GENERATION MARKETS EQUILIBRIUM ANALYSIS USING RESIDUAL DEMANDS

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Abstract: In this paper, Nash equilibriums of generation markets are investigated using a game theory application for simplified competitive electricity markets. The characteristic of equilibrium states in N-company spot markets modelled by uniform pricing auctions is analyzed and a new method for obtaining Nash equilibriums of the auctions is proposed. Spot markets are assumed to be operated as uniform pricing auctions and generation companies are assumed to submit their bids into the auctions in the form of a seal-bid. The uniform pricing auctions in this analysis are formulated as non-cooperative and static games in which generation companies correspond to the players of the game. The coefficient of the bidding function of company-*n* is assigned to the strategy of the player-*n* (company-*n*) and the payoff of player-*n* is defined as its profit from the uniform price auction. Based on the concept of residual demand, best response functions of each generation company in the N-company auctions are analytically derived. Finally, an efficient way to obtain all the possible equilibrium set pairs and to examine their feasibilities as Nash equilibriums are suggested. A simple numerical example with three generation companies is demonstrated to show the basic ideas of the proposed method.

Keywords: Nash games, Generation markets, Uniform price auctions, Equilibrium feasibility, Best response functions.

1. INTRODUCTION

The electric power industry is changing into a competitive marketplace for power transactions, and profitability is a primary concern to every market player (Schwarz, 2000; Nishimura, *et al*., 1993). Basically, the power transaction revenues of a generation firm are from bilateral contracts or the spot markets, which are usually formed as a sealed bid auction with uniform market prices (Singh, *et al.*, 1997; Rahmi, *et al*., 1999). In this environment, the optimal bidding strategy, to maximize the profit from the spot market, is indispensable to the generation firms.

In recent years, considerable amounts of research have been done in decision-making in competitive power markets, which are focused on the optimal bidding strategies for the competitive generation firms (Sheble, 1996; Ferrero, *et al*. 1997; Park, *at al*., 2001; Rudkevich, et al., 1998; Kim, *et al*., 2002). Privatized generation firms can decide their bidding strategies based on various theoretical or empirical studies, e.g., forecasts of market prices, evaluation of competitors' bidding behavior, and game theory applications (IEEE, 1999). Price based operations in an auction market structure are analyzed by Singh, *et al.*, (1997). Using the matrix game, where bidding strategy is represented with discrete quantities, decision-making processes in deregulated power systems are simulated (Rahmi, *et al*., 1999).

In this paper, the characteristics of equilibrium states in N-company spot markets modelled by uniform pricing auctions are analyzed and a new analytical method to obtain Nash equilibriums of the auction using residual demands is proposed. Generation companies are considered as the players of the spot market. Depending on the bids of the generation companies, market demands are allocated to each generation company.

The uniform pricing auctions in this analysis is formulated as non-cooperative and static games in which generation companies compete with each other as the players of the game. The coefficient of the bidding function of company-*n* is the strategy of the player-*n* (company-*n*) and the payoff of player-*n* is defined as its profit from the uniform price auction. The solution of this game can be obtained using the concept of the non-cooperative equilibrium based on the Nash idea. Based on the residual demand curve, the best response function of each generation company in the uniform pricing auction with N companies is derived, analytically.

In this paper, efficient ways to obtain all the possible equilibrium set pairs and to examine their feasibilities as equilibriums in N-player generation markets are proposed. A simple numerical example with three companies is demonstrated to show the basic idea of the proposed method. From this, the applicability of the proposed method to the real-world problem can be seen, even though it needs a further future analysis.

2. SIMPLIFIED MARKETS MODEL

2.1 Markets Model

It is assumed that spot markets are operated as uniform price auctions and that each generation company submits its bids into the auction in the form of a seal-bid.

The generation quantity allocated to and the maximum generation limit of generation company-*n* is denoted by q_n and $\bar{q}_n$, respectively. Generation costs and marginal costs of company-*n* are denoted by $C_n(q_n)$ and $C_n'(q_n)$, respectively. The bidding function of generation company-*n* is denoted by $B_n(q_n)$ and the profit of generation company-*n* is represented by Π_n . The total generation quantity covering all the generation companies, total system demand, and market clearing price are signified by q , d , and p , respectively. It can seen that $q = q_1 + \cdots + q_N$ where N is the number of generation companies and $\Pi_n = p \cdot q_n - C_n(q_n)$.

Basic assumptions are made on the market setups and some functions defined above. First, the number of generation companies in markets is *N*. Second, system demand is defined by an affine function such that it has an inverse given by $p = d^{-1}(q) = \alpha - \omega q$, where $\alpha > 0$ and $\omega \geq 0$. Third, $C_n(q_n)$ and $C_n'(q_n)$, $n \in \Omega,\ \Omega \triangleq \{1, 2, \cdots, N\}$, are defined by a quadratic and a linear function as $C_n(q_n) = 0.5 a_n (q_n)^2$ and $C_n'(q_n) = a_n q_n$, respectively. And, the bidding function of generation company-*n* is a linear form by $B_n(q_n) \triangleq b_n q_n,\ b_n > 0$.

2.2 Game Formulation.

As spot markets are modeled by uniform pricing auctions and the auction results are based on the interactions among companies in the spot markets, the auctions in this analysis can be formulated as a non-cooperative and static game in which generation companies correspond to players of the game. The coefficient of the bidding function of company-*n*, b_n , is the strategy of the player-*n* (company-*n*) and the payoff of player-*n* is defined as its profit from the uniform price auction. The solution of this game can be obtained using the concept of the non-cooperative equilibrium based on the Nash idea, and therefore, the solution strategies can be calculated as follows:

$$b_n^{Nash} = \underset{b_n}{argmax}\left(\Pi_n\right) \text{ for a given } b_m^{Nash}$$
$$\text{where } n \neq m \text{ and } n, m \in \Omega \qquad (1)$$

3. ANALYSIS OF NASH EQUILIBRIUMS

3.1 Classification of Companies

In equilibriums, companies can be classified into two distinct sets. Two equilibrium sets of companies are defined according to the generation quantity allocated to the individual companies as follows:

$$\begin{aligned} \Omega^U &= \{n \in \Omega \mid q_n < \bar{q}_n\} : \text{Set of unconstrained companies} \\ \Omega^C &= \{n \in \Omega \mid q_n = \bar{q}_n\} : \text{Set of constrained companies} \end{aligned} \qquad (2)$$

This classification of companies covers all the companies in the market and the classified sets are disjoint such that $\Omega = \Omega^U \cup \Omega^C$ and $\Omega^U \cap \Omega^C = \phi$. An explicit expression for the generation quantity allocated to each company can be obtained, depending on the set in which the company is included at equilibriums as follows:

$$q_n = p / b_n,\ n \in \Omega^U \text{ or } q_n = \bar{q}_n,\ n \in \Omega^C \qquad (3)$$

At given market demands, companies can be classified into two equilibrium states (i.e., set of unconstrained and constrained companies) according to the slope of their bidding function, b_n as follows:

$$\begin{cases} n \in \Omega^U, & b_n > \dfrac{p}{\bar{q}_n} = \dfrac{\alpha - \omega q}{\bar{q}_n} \\ n \in \Omega^C, & b_n \leq \dfrac{p}{\bar{q}_n} = \dfrac{\alpha - \omega q}{\bar{q}_n} \end{cases} \qquad (4)$$

where $q = \sum_{n \in \Omega} q_n = \sum_{n \in \Omega^U} p / b_n + \sum_{n \in \Omega^C} \bar{q}_n$.

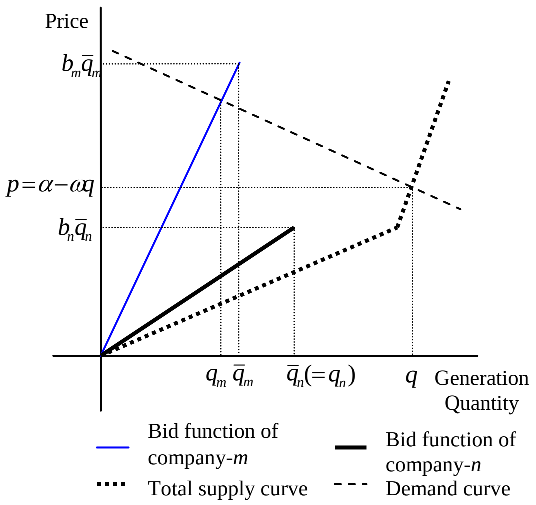

Fig. 1. Classification of companies $\left(n\in\Omega^C \text{ and } m\in\Omega^U\right)$

Figure 1 shows an example of equilibrium, in which company-*n* and company-*m* is in the set of constrained and unconstrained companies, respectively.

3.2 Best Response of Individual Companies Using Residual Demands

Based on the residual demand curve, the best response function of each generation company in the uniform pricing auction with N-company is obtained, analytically, in this paper. Suppose that an equilibrium set pair, $\left(\Omega^U,\Omega^C\right)$, is given.

First, the best response of company-*n*, $n\in\Omega^U$, can be determined as follows. In the inverse demand curve, ω can be interpreted as the parameter that determines how much demand's willingness-to-pay decreases when demand increase by one unit. In the same way, the strategy b_n, can be interpreted as the parameter that represents how much company-*n*'s willingness-to-earn increases when supply increases by one unit. However, it is more convenient to consider the reciprocal of these parameters since both consumers and suppliers respond to the price. That is, $1/\omega$ is the decreasing amount of demand when price increases by one unit and $1/b_n$ is the increasing amount of company-*n*'s supply when price increases by one unit.

Let Ω^U_{-n} denote equilibrium set of unconstrained companies except company-*n*, i.e., $\Omega^U_{-n}\triangleq\Omega^U-\{n\}$. Given strategies of other companies, b_o, $o\in\Omega^U_{-n}$, the market clearing price can be obtained as follows:

$$p=\alpha-\omega\sum_{o\in\Omega}q_o=\alpha-\omega\sum_{o\in\Omega^C}\bar{q}_o-\omega\sum_{o\in\Omega^U_{-n}}\left(\frac{p}{b_o}\right)-\omega q_n \qquad (5)$$

From the equation above, the residual demand of company-*n*, that is, the amount of demand which company-*n* is facing, can be obtained as follows:

$$p=\left(\alpha-\omega\sum_{o\in\Omega^C}\bar{q}_o\right)\cdot\left(1+\omega\sum_{o\in\Omega^U_{-n}}\frac{1}{b_o}\right)^{-1}-\left(1+\omega\sum_{o\in\Omega^U_{-n}}\frac{1}{b_o}\right)^{-1}\omega q_n$$
$$=\alpha_n^{res}-\omega_n^{res}q_n \qquad (6)$$

where $\alpha_n^{res}=\left(\alpha-\omega\sum_{o\in\Omega^C}\bar{q}_o\right)\cdot\left(1+\omega\sum_{o\in\Omega^U_{-n}}\frac{1}{b_o}\right)^{-1}$ and $\omega_n^{res}=\left(\frac{1}{\omega}+\sum_{o\in\Omega^U_{-n}}\frac{1}{b_o}\right)^{-1}$.

The equation above can be interpreted as the same way as in the original demand function. That is, $1/\omega_n^{res}$ is the amount by which demand decreases when price increases by one unit. It can be seen that this amount is related to the parameter in the original demand curve and strategies of generation companies.

From the interpretation of the demand curve above, it can be seen that if price increases by one unit, then original demand decreases by $1/\omega$ and supply of each company-*o* ($o\in\Omega^U_{-n}$) is increased by $1/b_o$.

Therefore, the residual demand decreases by the sum of the two amounts when price increases by one unit. This implies that $1/\omega_n^{res}$ can be represented as the sum of these two factors such as $\frac{1}{\omega_n^{res}}=\frac{1}{\omega}+\sum_{o\in\Omega^U_{-n}}\frac{1}{b_o}$.
For this residual demand, company-*n* can be regarded as a monopolist and its best response can be determined as the strategy of a monopolistic company as follows [Appendix]:

$$b_n^{BR}=\gamma_n+\left(\frac{1}{\omega}+\sum_{o\in\Omega^U_{-n}}\frac{1}{b_o}\right)^{-1} \qquad (7)$$

Next, the best response of company-*m*, $m\in\Omega^C$, which is in the equilibrium set of constrained companies, can be obtained as follows. From Figure 1, it can be seen that, for company-*m* in Ω^C, the best response at the equilibrium can be any value less than $p/\bar{q}_m$, where p is the market clearing price at the equilibrium. Therefore, from (5), market clearing price and the best response of company-*m* can be obtained as follows:

$$p=\left(\alpha-\omega\sum_{o\in\Omega^C}\bar{q}_o\right)\left(1+\omega\sum_{o\in\Omega^U}\frac{1}{b_o}\right)^{-1} \qquad (8)$$

$$b_m^{BR}=\left\{b_m \,\middle|\, b_m<\frac{p}{\bar{q}_m}=\frac{1}{\bar{q}_m}\left(\alpha-\omega\sum_{o\in\Omega^C}\bar{q}_o\right)\left(1+\omega\sum_{o\in\Omega^U}\frac{1}{b_o}\right)^{-1}\right\} \qquad (9)$$

3.3 Obtaining Candidates of Nash Equilibrium and Their Feasibilities Check

In the previous section, best responses of individual companies at a given equilibrium set pair $\left(\Omega^U,\Omega^C\right)$ are derived. In this section, a practical way to obtain all the possible equilibrium set pairs and to examine their feasibilities as Nash equilibriums is given. It is shown that how a feasible space for equilibrium set pairs is defined and the feasibilities of the individual equilibrium set pairs in the feasible space are checked as Nash equilibriums.

It can be seen that, at any given equilibrium set pair, a condition is needed to guarantee the existence of an equilibrium set of constrained companies at that pair. In the uniform pricing auction model described above, the lowest equilibrium market clearing price, $\underline{p}$, can be obtained when all companies are in the equilibrium set of constrained companies, while the highest equilibrium market clearing price, $\overline{p}$, can be obtained when all companies are in the equilibrium set of unconstrained companies.

However, it should be noted that company-*n* cannot be in the equilibrium set of constrained companies at any given equilibrium set pair if market clearing price at that equilibrium, p, is less than $\gamma_n \overline{q}_n$. That is, for an example, suppose that company-*n* is in the equilibrium set of constrained companies and market clearing price p is less than $\gamma_n \overline{q}_n$. Then, this assumption gives a relationship as $b_n \overline{q}_n \le p < \gamma_n \overline{q}_n$ and this inequality implies that the marginal revenue is less than the marginal cost of company-*n*. Therefore, company-*n* loses its money by selling its marginal generation unit.

In this case, the company will try to decrease its generation and it can only be done by choosing b_n large enough to transfer its state from set of constrained companies to unconstrained companies. Therefore, the best response of company-*n* cannot exist in the equilibrium set of constrained companies and this implies that any Nash equilibrium cannot be reached with company-*n*'s constrained equilibrium state. Therefore, the following condition is needed to guarantee the existence of an equilibrium set of constrained companies at a given equilibrium set pair:

$$p \ge \gamma_n \overline{q}_n, \text{ where } n \in \Omega^C \quad (10)$$

Let Ω^R denote a set of companies that can remain in the equilibrium set of constrained companies in a Nash equilibrium, in the sense of marginal profit (i.e., p is greater than $\gamma_n \overline{q}_n$). Then, a set of candidates of equilibrium set pairs, Λ, can be obtained and this set implies a feasible space for equilibrium set pairs for the problem. For each equilibrium set pair in the set, Λ, it will be checked whether it satisfies the condition in (10) and equilibrium set pairs satisfying (10) can be confirmed as Nash equilibriums after another examination which is presented in the following section. The number of elements of Λ, that is, the total number of candidates of equilibrium set pair, is the same as the number of subsets of Ω^R (i.e., $2^{|\Omega^R|}$, where $|\Omega^R|$ denotes to the number of elements of the set Ω^R).

The possibility of a candidate equilibrium set pair as Nash equilibrium can be checked using the condition described in (10). That is, for a given equilibrium set pair (Ω^U, Ω^C), it can be checked there is a possibility that this equilibrium set pair will be Nash equilibrium, by comparing $\max_{k \in \Omega^C}(\gamma_k \overline{q}_k)$ and the corresponding equilibrium market clearing price. Therefore, if market clearing price at the given equilibrium set pair is less than $\max_{k \in \Omega^C}(\gamma_k \overline{q}_k)$, the equilibrium set pair cannot result in Nash equilibrium and this pair will not considered in further examinations any longer. The procedure to obtain a set of feasible equilibrium set pairs and to check its possibility of being Nash equilibrium is summarized as follows:

1) Without loss of generality, it can be assumed that companies are ordered by the increasing order of the parameter $\gamma_n \overline{q}_n$, $n \in \Omega$, that is, $\gamma_n \overline{q}_n < \gamma_{n+1} \overline{q}_{n+1}$, since otherwise companies can be ordered.
2) Obtain the maximum market clearing price $\overline{p}$.
3) Find an index R such as $\gamma_R \overline{q}_R \le \overline{p} < \gamma_{R+1} \overline{q}_{R+1}$.
4) Obtain a set of companies, $\Omega^R = \{1, \cdots, R\}$.
5) Obtain a set of candidates of equilibrium set pairs, $\Lambda = \left\{ (\Omega^U, \Omega^C) \,|\, \Omega^U, \Omega^C \in 2^{\Omega^R}, \Omega^U \cap \Omega^C = \emptyset, \Omega^U \cup \Omega^C = \Omega \right\}$.
6) Check the feasibility of candidate equilibrium set pairs, by comparing market clearing price and $\max_{k \in \Omega^C}(\gamma_k \overline{q}_k)$ at the equilibrium set pairs.

4. NUMERICAL EXAMPLE

4.1 Market Data

In the numerical example, three generation companies are considered and the inverse demand function is given as $p = 25 - 0.01q$, where q is the total supply such as $q = q_1 + q_2 + q_3$. The parameter for each company is illustrated in Table 1.

4.2 Solution Procedure

Step 1; The given company index have been ordered already.

Step 2; Λ is obtained as follows: $\Lambda = \{\lambda_1, \cdots, \lambda_8\}$, where $\lambda_i = \{\Omega^U, \Omega^C\}$ is a candidate equilibrium set pair such as $\lambda_1 = \{\phi, \{1,2,3\}\}$, $\lambda_2 = \{\{1\}, \{2,3\}\}$, $\lambda_3 = \{\{2\}, \{1,3\}\}$, $\lambda_4 = \{\{3\}, \{1,2\}\}$, $\lambda_5 = \{\{1,2\}, \{3\}\}$, $\lambda_6 = \{\{1,3\}, \{2\}\}$, $\lambda_7 = \{\{2,3\}, \{1\}\}$, and $\lambda_8 = \{\{1,2,3\}, \phi\}$.

Table 1 Cost data of generation companies

Company	1	2	3
Cost coefficient γ	0.0219	0.0173	0.0111
Maximum Generation	400	600	1000

Step 3; Nash conditions for each candidate Nash equilibrium set pair are examined as follows:

1) $\lambda_1 = \{\phi, \{1,2,3\}\}$

Since every company is in the constrained state, the corresponding generation is its maximum generation. Therefore, total generation is the sum of maximum generations and the corresponding value is 2000. By the demand function, the market clearing price can be obtained as follows:

$$p = 25 - 0.01 \times 2000 = 5$$

The maximum marginal cost at the maximum generation level in Ω^C is acquired as company-3's marginal cost at the maximum generation level, 11.057. Based on (9), this candidate cannot be Nash, because the company-3's marginal cost at the maximum generation level is greater than the market clearing price *p*.

2) $\lambda_2 = \{\{1\}, \{2,3\}\}$

Company-1's strategy is determined as follows:

$$b_1 = \gamma_1 + \left(\frac{1}{\omega} + \sum_{o \in \Omega_{-1}^U} \frac{1}{b_o} \right)^{-1} = 0.0219 + 0.01 = 0.0319$$

The corresponding market clearing price is obtained as follows:

$$p = \left(\alpha - \omega \sum_{o \in \Omega^C} \bar{q}_o \right) \cdot \left(1 + \omega \sum_{o \in \Omega^U} \frac{1}{b_o} \right)^{-1} = 6.5549$$

The maximum marginal cost in Ω^C is 11.057 and this is greater than the price above. So, this cannot be Nash equilibrium because it does not meet (9).

3) Other candidate equilibrium set pairs except for λ_8 cannot result in Nash equilibrium either by similar analysis.

4) $\lambda_8 = \{\{1,2,3\}, \phi\}$

Each company's strategy is determined as follows:

$$b_1 = \gamma_1 + \left(\frac{1}{\omega} + \sum_{o \in \Omega_{-1}^U} \frac{1}{b_o} \right)^{-1} = 0.026809 ,$$

$$b_2 = \gamma_2 + \left(\frac{1}{\omega} + \sum_{o \in \Omega_{-2}^U} \frac{1}{b_o} \right)^{-1} = 0.022339$$

$$b_3 = \gamma_3 + \left(\frac{1}{\omega} + \sum_{o \in \Omega_{-3}^U} \frac{1}{b_o} \right)^{-1} = 0.01655$$

The corresponding market clearing price is obtained as follows:

$$p = \left(\alpha - \omega \sum_{o \in \Omega^C} \bar{q}_o \right) \cdot \left(1 + \omega \sum_{o \in \Omega^U} \frac{1}{b_o} \right)^{-1} = 10.31$$

There is no company in Ω^C, so this equilibrium meets the condition (9). Generation quantity which is allocated each generation company can be obtained from (3) as follows:

$$(q_1, q_2, q_3) = (384.5723,\ 461.5247,\ 622.9607)$$

Since the price and each company's generation are given, each company's payoff can be determined. The results are $\Pi_1 = 2342.8$, $\Pi_2 = 2917.6$, and $\Pi_3 = 4277.0$.

Step 4; Now the deviation of each company can be examined. For company-1, the reasonable deviation is to decrease its slope of bid curve and to be included in Ω^C. In this case, the deviated price is 10.2345, the deviated quantity is its maximum quantity 400 and the corresponding deviated payoff is 2339, which is less than the original payoff, 2342.8. For company-2, the reasonable deviation is to decrease its slope of bid curve and to be included in Ω^C. In this case, the deviated price is 9.6092, the deviated quantity is its maximum generation 600 and the corresponding deviated payoff is 2645.8, which is less than the original payoff, 2917.6. Company-3's reasonable deviation is also to decrease its slope of bid curve and to be included in Ω^C. In this case, the deviated price is 8.2388, the deviated quantity is its maximum generation 1000 and the corresponding deviated payoff is 2710.3, which is less than the original payoff, 4277. Since this candidate satisfy all the Nash conditions, this will result in Nash equilibrium and the following is the quantities of this Nash equilibrium:

$$\left(b_1^{Nash}, b_2^{Nash}, b_3^{Nash}\right) = (0.0268, 0.0223, 0.0165),$$

$$\left(q_1^{Nash}, q_2^{Nash}, q_3^{Nash}\right) = (384.5723,\ 461.5247,\ 622.9607)$$

Since every possible candidate for Nash equilibrium is explored, this Nash equilibrium is the unique Nash equilibrium.

4.3 Results

From the analysis above, the unique Nash equilibrium can be obtained. The corresponding Nash quantities for this numerical example are illustrated in Table 2.

Table 2 Nash Quantities

Company	1	2	3
b^{Nash}	0.0268	0.022339	0.01655
q^{Nash}	384.5723	461.5247	622.9607
Π^{Nash}	2342.8000	2917.6000	4277.0000
p^{Nash}		10.31	

5. CONCLUSION

In this paper, the characteristics of equilibrium states in N-company spot markets modeled by uniform pricing auctions are analyzed and new methods for obtaining Nash equilibriums of the auction are proposed. Based on the residual demand curve, an efficient way to obtain all the possible equilibrium set pairs and to examine their feasibilities as Nash equilibriums is suggested. A simple numerical example with three generation companies is demonstrated to show the basic idea of the proposed theory. As this paper is focused on the analytical study of equilibrium of N-company generation markets, the applicability of the proposed method to the real-world problem can be seen, theoretically.

However, in order to apply the proposed method to the real-world problem, further investigations on the realistic constraints in electricity markets are needed. For examples, quadratic cost functions with linear and constant terms should be considered. The solution procedure should also be discussed when there are many generation companies. They are limitations of this paper and can be studied more in the future study.

APPENDIX

Let a subscript *mc* denote a monopolistic company. If only one company is in the market, the following equality is satisfied since supply and demand must be balanced:

$$b_{mc} q_{mc} = \alpha - \omega q_{mc} \quad \text{(A-1)}$$

Now the monopolist's bidding strategy will be determined by maximizing his profit $\Pi_{mc}(b_{mc})$ as follows:

$$\max_{b_{mc}} \Pi_{mc}(b_{mc}) = \max_{b_{mc}} \left\{ b_{mc}(q_{mc})^2 - \frac{1}{2}\gamma_{mc}(q_{mc})^2 \right\}$$
$$= \max_{b_{mc}} \left(\frac{\alpha}{b_{mc}+\omega} \right)^2 \left(b_{mc} - \frac{1}{2}\gamma_{mc} \right) \quad \text{(A-2)}$$

By the necessary condition for the optimality, the following equality must be satisfied at the optimal b_{mc}^*:

$$\frac{d\Pi_{mc}}{db_{mc}}\left(b_{mc}^*\right) = \alpha^2 \frac{\omega + \gamma_{mc} - b_{mc}^*}{\left(b_{mc}^* + \omega\right)^3} = 0 \quad \text{(A-3)}$$

Since $b_{mc}^* + \omega > 0$, the monopolistic company's optimal decision for his bidding is determined as $b_{mc}^* = \omega + \gamma_{mc}$.

Next, a market with perfect competition can be considered. Let a superscript pc denote this perfect competition situation. In this case, since every company can be regarded as a price-taker, the market clearing price under perfect competition, denoted by $\bar{p}$, is can be assumed as constant. Therefore, from the optimality condition on the profit of company-*n*, the optimal bidding strategy of company-*n* in a perfectly competitive market, b_n^{pc*}, can be obtained as follows:

$$\frac{d\Pi_n^{pc}}{dq_n^{pc}} = \frac{d}{dq_n^{pc}}\left(\bar{p}\cdot q_n^{pc} - \frac{1}{2}\gamma_n (q_n^{pc})^2 \right) = \bar{p} - \gamma_n q_n^{pc} = 0;$$
$$b_n^{pc*} = \frac{\bar{p}}{q_n^{pc}} = \frac{\gamma_n q_n^{pc}}{q_n^{pc}} = \gamma_n \quad \text{(A-4)}$$

REFERENCES

A. Rudkevich, M. Duckworth and R. Rosen (1998). Modeling electricity pricing in a deregulated generation industry: the potential for oligopoly pricing in a poolco. *Energy Journal*, **Vol. 19, No. 3**, pp. 19-48.

F. A. Rahimi and A. Vojdani (1999). Meet the emerging transmission market segment. *IEEE Computer Applications in Power*, **Vol.12, No.1,** pp. 26-32.

F. Nishimura, R. D. Tabors, M. D. Ilic, and J. R. Lacalle-Melero (1993). Benefit Optimization of Centralized and Decentralized Power Systems in a Multi-Utility Environment. *IEEE Trans. on Power Systems*, **Vol. 8, No. 3**, pp. 1180-1186.

G. B. Sheble (1996). Price based operation in an auction market structure. *IEEE Trans. on Power Systems*, **Vol.11, No.4**, pp. 1770-1777.

H. Singh, S. Hao, and A. Papalexopoulos (1997). Power auctions and network constraints. *Proceedings of the Thirtieth Hawaii International Conference on System Sciences*, **Vol. 5**, pp. 608-614.

IEEE (1999). IEEE Tutorial on Game Theory Applications in Electric Power Markets.

Jong-Bae Park, Balho H. Kim, Jin-Ho Kim, Manho Joung, and Jong-Keun Park (2001). A Continuous Strategy Game for Power Transactions Analysis in Competitive Electricity Markets. *IEEE Trans. on Power Systems*, **Vol. 16, No. 4**, pp. 847-855.

Jin-Ho Kim, Jong-Bae Park, Jong-Keun Park, and Balho H. Kim (2002). A New Approach to Maintenance Scheduling Problems Based on Dynamic Game Theory. *KIEE International Transactions on Power Engineering*, **Vol. 12A, No. 2**, pp. 73-79.

J. Schwarz (2000). Overview of the EU Electricity Directive. *IEEE Power Engineering Review*, **Vol. 20, No. 4**, pp 134-141

R. W. Ferrero, S. M. Shahidehpour, and V.C.Ramesh, (1997). Transaction analysis in deregulated power systems using game theory. *IEEE Trans. on Power Systems*, **Vol.12, No.3,** pp. 1340-1347.