DISCRETE TAKAGI-SUGENO FUZZY MODELS: REDUCED NUMBER OF STABILIZATION CONDITIONS

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Abstract – Takagi-Sugeno's fuzzy models enable to represent a wide class of non linear models in a compact set of the state variables. According to this representation stabilization conditions can be obtained and are usually written as Linear Matrix Inequalities. Since the obtained conditions are only sufficient, current researches try to lower the conservatism of the results. In this paper several matrix properties are used with the help of the elimination lemma for discrete TS models.

Keywords: Non linear systems, Discrete Takagi-Sugeno fuzzy models, LMI, elimination lemma.

1. INTRODUCTION

Since nearly twenty years, Takagi-Sugeno's fuzzy models (Takagi and Sugeno, 1985) have been used to model and control non linear systems. Stability and stabilization are mainly based on Lyapunov functions (Wang, et al., 1996; Ma et al, 1998; Tanaka, et al, 1998; Yoneyama et al, 2000). These latter are usually quadratic. Sometimes piecewise quadratic functions are used (Johansson et al 1999; Feng and Wang, 2001). There are also some results using non linear functions, in the continuous case (Blanco et al, 2001; Tanaka et al, 2001) and in the discrete case (Guerra and Vermeiren, 2004). Nevertheless in this case, the complexity of the LMI problem has been seriously increased.

In every case, the number of conditions put in the form of LMI increases highly as the number of models increases. Usually the number of LMI is about $r(r+1)/2$ with r the number of linear models of the TS fuzzy model.

Several approaches have been developed to lower the conservatism of the conditions. One approach is based on reducing the number of models (Lauber, 2003; Taniguchi et al, 2001), another one uses matrix properties to reduce the conservatism of the conditions themselves (Guerra et al, 2003). Results presented in this paper follow the latter idea.

The paper is organized as follows. The second part recalls useful mathematical tools. The third part presents the new conditions and part fourth compares various conditions on an example.

2. TOOLS

2.1 Basic conditions

Let be a Takagi-Sugeno's fuzzy model (Takagi and Sugeno, 1985) with r the number of rules, $x(t)=\left[x_1(t), x_2(t), \cdots, x_n(t)\right]^T$ the state vector, $u(t)$ the control signal, $y(t)$ the output, and $z(t)$ the premises variables. The fuzzy model is given by:

$$\begin{cases} x(t+1)=\sum_{i=1}^{r} h_i\left(z(t)\right)\left(A_i x(t)+B_i u(t)\right) \\ y(t)=\sum_{i=1}^{r} h_i\left(z(t)\right)C_i x(t) \end{cases} \quad (1)$$

The non-linearities of the global model are due to the terms $h_i\left(z(t)\right)\geq 0$, with the convex sum property, i.e. $\sum_{i=1}^{r} h_i\left(z(t)\right)=1$. In this paper is assumed that, $\forall i$ pairs $\left(A_i, B_i\right)$ are controllable.

The usual control law used to stabilize model (1) is called a PDC (Parallel Distributed Compensation) and is given by (Wang et al, 1996):

$$u(t) = -\sum_{i=1}^{r} h_i(z(t)) F_i x(t) \qquad (2)$$

The design of $u(t)$ requires to obtain the feedback gains F_i. Several problems can be addressed, robustness, performances and so on (Tanaka et al. 1998) (Liu and Zhang 2003). LMI tools (Boyd et al, 1994) are often a very convenient way to solve these problems. Let be a quadratic Lyapunov function $V = x^T P x$ with $P > 0$. Then with $X = P^{-1}$ and $N_i = F_i P^{-1}$ is defined:

$$\Upsilon_{ij} = \begin{bmatrix} -X & (*) \\ A_i X - B_i N_j & -X \end{bmatrix} \qquad (3)$$

The most basic conditions of stabilization are presented theorem 1.

Theorem 1 (Wang et al, 1996) : Model (1) is globally asymptotically stable in closed loop with the control law (2) and the Υ_{ij} defined in (3) if there exist: $X > 0$, N_i, $i, j \in \{1, \ldots, r\}$ such that:

$$\forall i \quad \Upsilon_{ii} < 0 \qquad (4)$$

$$\forall i, j \quad i < j \quad \Upsilon_{ij} + \Upsilon_{ji} < 0 \qquad (5)$$

According to the work of (Guerra and Vermeiren, 2004, Guerra et al, 2003) the following notation is defined. For scalar functions $h_i(z) \ge 0$ and U_i, $i \in \{1, ..., r\}$ matrices of the same dimension, we note: $U_z = \sum_{i=1}^{r} h_i(z) U_i$.

Similarly $U_{zz} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z) h_j(z) U_{ij}$ (6)

2.2 Properties

The following properties are useful to establish the main result.

Lemma 1 (Congruence): Let X be a full rank matrix. If $Y > 0$ then:

$$XYX^T > 0 \qquad (7)$$

Lemma 2: (Shur's complement Boyd et al, 1994): Matrices X, Y and R being of appropriate sizes, we have:

$$\begin{cases} Y - XR^{-1}X^T > 0 \\ R > 0 \end{cases} \Leftrightarrow \begin{bmatrix} Y & (*) \\ X^T & R \end{bmatrix} > 0 \qquad (8)$$

(*) represents all terms induced by symmetry in a symmetric matrix.

Lemma 3: The two next problems are equivalent:

(i) Find $P = P^T$ such that $T + A^T P A < 0$ (9)

(ii) Find $P = P^T$, L, G such that:

$$\begin{bmatrix} T + A^T L^T + LA & (*) \\ -L^T + G^T A & P - G - G^T \end{bmatrix} < 0 \qquad (10)$$

It is a generalization of a lemma proposed in (Peaucelle et al, 2000) that generalizes (Oliverira et al, 1999).

Proof:

(ii) implies (i): $\begin{bmatrix} I & A^T \end{bmatrix}$ being a full row matrix using the congruence, lemma 1, gives the result.

(i) implies (ii): consider $L = 0.5 A^T P$ and $G = 0.5(P + P')$ with $P' > 0$ an unspecified matrix. Thus the condition (i) becomes:

$$\begin{bmatrix} -T - A^T P A & (*) \\ -0.5 P' A & P' \end{bmatrix} > 0 \qquad (11)$$

Applying the Shur's complement (8) gives: (11) is equivalent to: $-T - A^T P A - 0.25 A^T P' A > 0$. Since $-T - A^T P A > 0$ by hypothesis, an enough small $P' > 0$ such that (11) is satisfied can always be defined.

This lemma can be extended to matrices defined by blocks. For example:

(i) Find $P = P^T$ such that $\begin{bmatrix} T_1 + A^T P A & (*) \\ T_2 & T_3 \end{bmatrix} < 0$ (12)

(ii) Find $P = P^T$, L_1, L_2 and G such that

$$\begin{bmatrix} T_1 + A^T L_1^T + L_1 A & (*) & (*) \\ T_2 + L_2 A & T_3 & (*) \\ -L_1^T + G^T A & -L_2^T & P - G - G^T \end{bmatrix} < 0 \qquad (13)$$

Remark 1: (12) can be recovered from (13) using the congruence with the row full rank matrix $\begin{bmatrix} I & 0 & A^T \\ 0 & I & 0 \end{bmatrix}$.

Lemma 4. (Peaucelle et al, 2000) The two next problems are equivalent:

(i) Find $P > 0$, such that: $T + A^T P + PA < 0$ (14)

(ii) Find $P > 0$, L, G such that:

$$\begin{bmatrix} T + A^T L^T + LA & (*) \\ P - L^T + G^T A & -G - G^T \end{bmatrix} < 0 \qquad (15)$$

This lemma is the pending of lemma 3 for the continuous case. Similarly, it can be extended to matrices defined by blocks, for example there is equivalence between:

(i) Find $P > 0$ such that

$$\begin{bmatrix} T_1 + A^T P + PA & (*) \\ T_2 & T_3 \end{bmatrix} < 0 \quad (16)$$

(ii) Find $P > 0$, L_1, L_2 and G such that

$$\begin{bmatrix} T_1 + A^T L_1^T + L_1 A & & (*) \\ T_2 + L_2 A & T_3 & \\ P - L_1^T + G^T A & -L_2^T & -G - G^T \end{bmatrix} < 0 \quad (17)$$

Several relaxations of conditions (4) and (5) have been defined in the literature. The main idea is to relax the crossed terms $\Upsilon_{ij} + \Upsilon_{ji}$ by introducing a new LMI depending on the whole terms $\Upsilon_{ij} + \Upsilon_{ji}$ and Υ_{ii}. First results were proposed by Kim and Lee (Kim and Lee, 2000). They were extended in (Liu and Zhang, 2004), and we will use this latter approach. The work presented in (Teixeira et al, 2003) can also be quoted, but it implies a serious increase of the number of variables involved in the problem.

Lemma 5. (Liu and Zhang, 2004) Consider matrices Υ_{ij}, the condition:

$$\sum_{i=1}^{r} h_i^2(z)\Upsilon_{ii} + \sum_{i=1}^{r}\sum_{i=1}^{r} h_i h_j(z)\left(\Upsilon_{ij} + \Upsilon_{ji}\right) < 0 \quad (18)$$

is true if there exists Q_i and $Q_{ij} = Q_{ji}^T$ $(j > i)$ such as the following conditions are satisfied:

$$\forall i \qquad \Upsilon_{ii} + Q_i < 0 \quad (19)$$

$$\forall i, j \quad i < j \qquad \Upsilon_{ij} + \Upsilon_{ji} + Q_{ij} + Q_{ij}^T \le 0 \quad (20)$$

$$\begin{bmatrix} Q_1 & (*) & & (*) \\ Q_{ij} & Q_i & & \\ \vdots & & \ddots & (*) \\ Q_{1r} & \cdots & Q_{(r-1)r} & Q_r \end{bmatrix} > 0 \quad (21)$$

Lemma 6. (Boyd et al, 1994).
Consider the following condition:

$$G(z) + U(z) X V^T(z) + V(z) X^T U^T(z) > 0 \quad (22)$$

with z and X two variables. U and V do not depend on X. Moreover X must be an unspecified matrix with no constraint. Then (22) is equivalent to:

$$\begin{cases} G(z) - \sigma U(z) U^T(z) > 0 \\ G(z) - \sigma V(z) V^T(z) > 0 \end{cases} \quad (23)$$

with z the first variable, and $\sigma \in \mathbb{R}$.

This result is based on the Finsler's lemma and enables to obtain an equivalent problem with a reduced complexity, since we replace an unknown matrix by an unknown scalar.

This lemma has two simplified versions:
If either U or V is the Identity matrix, then its corresponding condition can be removed in (23).
If we have the simplified problem:

$$\begin{bmatrix} G_{11} & (*) \\ G_{21} & G_{22} \end{bmatrix} + UX \begin{bmatrix} I \\ 0 \end{bmatrix}^T + \begin{bmatrix} I \\ 0 \end{bmatrix} X^T U^T > 0, \quad \text{then} \quad (23)$$

reduces to: $G(z) - \sigma U(z) U^T(z) > 0$ and $G_{22}(z) > 0$.

Lemma 7 (Inversion matrix lemma). Let be A, B, C, D matrices of appropriate dimension. Then:

$$\left(A + BCD\right)^{-1} = A^{-1} - A^{-1}B\left[C^{-1} + DA^{-1}B\right]^{-1} DA^{-1} \quad (24)$$

The best previous conditions to guarantee the stability of the closed loop for discrete fuzzy models are recalled in the next theorem. With the same notations as previously for theorem 1:

Theorem 2 (Liu and Zhang, 2003) : Fuzzy model (1) is globally asymptotically stable in closed loop with control law (2) and the Υ_{ij} defined in (3) if there exists matrices: $X > 0$, N_i, $Q_i > 0$, $Q_{ij} = Q_{ji}^T$ $(j > i)$, $i, j \in \{1, \ldots, r\}$ such that: (19), (20) and (21) hold.

Remark 2: Theorem 2 includes conditions of theorem 1.

Remark 3: The number of LMI to check with theorem 2 (excepted condition (21)) is equal to $r(r+1)/2$.

3. MAIN RESULT

Theorem 3: The fuzzy model (1) is globally asymptotically stable in closed loop with control law (2) if there exists matrices: $X > 0$, U, T, L_1 and L_2 such that:

$$\beta = T + T^T - X > 0 \quad (25)$$

and for $i \in \{1, \ldots, r\}$:

$$\begin{bmatrix} X & (*) & (*) \\ -A_i T - B_i L_1^T & \beta - B_i L_2^T - L_2 B_i^T & (*) \\ L_1^T & L_2^T - U^T B_i^T & U + U^T - \sigma I \end{bmatrix} > 0 \quad (26)$$

Moreover, the expression of the control law is:

$$u = -\left[B_z^T \beta^{-1} B_z\right]^{-1} B_z^T \beta^{-1} A_z x \quad (27)$$

Proof: The variation of the quadratic Lyapunov function along the trajectories of the closed loop model, i.e.: $\Delta V(k) = V(k+1) - V(k) < 0$ gives:

$$(A_z x + B_z u)^T P (A_z x + B_z u) - P < 0 \quad (28)$$

By applying lemma 3 with $L = 0$, (28) is equivalent to:

$$\begin{bmatrix} P & -\left(A_z - B_z F_z\right)^T G \\ -G^T\left(A_z - B_z F_z\right) & G + G^T - P \end{bmatrix} > 0 \qquad (29)$$

G is invertible since the last block of (29) gives: $G + G^T - P > 0$. Thus by congruence with the full-rank matrix $\begin{bmatrix} G^{-T} & 0 \\ 0 & G^{-T} \end{bmatrix}$ (29) is equivalent to:

$$\begin{bmatrix} G^{-T}PG^{-1} & -G^{-T}A_z^T + G^{-T}F_z^T B_z^T \\ -A_z G^{-1} + B_z F_z G^{-1} & G^{-1} + G^{-T} - G^{-T}PG^{-1} \end{bmatrix} > 0 \qquad (30)$$

thus defining the new variables $G^{-T}PG^{-1} = X > 0$, $G^{-1} = T$ and $F_z G^{-1} = N_z$, (30) can be written as:

$$\begin{bmatrix} X & -T^T A_z^T + N_z^T B_z^T \\ -A_z T + B_z N_z & T + T^T - X \end{bmatrix} > 0 \qquad (31)$$

or:

$$\begin{bmatrix} X & (*) \\ -A_z T & T + T^T - X \end{bmatrix} + \begin{bmatrix} 0 \\ B_z \end{bmatrix} N_z \begin{bmatrix} I \\ 0 \end{bmatrix}^T + \begin{bmatrix} I \\ 0 \end{bmatrix} N_z^T \begin{bmatrix} 0 \\ B_z \end{bmatrix}^T > 0$$

According to the simplified version of lemma 6, the elimination lemma gives two following conditions:

$$T + T^T - X > 0$$

$$\begin{bmatrix} X & (*) \\ -A_z T & T + T^T - X - \sigma B_z B_z^T \end{bmatrix} > 0 \qquad (32)$$

From the last block of (32) we can see that if (32) is verified then it exists at least one $\sigma < 0$. This remark will be useful for the end of the proof.

Now lemma 4 is applied on the term $B_z B_z^T$ to finally get the condition:

$$\begin{bmatrix} X & (*) & (*) \\ -A_z T - B_z L_1^T & \begin{pmatrix} T + T^T - X \\ -B_z L_2^T - L_2 B_z^T \end{pmatrix} & (*) \\ L_1^T & L_2^T - U^T B_z^T & U + U^T - \sigma I \end{bmatrix} > 0 \qquad (33)$$

Notice that inequality (32) can be recovered from (33) by using the congruence with the row full rank matrix: $\begin{bmatrix} I & 0 & 0 \\ 0 & I & B_z \end{bmatrix} > 0$.

Conditions (33) are respected with conditions of theorem 3. Now we can turn back to the control law. Applying Shur's complement on equations (31) and (32) gives:

$$X - \left(TA_z^T - N_z^T B_z^T\right)\beta^{-1}\left(A_z T - B_z N_z\right) > 0, \qquad (34)$$

$$X - T^T A_z^T \left(\beta - \sigma B_z B_z^T\right)^{-1} A_z T > 0 \qquad (35)$$

If the theorem 3 conditions (25) and (26) are verified, it ensures that (35) holds. We need now to prove that (34) also holds. The proof is based on the inversion matrix lemma (24). Applied to (35) we obtain:

$$\begin{aligned} & X - T^T A_z^T \Psi A_z T > 0 \\ & \Psi = \beta^{-1} - \beta^{-1} B_z \left(-\sigma^{-1} I + B_z^T \beta^{-1} B_z\right)^{-1} B_z^T \beta^{-1} \end{aligned} \qquad (36)$$

Then (34) holds if it exists N_z satisfying:

$$N_z^T B_z^T \beta^{-1} A_z T + T^T A_z^T \beta^{-1} B_z N_z - N_z^T B_z^T \beta^{-1} B_z N_z \geq$$
$$T^T A_z^T \beta^{-1} B_z \left(-\sigma^{-1} I + B^T \beta^{-1} B_z\right)^{-1} B_z^T \beta^{-1} A_z T$$

or equivalently if it exists F_z satisfying:

$$\begin{aligned} & F_z^T B_z^T \beta^{-1} A_z + A_z^T \beta^{-1} B_z F_z - F_z^T B_z^T \beta^{-1} B_z F_z \geq \\ & A_z^T \beta^{-1} B_z \left(-\sigma^{-1} I + B^T \beta^{-1} B_z\right)^{-1} B_z^T \beta^{-1} A_z \end{aligned} \qquad (37)$$

Introducing the control law (27) gives:

$$\begin{aligned} & A_z^T \beta^{-1} B_z \left(B^T \beta^{-1} B_z\right)^{-1} B_z^T \beta^{-1} A_z \geq \\ & A_z^T \beta^{-1} B_z \left(-\sigma^{-1} I + B^T \beta^{-1} B_z\right)^{-1} B_z^T \beta^{-1} A_z \end{aligned} \qquad (38)$$

Since it exists $\sigma < 0$, (38) holds.

Remark 4: The number of LMI has been reduced from $r(r+1)/2$ to $r+1$. We have to stress that no relaxation principle, such as with lemma 5, is required anymore. Indeed there is no double sum in (33).

Remark 5: Due to the expression of the control law (27) it becomes impossible to use a pole placement approach to obtain feedback gains and to search after a $P > 0$ for the Lyapunov function. This is done for example in (Teixeira el al, 2003) where the stabilization problem is replaced by a stability problem.

4. REGULATOR PROBLEM

We want to minimize the following criterion:

$$u = \arg\min\left(\sum_{o}^{\infty}\left(x^T Q x + u^T R u\right)\right) \qquad (39)$$

An upper bound of this criterion is given solving the following problem.

Theorem 4: The fuzzy model (1) is globally asymptotically stable in closed loop with control law (2) and an upper bound of (39) is guaranteed if there exists matrices: $X > 0$, U, T, L_1 and L_k such that:

$\min : \gamma$, subject to $\begin{bmatrix} T + T^T - X & x_0 \\ x_0^T & \gamma \end{bmatrix} > 0$ for $i \in \{1, \ldots, r\}$,

$$\begin{bmatrix} X & & & & (*) \\ -A_iT - B_iL_1^T & T+T^T - X - B_iL_2^T - L_2B_i^T & & & \\ T & -L_3B_i^T & Q^{-1} & & \\ 0 & \sigma B_z^T - L_4B_i^T & 0 & R^{-1}-\sigma I & \\ L_1^T & L_2^T - U^T B_i^T & L_3^T & L_4^T & U + U^T - \sigma I \end{bmatrix} > 0 \quad (40)$$

With $P = X^{-1}$, the control law is given by,:

$$u = -\left(B_z^T P B_z + R\right)^{-1} B_z^T P A_z x \quad (41)$$

Proof The inequality:

$$\Delta V + x^T Q x + u^T R u < 0 \quad (42)$$

gives

$$(A_z x - B_z u)^T P (A_z x - B_z u) - P + x^T Q x + u^T R u < 0 \quad (43)$$

Minimizing with the variable u leads to :

$$-B_z^T P A_z x - B_z^T P A_z x + 2\left(B_z^T P B_z + R\right) u = 0$$

so once the matrix P is known, the control law is given by:

$$u = -\left(B_z^T P B_z + R\right)^{-1} B_z^T P A_z x \quad (44)$$

Now by summing equation (42) between 0 and k, it is obvious that $V(0)$ is an upper bound of (39).

The same usual operations applied to (43) gives, with the new variables $G^{-T} P G^{-1} = X > 0$, $G^{-1} = T$ and $F_z G^{-1} = N_z$:

$$\begin{bmatrix} X - T^T Q T - N_z^T R N_z & -T^T A_z^T + N_z^T B \\ -A_z T + B_z N_z & T + T^T - X_z \end{bmatrix} > 0$$

Then the Shur's complement is applied to get:

$$\begin{bmatrix} X & & & (*) \\ -A_z T & T + T^T - X_z & & \\ T & 0 & Q^{-1} & 0 \\ 0 & 0 & 0 & R^{-1} \end{bmatrix} + \begin{bmatrix} 0 \\ B_z \\ 0 \\ I \end{bmatrix} N_z \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix} N_z^T \begin{bmatrix} 0 \\ B_z \\ 0 \\ I \end{bmatrix}^T > 0$$

The elimination lemma gives two conditions. The first is $T + T^T - X > 0$. The second is:

$$\begin{bmatrix} X & & & (*) \\ -A_z T & T + T^T - X - \sigma B_z B_z^T & & \\ T & 0 & Q^{-1} & \\ 0 & \sigma B_z^T & 0 & R^{-1} - \sigma I \end{bmatrix} > 0$$

Lemma 4 gives conditions (40).

Now the optimization criteria can be studied. Recall that $V(0) = x_0^T P x_0$ has to be minimized. So we search the smallest γ such that:

$x_0^T P x_0 < \gamma$. With the decision matrices this gives $x_0^T T^{-T} X T^{-1} x_0 < \gamma$.

Lemma 2 is applied with $L = 0$ to get:

$$\begin{bmatrix} G + G^T - X & -G T^{-1} x_0 \\ -x_0^T T^{-T} G^T & \gamma \end{bmatrix} > 0$$

With $T^{-1} = G$ we obtain the criteria to minimize.

5. RESULTS

We will take the example 2 of (Guerra and Vermeiren, 2004) for which there is no solution with theorem 2 (with a quadratic Lyapunov function). The system is described by two rules defined by the matrices:

$$A_1 = \begin{bmatrix} -0.5 & 2 \\ -0.1 & 0.5 \end{bmatrix},\ A_2 = \begin{bmatrix} -0.9 & 0.5 \\ -0.1 & -1.7 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 4.1 \\ 4.8 \end{bmatrix},\ B_2 = \begin{bmatrix} 3 \\ 0.1 \end{bmatrix}$$

Previous theorems do not rely on functions h_i, which play no role in the comparison. For the sake of plotting results for a dynamical model, we have chosen:

$$h_1 = \frac{1}{2\pi}\left(\pi/2 - Arc\tan(x_1)\right),$$

$$h_2 = \frac{1}{2\pi}\left(\pi/2 + Arc\tan(x_1)\right)$$

A solution with the new conditions is found with the matrix P defined by: $P = \begin{bmatrix} 0.32 & 2.65 \\ 2.65 & 27.2 \end{bmatrix}$. The next figures show the evolutions of the states x_1, x_2 and the control signal u when $x_0 = \begin{bmatrix} 10 & 2 \end{bmatrix}^T$.

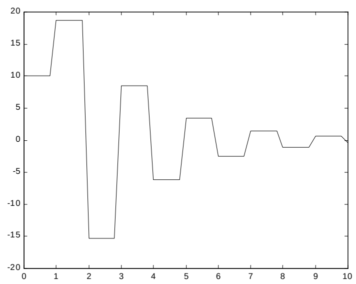

Fig. 1. evolution of $x_1(t)$

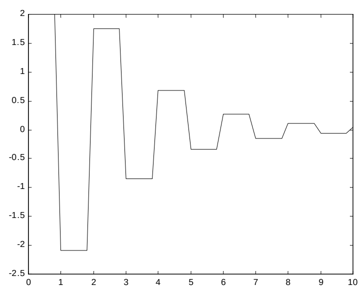

Fig. 2. evolution of $x_2(t)$

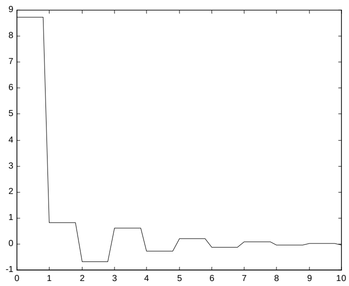

Fig. 3. evolution of $u(t)$

6. CONCLUSION

The study presented in this paper tries to reduce the conservatism of the conditions by lowering the number of conditions while still keeping all the degrees of freedom. The number of decision variables has been reduced, and thus the complexity of the LMI problem is smaller. Results show that we were able to obtain such results without raising the conservatism of our conditions. The elimination lemma and several matrix transformations were used for this purpose.

REFERENCES

Blanco Y., Perruquetti W. and Borne P. (2001), "Nonquadratic stability of non linear systems in the Takagi-Sugeno form", in *Proc. ECC*, Porto, Portugal, 2001.

Boyd S., Ghaoui L.El., Féron E. and Balakrishnan V. (1994), *Linear Matrix Inequalities in system and control theory*, Studies in Applied Mathematics, SIAM, Philadelphia PA.

De Oliveira J., Bernussou J. and Geromel J.C. (1999), "A new discrete-time robust stability condition", *Systems and Control Letters*, vol. 37, pp. 261-265.

Feng G. and Wang L. (2001), "Controller synthesis of fuzzy dynamic systems based on piecewise Lyapunov functions", in *Proc. IEEE Conf. On Fuzzy Systems*, Melbourne, Australia, 2001.

Johansson T., Rantzer A. and Arzen K. (1999), "Piecewise quadratic stability of fuzzy systems", *IEEE Trans. Fuzzy Systems*, vol. 7, pp. 713-722.

Guerra T.M., Ksontini M. and Delmotte F. (2003), "Some new relaxed conditions of quadratic stabilization for continuous Takagi-Sugeno fuzzy models", in *Proc. IMACS/IEEE CESA'2003*, Lille, France.

Guerra T.M. and Vermeiren L. (2004), "LMI-based relaxed non-quadratic stabilization conditions for nonlinear systems in Takagi-Sugeno's form", *Automatica*, vol. 40 (5), pp. 823-829.

Kim E. and Lee H. (2000), "New Approaches to Relaxed Quadratic Stability Condition of Fuzzy Control Systems", *IEEE Trans. on Fuzzy Systems*, vol. 8 (5), pp. 523-533.

Lauber J. (2003), "Moteur à allumage commandé avec EGR : modélisation et commande non linéaires", LAMIH-SF, Thesis of Université de Valenciennes et du Hainaut-Cambrésis, France, In french.

Liu X. and Zhang Q. (2003), "New approaches to H_∞ controller designs based on fuzzy observers for TS fuzzy systems via LMI", *Automatica*, vol. 39 (9), pp. 1571-1582.

Ma X.J., Sun Z.Q. and He Y.Y. (1998), "Analysis and design of fuzzy controller and fuzzy observer", *IEEE trans. on Fuzzy Systems*, vol. 6 (1), pp. 41-50.

Peaucelle D., Arzelier D., Bachelier O. and Bernussou J. (2000), "A new robust -stability condition for real convex polytopic uncertainty", *Systems and Control letters*, vol. 40 (1), pp. 21-30.

Takagi T. and Sugeno M. (1985), "Fuzzy identification of systems and its applications to modeling and control", *IEEE Trans. on Systems Man and Cybernetics*, vol. 15 (1), pp. 116-132.

Tanaka K., Ikeda T. and Wang H.O. (1998), "Fuzzy regulators and fuzzy observers: Relaxed stability conditions and LMI-based designs", *IEEE Trans. on Fuzzy Systems*, vol. 6 (2), pp. 1-16.

Tanaka K., Hori T. and Wang H.O. (2001), "A fuzzy Lyapunov approach to fuzzy control system design", in *Proc. ACC 2001*, Washington, USA.

Taniguchi T., Tanaka K., Ohtake H. and Wang H.O. (2001), "Model construction, rule reduction, and robust compensation for generalized form of Takagi-Sugeno fuzzy systems", *IEEE Trans. on Fuzzy Systems*, vol. 9 (4), pp. 525-537.

Teixeira M., Assuncao E. and Avellar R. (2003), "On relaxed LMI-based Design for Fuzzy regulators and fuzzy observers", *IEEE Trans. on Fuzzy Systems*, vol. 11 (5), pp. 613-623.

Tuan H.D., Apkarian P., Narikiyo T. and Yamamoto Y. (2001), "Parameterized linear matrix inequality techniques in Fuzzy control system design", *IEEE Trans. on Fuzzy Systems*, vol. 9 (2), pp. 324-332.

Wang H.O., Tanaka K. and Griffin M. (1996), "An Approach to Fuzzy Control of Nonlinear Systems : Stability and Design Issues", *IEEE Trans. on Fuzzy Systems*, vol. 4 (1), pp. 14-23.

Yoneyama J., Nishikawa M., Katayama H. and Ichikawa H. (2000), "Output stabilization of Takagi-Sugeno fuzzy systems", *Fuzzy Sets and Systems*, vol. 111, pp. 253-266.