id}^k}{(p - x_{id}^k)} \tag{13}$$

Then $\rho_i^k \ge \left\| S_{\max}^n, S_{\min}^n \right\|_2$, where $S_{\max}^n$, $S_{\min}^n$ are upper and lower bound of searching space S^n separately. If $\eta \cdot \varphi^{k+1}$ meets Eq.(13), then $S^n \subset M_i^k$, furthermore $S^n \subset \bigcup_{i=1}^{s} M_i^k$, $\upsilon(\bigcup_{i=1}^{s} M_i^k \cap S) = \upsilon(S)$. So $\forall A \subset S^n$, $\prod_{k=0}^{\infty}[1-\mu_k(A)] = 0$ and (***H2***) are satisfied,

According to Lemma 2, BPSO is a global search algorithm. □

4. EXPERIMENT RESULTS

Vehicle running on various kinds of road at different speeds, there are some vibratory disturbance in the vehicle, caused by the road surface roughness. Modelling the vehicle vibratory response is important to the future work for researching the characteristic of road irregularity motivation.

The time series data $\{x_i(n)\}$ analyzed in this section is collected from a servo platform and the sampling time is 66ms, The PSD $P_x(\omega)$ of the disturbance data vector is estimated by the Yule-Walker AR method, which assumes the data is output of a linear system driven by white noise, whose variance is σ_ω^2. The method estimates the PSD by first choosing the AR order, which is determined via single value decomposition in this paper and is 12; secondly estimating the parameters of the linear system that hypothetically generates the signal; finally the PSD can be computed as follows (Zhang,X.-D, 1995):

$$P_x(\omega) = \frac{\sigma_\omega^2}{\left|A(e^{j\omega})\right|^2} \tag{14}$$

The Yule-Walker equations can be presented as below:

$$\begin{bmatrix} R_x(0) & R_x(1) & \dots & R_x(p) \\ R_x(1) & R_x(0) & \dots & R_x(p-1) \\ \dots & \dots & \dots & \dots \\ R_x(p) & R_x(p-1) & \dots & R_x(0) \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -a_{p1} \\ \dots \\ -a_{pp} \end{bmatrix} = \begin{bmatrix} \sigma_\omega^2 \\ 0 \\ \dots \\ 0 \end{bmatrix} \tag{15}$$

$R_x(\tau) = E\{x(n)x(n+\tau)\}$ is the autocorrelation coefficients. It is a hard work to calculate the inverse matrix to obtain the parameters a_k and σ_ω^2. The Levinson-Durbin algorithm is in general use and can work efficiently. Here, standard PSO, SAPSO (Feng Pan, 2005), GA and BPSO are used to solve the Yule-Walker equations. The parameter values is given in Tables 1

Table 1. Parameters Setting

Variable	Value
Swarm Size	20
Inertia Weight	0.79
Acceleration Factors	1.4
Scaled factor	1
Initial Value	[-1 1]
Error Goal	10^{-3}
$DV_{\max}$	1
$DV_{\min}$	10^{-4}

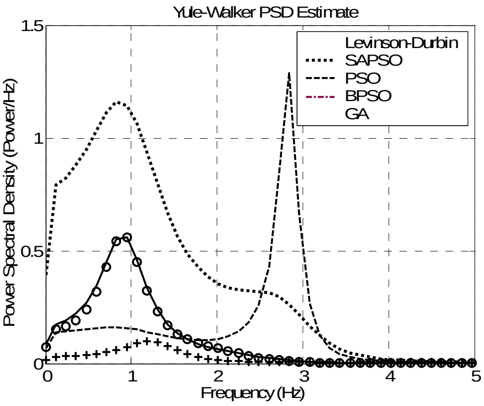

Fig1 PSD curve comparison

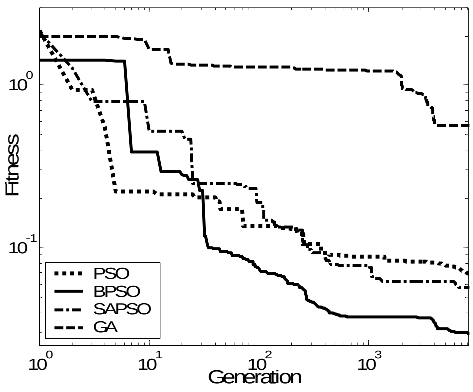

Fig2 Comparison of Error Curve

The PSD curves are plotted in Fig1 (plotting the PSD in units of power per units of frequency). The circle corresponding to the BPSO almost completely coincides with the solid line solved by Levinson-Durbin method, and is much more accurate than the curves of other three methods. The error curve in comparison of the algorithms is shown in Fig2.It is obviously that the convergence speeds of PSO and SAPSO are faster and the searching ability is better than that of BPSO at the beginning of optimization. The reason is that BPSO will repeat the "expand-shrink" process, but not shrink only. The searching ability of standard PSO will decrease if there is no new information introduced, however, BPSO can guarantee a harmonious relation of swarm diversity. The swarm of BPSO expands until overlap the whole searching space, after that, the swarm shrink to a lower limit. This process will repeat till meet the terminate condition. The modified update rules of BPSO promote the swarm searching ability.

5、DISCCUSION

The parameters and update rules of standard PSO have made it easy to fall into stagnate, unless the swarm can provide new information incessantly. In this paper, the stability conditions of PSO parameters are explored and BPSO is proposed which can adaptively adjust parameters and is proved its global convergence. Experiment demonstrates the validity of BPSO.

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