

ACTIVE VIBRATION REJECTION IN STEEL ROLLING MILLS

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Abstract: This contribution is concerned with vibration rejection by nonlinear control techniques in steel industry, particularly in rolling mill plants. The high quality requirements of rolled products, especially in relation to the thickness tolerances, mean a challenging task also from a control point of view. The key part of this contribution deals with so-called wrapper rolls as well as chatter vibration phenomena. With regard to the control design a mathematical model based on physical considerations is introduced where the essential nonlinearities of the system are taken into account. For the purpose of vibration rejection two nonlinear control concepts based on energy considerations and the theory of differential flatness are presented. *Copyright ©2005 IFAC*

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1. INTRODUCTION

The high quality requirements of steel rolling industry products mean a challenging task not at least from a control point of view. Especially the observance of the restrictive thickness tolerances and surface properties of rolled products demand effective vibration rejection concepts. Since there is also a strong economic objective to increase the efficiency of mill plants these vibration phenomena gain more and more in importance.

In this contribution two common vibration problems of a rolling process are investigated, namely vibrations of a so-called wrapper roll and third octave chatter. The arrangement of a wrapper roll is typically used for coiling the rolled strip. This system is poorly damped and in order to avoid

damages of the coiled strip this fact has to be taken into account for a controller design. The phenomenon of third octave chatter is especially observed in thin product cold rolling plants. Characteristic for this form of rolling mill chatter is a vertical vibration of the rolls as well as of the mill housing in the frequency range of the musical third octave. The consequences of these vibrations vary from rejected products to damages of the rolling plant and consequently lead to lasting production delays.

The presented active vibration rejection concepts for the discussed mill plant problems rely on model-based nonlinear control techniques. In Section 2 the wrapper roll vibrations are treated by considering the port controlled Hamiltonian representation of the dynamic system and an

energy-based controller design. The active rejection of third octave chatter which is investigated in Section 3 uses the theory of differential flatness whereby the trajectory tracking problem can be solved in a straightforward manner.

2. WRAPPER ROLLS

In many rolling mills the coiling and uncoiling of the steel strip is supported by so-called wrapper rolls. These rolls are hydraulically pressed against the coil and should guarantee a homogenous distance of the adjacent strip layers. The wrapper roll is lifted off the coil and moved back again each time the starting point of the coil passes by. As already mentioned these wrapper assemblies are poorly damped and therefore, it is highly desirable to implement a robust position control law with additional damping for that task. The controller should not rely on the velocity signal because separate velocity sensors are usually not mounted. Additionally, it is not easy to obtain a good approximation from the position signal since it is often corrupted due to measurement and quantization noise.

The relevant details of the wrapper assembly are depicted in figure 1. The mechanical wrapper

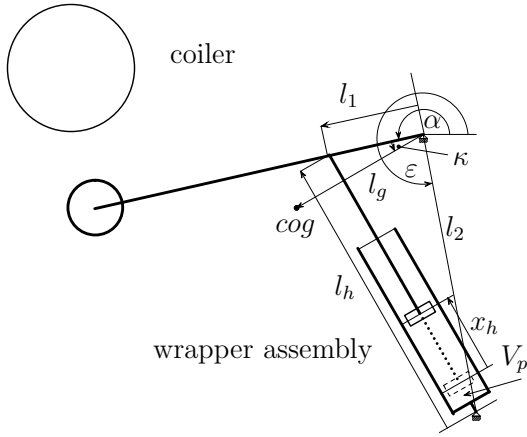


Fig. 1. A hydraulically actuated wrapper.

system belongs to the class of Euler-Lagrange (EL) systems with Lagrangian

$$L = \frac{1}{2}\Theta\dot{q}^2 - V, \quad V = m_l g l_g \sin(q + \kappa)$$

with the generalized coordinate $q = \alpha$, the constant moment of inertia Θ – the moment of inertia of the actuator and its mass is neglected –, the mass of the wrapper assembly m_l , the constant of gravity g and the relative coordinates of the center of gravity (cog) (l_g, κ) . It has to be mentioned that the assumption of constant inertia does not mean any restriction for the approach presented in the following, it is also applicable for systems with an inertia depending on the generalized coordinate.

The corresponding Euler-Lagrange equation augmented by a damping term with damping constant d and by the hydraulic force F_h as fictitious input reads as

$$\Theta\ddot{q} = -\partial_q V - d\dot{q} + \partial_q x_h(q) F_h - M_l,$$

with the piston position

$$x_h(q) = \sqrt{l_1^2 + l_2^2 - 2l_1 l_2 \cos(\varepsilon - q)} - l_{h0},$$

$l_{h0} = l_h|_{x_h=0}$, and an unknown but constant external load torque M_l . Here and in the following ∂_q indicates the partial derivative with respect to q . Let $\partial_q x_h(q) \neq 0$ be met in the domain under consideration.

In general, the hydraulic actuator has a double acting piston and one may assume that each chamber is rigidly connected to a three-land-four-way spool valve. Often, the dynamics of the compensated servo valves are much faster than those of the other parts of the hydraulic adjustment system. Therefore, one can neglect the valve dynamics and consider the valve volume flow $q_{v,i}$ as the plant input. As already shown in (Grabmair *et al.*, 2004) the overall system has port controlled Hamiltonian (PCH) structure with the fluid masses in both chambers as Casimir functions, i.e., invariants of the system independent of H . In the following the canonical coordinates $x = (q, p, z_{h1}, z_{h2})$ with $p = \Theta\dot{q}$, $F_h = p_{h1}A_1 - p_{h2}A_2$,

$$z_{h1} = F_h + \sum_{i=1}^2 (-1)^{i-1} EA_i \ln(V_{hi}V_{pi}^{-1}) \quad (1)$$

$$z_{h2} = p_{h2}A_2 + EA_2 \ln(V_{h2}V_{p2}^{-1})$$

will be used. $E > 0$ indicates the constant hydraulic bulk modulus. The effective piston areas are denoted by A_i , the chamber offset volumes by V_{pi} , the chamber pressures and volumes by p_{hi} and V_{hi} , with $i = 1, 2$ and $V_{h1} = V_{p1} + A_1 x_h(q)$, $V_{h2} = V_{p2} - A_2 x_h(q)$. The PCH structure, see (Grabmair *et al.*, 2003), is then given by

$$\begin{aligned} \dot{x} &= (J - R) \partial_x H^T + Gu \\ y &= G^T \partial_x H^T \end{aligned}$$

with Hamiltonian

$$\begin{aligned} H &= H_h + \frac{1}{2}\Theta^{-1}p^2 + V \\ H_h &= \sum_{i=1}^2 EV_{hi} (\ln(V_{hi}V_{pi}^{-1}) - 1) + \\ &V_{p1} (p_{h0} + E) e^{\frac{z_{h1} + z_{h2}}{A_1 E} - \frac{p_{h0}}{E}} + \\ &+ V_{p2} (p_{h0} + E) e^{\frac{z_{h2}}{A_2 E} - \frac{p_{h0}}{E}} + \\ &- z_{h1} x_h - (z_{h1} + z_{h2}) \frac{V_{p1}}{A_1} - z_{h2} \frac{V_{p2}}{A_2}, \end{aligned} \quad (2)$$

the input matrix G for the input $u = (u_1, u_2)$

$$G^T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad u_2 = \frac{EA_2}{V_{p2} - A_2 x_h(q)} q_{v2}$$

$$u_1 = \frac{EA_1}{V_{p1} + A_1 x_h(q)} q_{v1} - \frac{EA_2}{V_{p2} - A_2 x_h(q)} q_{v2},$$

and the canonical skew symmetric structure matrix J and the positive semidefinite dissipation matrix R with $R_{22} = d$ and zero otherwise. p_{h0} denotes some offset pressure.

Now, one can construct an energy-based controller, which maintains the plants PCH structure consisting of the wrapper part $x_1 = (q, p, z_{h1})$ and a decoupled part $x_2 = (z_{h2})$, and asymptotically stabilizes the equilibrium $\check{x} = (\check{q}, 0, z_{h1}(\check{q}), \check{z}_{h2})$ by choosing

$$\begin{aligned} u_1 &= -k_{p1} (z_{h1} - z_{h1}(\check{q})) \\ u_2 &= -k_{p2} (z_{h2} - \check{z}_{h2}). \end{aligned} \quad (3)$$

However, in hydraulic actuators usually there is no velocity signal of the piston available and additionally M_i and thus \check{F}_h , i.e., the desired value of F_h , is unknown. In order to tackle this problem, one can introduce a dynamical extension of second order

$$\begin{aligned} \dot{\hat{x}}_{obs} &= \begin{bmatrix} \lambda_1 + \lambda_2 & -1 \\ \lambda_1 \lambda_2 & 0 \end{bmatrix} \hat{x}_{obs} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_{obs} + \\ &\begin{bmatrix} -((\lambda_1 + \lambda_2)^2 - \lambda_1 \lambda_2) \Theta - ((\lambda_1 + \lambda_2) d) \\ -\lambda_1 \lambda_2 ((\lambda_1 + \lambda_2) \Theta + d) \end{bmatrix} \bar{q} \\ \begin{bmatrix} \hat{p} \\ \hat{u}_{obs} \end{bmatrix} &= \hat{x}_{obs} + \begin{bmatrix} -(\lambda_1 + \lambda_2) \Theta - d \\ -\lambda_1 \lambda_2 \Theta \end{bmatrix} \bar{q}, \end{aligned} \quad (4)$$

by means of a reduced disturbance observer for $\frac{d}{dt} \check{F}_h = 0$ with

$$\begin{aligned} u_{obs} &= \partial_q V + \partial_q x_h(q) F_h \\ \hat{F}_h &= \frac{\hat{u}_{obs} + \partial_q V}{\partial_q x_h(q)} \end{aligned}$$

and the chosen stable eigenvalues λ_1, λ_2 of the observer error dynamics. Here and in the following, $\bar{x} = x - \check{x}$ indicates coordinates relative to the equilibrium \check{x} . Then, the observer error dynamics with $e_1 = \hat{p} - p$ and $e_2 = \hat{u}_{obs} - u_{obs}$ are given by

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \lambda_1 + \lambda_2 & -1 \\ \lambda_1 \lambda_2 & 0 \end{bmatrix}}_{A_{obs}} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}.$$

Additionally, there exists a positive definite solution P^{-1} of the Lyapunov type equation

$$P^{-1} A_{obs}^T + A_{obs} P^{-1} + 2Q = 0$$

for any positive definite Q . This leads to the decomposition $A_{obs} = (J_o - Q) P$ with $J_o = (A_{obs} + QP) P^{-1}$.

Now, one chooses an overall desired PCH structure (J_a, R_a, H_a) for the extended state $\bar{x}_e =$

$(\bar{q}, p, \bar{z}_{h1}, e_1, e_2, \bar{z}_{h2})$ with some constants $\Gamma_1, \Gamma_2 > 0$, a positive definite augmented Hamiltonian

$$\begin{aligned} H_a &= \frac{p^2}{2\Theta} + V - \frac{\partial_q V(q)|_{q=\check{q}}}{\partial_q x_h(q)|_{q=\check{q}}} (x_h - x_h(\check{q})) \\ \Gamma_1 \frac{\bar{z}_{h1}^2}{2} &+ E \int_{x_h(\check{q})}^{x_h(\check{q}+\bar{q})} \left(A_1 \ln \left(\frac{V_{h1}(\check{x}_h + \tau)}{\check{V}_{h1}} \right) + \right. \\ &\left. A_2 \ln \left(\frac{\check{V}_{h2}}{V_{h2}(\check{x}_h + \tau)} \right) \right) d\tau + \Gamma_2 \frac{\bar{z}_{h2}^2}{2} + \frac{1}{2} e^T P e \end{aligned}$$

and

$$\begin{aligned} J_a &= \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & \beta + \gamma & 0 & 0 \\ 0 & -\beta - \gamma & 0 & -\delta^T & 0 \\ 0 & 0 & \delta & J_o & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \tilde{\delta} = \begin{bmatrix} \tilde{\delta}_1 \\ \tilde{\delta}_2 \end{bmatrix} \\ R_a &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & d & -\beta + \gamma & 0 & 0 \\ 0 & -\beta + \gamma & \frac{k_{p1}}{\Gamma_1} & \delta^T & 0 \\ 0 & 0 & \delta & Q & 0 \\ 0 & 0 & 0 & 0 & \frac{k_{p2}}{\Gamma_2} \end{bmatrix}, \quad \delta = P^{-1} \tilde{\delta} \end{aligned}$$

with

$$\begin{aligned} \beta &= \frac{\partial_q x_h(q)|_{q=\check{q}+\bar{q}}}{2\Gamma_1}, \quad \gamma = \Theta \frac{k_d}{2} \\ \tilde{\delta}_1 &= -\frac{k_d}{2}, \quad \tilde{\delta}_2 = -\frac{k_p}{2 \partial_q x_h(q)|_{q=\check{q}+\bar{q}}}. \end{aligned}$$

This choice guarantees local asymptotic stability of the closed loop system due to

$$\dot{H}_a = -\partial_{\bar{x}_e} H_a R_a \partial_{\bar{x}_e} H_a^T$$

and LaSalle's principle if $k_{p1}, k_{p2}, k_d > 0$ and if

$$\frac{k_{p1} d}{\Gamma_1} - \left(\frac{\partial_q x_h(q)|_{q=\check{q}+\bar{q}}}{2\Gamma_1} - \Theta \frac{k_d}{2} \right)^2 \geq 0$$

is fulfilled. These conditions are obtained by decomposing R_a into the sum of positive semidefinite matrices. The obtained control law is given by

$$\begin{aligned} u_1 &= -k_{p1} \left(z_{h1} - \frac{1}{\partial_q x_h(q)} (\hat{u}_{obs} + \partial_q V) \right) \\ &\quad - k_{p1} E A_1 \ln \left(\frac{V_{h1}}{V_{p1}} \left(\frac{V_{p2}}{V_{h2}} \right)^{\frac{A_2}{A_1}} \right) \Bigg|_{q=\check{q}} - k_d \Theta \hat{p} \\ u_2 &= -k_{p2} \bar{z}_{h2} \end{aligned}$$

with the observer equations from (4) and the transformations (1). The tuning parameters of the controller are the proportional gains k_{p1}, k_{p2} , the damping gain k_d and the eigenvalues of the observer. The performance of the energy-based dynamic controller with damping compared to the pure static controller (3) is demonstrated by the

simulations in figure 2. At the scaled time value 0.5 an external load torque is applied, which is removed at 1.5.

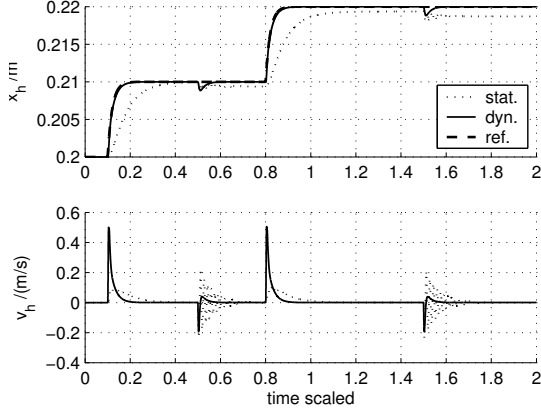


Fig. 2. Wrapper position (x_h) and velocity (v_h) responses with static and dynamic controller.

3. THIRD OCTAVE CHATTER

The occurrence of third octave chatter vibrations is typically found in multi-stand cold rolling mills where the strip deformation takes place under a considerable strip tension. An experimentally proven method of third octave chatter prevention proposed in literature (see, e.g., (Boulton *et al.*, 2000)) is essentially based on an operating speed reduction. But this method is undesirable since it causes less production efficiency of the plant. Therefore, this section is focused on a model-based active rejection of third octave chatter by nonlinear control.

3.1 Roll-gap Model

Under the action of the roll force F_r and the entry and exit strip tensions σ_{en} , σ_{ex} the strip is deformed in the roll-gap elasto-plastically in order to achieve the desired output thickness h_{ex} , see figure 3. The strip entering the roll-gap moves slower than the work roll surface (backward slip), such that due to the frictional and normal stresses arising in the roll/strip interface plastic deformation of the strip occurs after a short elastic compression zone. After passing the so-called neutral point, where the strip speed coincides with the velocity of the roll, the frictional stresses change their direction due to the occurrence of forward slip. This implies a decrease of the contact stresses unless the plastic deformation of the strip stops and an elastic recovery zone occurs at the exit domain of the roll-gap.

The normal and frictional stresses acting in the roll/strip interface also result in an elastic deformation of the work rolls, see again figure 3. In the

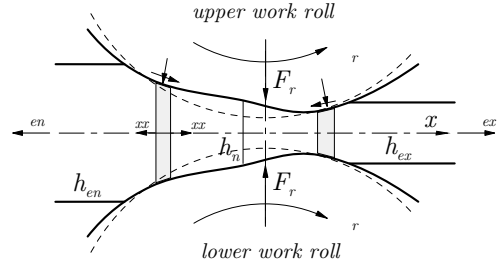


Fig. 3. Scheme of the roll-gap (the indicated directions of the load are related to the strip).

case of cold rolling, the assumption of a circular roll-gap shape under the action of the rolling load, though with a larger so-called equivalent radius, is appropriate, see, e.g., (Bland *et al.*, 1951). However, for the case of temper and thin strip rolling, this approximation is no longer valid and the elastic work roll deformations have to be considered in detail. These models are usually referred to as non-circular arc roll-gap models, see, e.g., (Fleck *et al.*, 1992) and either include the elastic halfspace solution or Jortner's solution (Jortner *et al.*, 1960) for the radial displacement field due to a piecewise constant normal load in order to account for the roll deformations. A different approach to cope with this problem is proposed in (Fuchshumer *et al.*, 2004). Motivated from a control point of view, the displacement fields are approximated in the sense of the Rayleigh-Ritz method. This yields a finite-dimensional approximation of the roll deformation problem.

As the dynamics of the processes taking place in the roll-gap are considerably faster than the dynamics of the mill stand and the hydraulic adjustment system, it is appropriate to set up a quasi-static roll-gap model. The resulting model is represented by a set of implicit algebraic equations

$$f_{RFM}(F_r, h_{en}, h_{ex}, \sigma_{en}, \sigma_{ex}) = 0 \quad (5)$$

relating the strip entry and exit thicknesses h_{en} , h_{ex} , the tensions σ_{en} , σ_{ex} , the geometry and material parameters, the roll force F_r and the slip at the entry and exit point of the roll-gap.

3.2 Mill Stand Interconnection Model

In multi-stand rolling mills the essential coupling of adjacent mill stands is given by a strip element of length L_i and cross-section $A_{s,i}$, see figure 4, which is modelled as a massless linear-elastic spring with Young's modulus E_s since the eigenfrequencies of the distributed parameter system are much higher than the dynamics of the considered system. The so-called looper is acting on the strip in order to adjust the strip length between adjacent mill stands, see again figure 4. For modelling it is assumed that the looper is permanently in contact with the strip and that

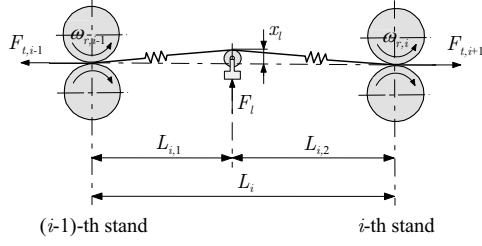


Fig. 4. Coupling of adjacent mill stands.

the rotational inertia and friction effects of the looper roll can be neglected. Therefore, with the abbreviations

$$\alpha_1 = \frac{h_{n,i-1}}{h_{ex,i-1}} R_{i-1}, \quad \alpha_2 = \frac{h_{n,i}}{h_{en,i}} R_i$$

$$\alpha_3 = x_l \left(\frac{1}{\sqrt{L_{i,1}^2 + x_l^2}} + \frac{1}{\sqrt{L_{i,2}^2 + x_l^2}} \right)$$

and the strip stiffness $c_{s,i} = \frac{E_{s,i} A_{s,i}}{L_i}$ the equations of motion of the looper read as

$$\dot{x}_l = v_l \quad (6a)$$

$$\dot{x}_s = \alpha_2 \omega_i - \alpha_1 \omega_{i-1} + \alpha_3 v_l \quad (6b)$$

$$m_l \dot{v}_l = F_l - c_{s,i} x_s \alpha_3 - m_l g - d_l v_l, \quad (6c)$$

where x_l , v_l denote the displacement and the velocity of the looper, x_s the elongation of the linear elastic strip element, F_l the force acting on the looper, m_l the mass of the looper and R_i , ω_i the radius and the angular velocity of the work rolls of the i -th mill stand. Furthermore, the dynamics of the main drives follow under the assumption of symmetry of upper and lower rolls for the $(i-1)$ -th stand as

$$I_{r,i-1} \dot{\omega}_{r,i-1} = M_{r,i-1} - M_{d,i-1} - d_{r,i-1} \omega_{r,i-1} - (F_{t,i-1} - c_{s,i} x_s) R_{i-1} \quad (7)$$

with the inertia of the rolls reduced to the shaft of the work roll I_r , the torque of the main drives M_r , the torque of strip deformation M_d , the coefficient of viscous friction d_r and the force according to the strip tension F_t . In analogy one obtains

$$I_{r,i} \dot{\omega}_{r,i} = M_{r,i} - M_{d,i} - d_{r,i} \omega_{r,i} - (c_{s,i} x_s - F_{t,i+1}) R_i \quad (8)$$

for the drive dynamics of the i -th stand.

The interaction of the roll force, the strip tensions and the strip thicknesses, described by (5) and (6b), allow an explanation for the occurrence of third octave chatter by an instability of the mechanical interconnected rolling mill stand. The occurrence of the instability clearly depends on the operation point of the mill plant. For a more detailed discussion of this observation the reader is referred to, e.g., (Holl *et al.*, 2004).

In order to illustrate a chatter initiation in simulation a disturbance in the strip entry thickness

of $10 \mu\text{m}$ (e.g., due to a welding seam) is assumed. In figure 5 these simulation results are compared with measurements of a plant where third octave chatter was observed.

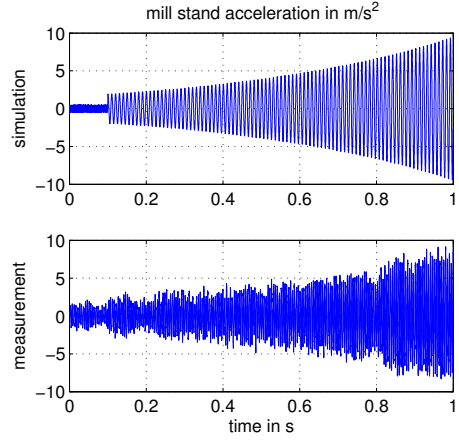


Fig. 5. Simulation and measurement results of a third octave chatter initiation.

3.3 Flatness-based Control

For an active chatter rejection it is necessary to prevent that the mechanical feedback established by the strip causes an unstable operating point of the mill plant. This can be obtained by the control of the strip tension with the looper as well as with the main mill drives as control inputs $u = (F_l, M_{r,i-1}, M_{r,i})$. The mathematical model (6)-(8) formulated in state space representation with the state variables $x = (x_l, x_s, v_l, \omega_{r,i-1}, \omega_{r,i})$ can be written as a dynamic system of the form $\dot{x}^{\alpha_x} = f^{\alpha_x}(x, u)$, $\alpha_x = 1, \dots, n$, with m inputs and the output functions $y^{\alpha_y} = c^{\alpha_y}(x)$, $\alpha_y = 1, \dots, m$. It is well known in control theory that such a nonlinear system is state feedback equivalent to a linear time-invariant one, if and only if it has some vector relative degree (r_1, \dots, r_m) at x_0 such that $\sum_{i=1}^m r_i = n$ holds, cp., e.g., (Isidori, 1995). With regard to a flatness-based controller design the quest for a flat output can be accomplished by the fact that a system that has the property that it is state feedback equivalent to a linear time-invariant one implies the property of differential flatness and the associated output y is also a flat output, (Fliess *et al.*, 1995), (Rudolph, 2003).

It is not difficult to show that the mathematical model of the coupled mill plant (6)-(8) has a vector relative degree $(2, 1, 2)$ with the output functions

$$h_1 = \sigma_{s,i} = E_{s,i} x_s / L_i$$

$$h_2 = \omega_{r,i} \quad (9)$$

$$h_3 = x_s - \sqrt{L_{i,1}^2 + x_l^2} - \sqrt{L_{i,2}^2 + x_l^2}$$

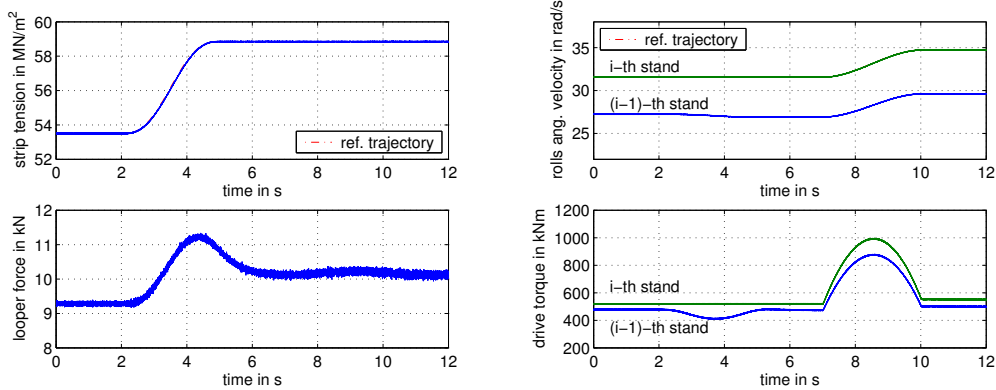


Fig. 6. On active chatter rejection by nonlinear flatness-based control.

for all $x_l \neq 0$ and therefore, the system is flat with the flat output $y = (h_1, h_2, h_3)$. It has to be mentioned that for control only the domain $x_l > 0$ is relevant since for modelling it was assumed that the strip is in permanent contact with the looper roll.

The concept of differential flatness allows in addition a systematic approach for solving the trajectory tracking problem. Under consideration of the initial value the desired trajectories y_d for the components of the flat output are chosen and therefore, the corresponding trajectories of the control inputs are calculated. For the asymptotic stabilization of the trajectory tracking error $e_i = y_i - y_{i,d}$, $i = 1, \dots, m = 3$, the dynamics of the tracking error are adjusted by linear, time-invariant differential equations. It is worth mentioning that the torque required for strip deformation is known only insufficiently with respect to an exact input-to-state linearization. Therefore, to obtain stationary accuracy for the flat output an integral term is added to the control law.

Finally, in figure 6 it is demonstrated in simulation that by means of the proposed flatness-based control approach chatter does not occur for relevant operating points. Moreover, an increase of the strip tension as well as of the mill plant operating speed is illustrated by means of a reference trajectory, respectively. The authors ask for understanding that the data used for this as well as the previous results can not be stated due to observance of secrecy.

4. CONCLUSIONS

This contribution deals with two applications of model-based nonlinear control in rolling industry. By energy considerations of a mechanical Euler-Lagrange system driven by a hydraulic actuator the vibration problem of a so-called wrapper roll is investigated. For an active rejection of the third octave chatter phenomenon, which is caused by an instability of the mechanical rolling mill plant,

a flatness-based control approach is discussed. Finally, it has to be mentioned that the presented methods are protected by patent.

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