RANK-CONSTRAINED LMI APPROACH TO MIXED H_2/H_∞ STATIC OUTPUT FEEDBACK CONTROLLERS

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Abstract:
This paper deals with a numerical method for the design of mixed H_2/H_∞ static output feedback controllers. We first formulate the problem as a new type of rank-constrained linear matrix inequalities (LMIs). Then, the LMI optimization problem subject to a rank condition is tackled by the recently developed penalty function method, where a linear penalty function is introduced for the nonconvex rank constraint. The overall procedure results in solving a series of convex optimization problems. With an increasing sequence of the penalty parameter, the solution of the penalized optimization problem moves towards the feasible region of the original nonconvex problem. Comparisons with previous research are performed to illustrate the proposed method.

Keywords: Linear matrix inequality (LMI), penalty method, rank constraint, static output feedback (SOF)

1. INTRODUCTION

Recently, a semidefinite program formulation applicable to static output feedback (SOF) stabilization has been proposed (Mesbahi, 1999); however, most SOF control problems including mixed H_2/H_∞ control still remain open.

The purpose of mixed H_2/H_∞ control guarantees optimal closed-loop performance while maintaining a prescribed level of robustness (Bernstein and Haddad, 1989; Khargonekar and Rotea, 1991). Considering performance and robustness simultaneously often arises in many control fields, but no analytic solution exists to date.

The conventional representation of H_2/H_∞ SOF problems based on the celebrated elimination lemma (Boyd *et al.*, 1994; Gahinet and Apkarian, 1994; Skelton *et al.*, 1997) leads to a linear matrix inequality (LMI) optimization problem subject to a nonconvex algebraic or rank constraint on the Lyapunov variables (Leibfritz, 2001). The use of a single Lyapunov matrix in multiobjective control is known to produce conservative results. To reduce the degree of conservatism, several methods have been proposed in the LMI framework (Arzelier and Peaucelle, 2002; Halder and Kailath, 1999; Shimomura and Fujii, 1999).

Meanwhile, to solve nonconvex rank-constrained LMI problems, several global and local methods have been presented during the last decade (Goh *et al.*, 1994; Grigoriadis and Skelton, 1996; Ghaoui *et al.*, 1997; Fazel *et al.*, 2003).

More recently, a *partially augmented Lagrangian* (PAL) method (Apkarian *et al.*, 2003) has been developed. This second-order method has a superior convergence property over the local methods, but the implementation of the algorithm is not easy since the gradient and Hessian of the objective function must be derived for the Newton-type method.

In this paper, mixed H_2/H_∞ SOF problems are converted to a new type of rank-constrained LMI problems. The rank condition here is not imposed on the Lyapunov matrix but imposed on the slack matrix; thus the proposed method can be applied to simultaneous stabilization, polytopic uncertain plant models and multi-objective control problems. For the SOF stabilization problem, a similar method was addressed by (Peaucelle *et al.*, 2002). After formulating the problem, we discuss the newly developed computation method for solving rank-constrained LMI optimization problems (Kim *et al.*, 2004).

The remainder of the paper is organized as follows. In Section 2 and 3, we state SOF problems in terms of rank-constrained LMIs. Section 4 briefly describes the penalty function method for general rank-constrained problems. In Section 5, the practical implementation of the algorithm is given. Section 6 shows some numerical experiments.

The notation is quite standard. I denotes the identity matrix. A^T means the transpose of the matrix A. The trace and the rank of a matrix A are denoted by $\mathrm{tr}(A)$ and $\mathrm{rank}(A)$, respectively. $A \succ 0$ (respectively, $A \succeq 0$) means that the matrix A is symmetric and positive definite (respectively, semidefinite). For long matrix expressions, $(\star)^T AX$ means $X^T AX$. The notation T_{zw} denotes the transfer function from w to z.

2. STATIC OUTPUT FEEDBACK STABILIZATION

Let us consider the SOF stabilization with a performance channel shown in Fig. 1. The state-space representation of the system is described by

$$\begin{pmatrix} \dot{x} \\ \hline z \\ y \end{pmatrix} = \left(\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & 0 \end{array}\right) \begin{pmatrix} x \\ \hline w \\ u \end{pmatrix}, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $w \in \mathbb{R}^{n_w}$ is the exogenous input, $u \in \mathbb{R}^{n_u}$ is the control input, $z \in \mathbb{R}^{n_z}$ is the output to be regulated, and $y \in \mathbb{R}^{n_y}$ is the output.

We seek a static control law $u = Ky$ such that the closed-loop system becomes stable and satisfies the performance specification for all $T > 0$,

$$\int_0^T \begin{pmatrix} z \\ w \end{pmatrix}^T \begin{pmatrix} Q & R \\ R^T & S \end{pmatrix} \begin{pmatrix} z \\ w \end{pmatrix} dt < 0 . \tag{2}$$

The following lemma states a new rank-constrained LMI formulation for the design of a static controller with a quadratic performance specification.

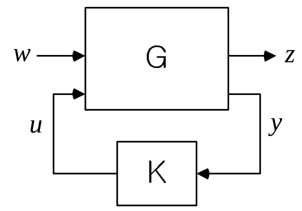

Fig. 1. Static output feedback control with a performance channel.

Lemma 1. For the system in Fig. 1 represented by (1), there exists a stabilizing static controller $u = Ky$ such that the performance condition (2) holds if and only if there exist matrices $P \succ 0, W \succeq 0$ satisfying the LMI subject to the rank condition,

$$\begin{pmatrix} \star \\ \star \\ \hline \star \\ \star \end{pmatrix}^T \left(\begin{array}{cc|cc} 0 & P & 0 & 0 \\ P & 0 & 0 & 0 \\ \hline 0 & 0 & Q & R \\ 0 & 0 & R^T & S \end{array}\right) \begin{pmatrix} I & 0 & 0 \\ A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ 0 & I & 0 \end{pmatrix} \prec \begin{pmatrix} \star \\ \star \end{pmatrix}^T W \begin{pmatrix} C_2 & D_{21} & 0 \\ 0 & 0 & I \end{pmatrix} \tag{3}$$

$$\mathrm{rank}(W) = n_u . \tag{4}$$

If W satisfying (3) and (4) is found, K can be computed by solving the LMI in the variable K,

$$\begin{pmatrix} W_1 + W_2 K + K^T W_2^T & K^T W_3 \\ W_3 K & -W_3 \end{pmatrix} \preceq 0, \tag{5}$$

where

$$W = \begin{pmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{pmatrix} .$$

Proof. Define Ω as

$$\Omega = \begin{pmatrix} \star \\ \star \end{pmatrix}^T \begin{pmatrix} 0 & P \\ P & 0 \end{pmatrix} \begin{pmatrix} I & 0 & 0 \\ A & B_1 & B_2 \end{pmatrix} + \begin{pmatrix} \star \\ \star \end{pmatrix}^T \begin{pmatrix} Q & R \\ R^T & S \end{pmatrix} \begin{pmatrix} C_1 & D_{11} & D_{12} \\ 0 & I & 0 \end{pmatrix} .$$

Then, by virtue of Lyapunov stability theory and Finsler's lemma, the existence condition of an SOF controller can be expressed as the following LMI in $P \succ 0$ and K (Arzelier and Peaucelle, 2002)

$$\begin{pmatrix} \star \\ \star \\ \star \end{pmatrix}^T \Omega \begin{pmatrix} I & 0 \\ 0 & I \\ KC_2 & KD_{21} \end{pmatrix} \prec 0, \tag{6}$$

which is equivalent to

$$\Omega \prec \mu \begin{pmatrix} C_2^T K^T \\ D_{21}^T K^T \\ -I \end{pmatrix} \begin{pmatrix} KC_2 & KD_{21} & -I \end{pmatrix} \Leftrightarrow$$

$$\Omega \prec \mu \begin{pmatrix} \star \\ \star \end{pmatrix}^T \begin{pmatrix} K^T K & -K^T \\ -K & I \end{pmatrix} \begin{pmatrix} C_2 & D_{21} & 0 \\ 0 & 0 & I \end{pmatrix}, \tag{7}$$

where $\mu > 0$. From (7), we can easily obtain (3), (4) and (5).

In the case of H_∞ controllers, the performance matrices are given by $Q = I, S = -\gamma^2 I, R = 0$. Also,

H_2 optimal controllers can be described in a similar manner.

Lemma 2. For system (1) with $D_{11} = D_{21} = 0$, we can find a static control law such that the H_2 performance of the closed-loop system is $||T_{zw}||_2 < \gamma_2$ if and only if there exist matrices $P_2 \succ 0, W \succeq 0$ satisfying,

$$\mathrm{tr}(B_1^T P_2 B_1) \le \gamma_2^2, \tag{8}$$

$$\begin{pmatrix}\star\\ \star\\ \hline \star\end{pmatrix}^T \left(\begin{array}{cc|c} 0 & P_2 & 0\\ P_2 & 0 & 0\\ \hline 0 & 0 & I\end{array}\right)\left(\begin{array}{cc} I & 0\\ A & B_2\\ \hline C_1 & D_{12}\end{array}\right)$$
$$\prec \begin{pmatrix}\star\\ \star\end{pmatrix}^T W \begin{pmatrix} C_2 & 0\\ 0 & I\end{pmatrix}, \tag{9}$$

$$\mathrm{rank}(W) = n_u. \tag{10}$$

Remark 3. The advantage of Lemmas 1 and 2 lies in that no constraints are imposed on the Lypunov matrix P. Thus, we can use separate Lyapunov matrices for polytopic plants or multiobjective control syntheses to reduce conservatism.

3. MIXED H_2/H_∞ STATIC OUTPUT CONTROL

In this section, we present a rank-constrained LMI approach to the mixed H_2/H_∞ static control problem shown in Fig. 2, whose state-space representation is

$$\begin{pmatrix}\dot{x}\\ \hline z_\infty\\ z_2\\ y\end{pmatrix} = \left(\begin{array}{c|ccc} A & B_0 & B_1 & B_2\\ \hline C_0 & D_{00} & 0 & D_{02}\\ C_1 & 0 & 0 & D_{12}\\ C_2 & D_{20} & 0 & 0\end{array}\right)\begin{pmatrix} x\\ \hline w_\infty\\ w_2\\ u\end{pmatrix} \tag{11}$$

$$u = Ky, \tag{12}$$

where all notations have the same meaning as in (1). The channel (w_∞, z_∞) is for the robustness condition of the system, and the channel (w_2, z_2) for the optimal H_2 performance of the closed-loop system. The mixed H_2/H_∞ SOF problem for system (11) can be written as follows.

Problem 4. For a given $\gamma_\infty > 0$, find a static control law $u = Ky$ that minimizes $||T_{z_2 w_2}||_2$ subject to $||T_{z_\infty w_\infty}|| < \gamma_\infty$.

Based on the formulation of the previous section, we use two Lyapunov matrices for H_2 and H_∞ channels. To find a single control gain, a common W matrix is chosen at the expense of some conservatism. The resulting problem to be solved reduces to

$$\begin{array}{cc} \min\limits_{P_2\succ 0, P_\infty \succ 0, W\succeq 0} & \mathrm{tr}(B_1^T P_2 B_1)\\ \text{subject to} & \text{LMI (9)}, \end{array}$$

$$\begin{pmatrix}\star\\ \star\\ \hline \star\\ \star\end{pmatrix}^T \left(\begin{array}{cc|cc} 0 & P_\infty & 0 & 0\\ P_\infty & 0 & 0 & 0\\ \hline 0 & 0 & I & 0\\ 0 & 0 & 0 & -\gamma^2 I\end{array}\right)\left(\begin{array}{ccc} I & 0 & 0\\ A & B_0 & B_2\\ \hline C_0 & D_{00} & D_{02}\\ 0 & I & 0\end{array}\right)$$
$$\prec \begin{pmatrix}\star\\ \star\end{pmatrix}^T W \begin{pmatrix} C_2 & D_{20} & 0\\ 0 & 0 & I\end{pmatrix}, \tag{13}$$

$$\mathrm{rank}(W) = n_u. \tag{14}$$

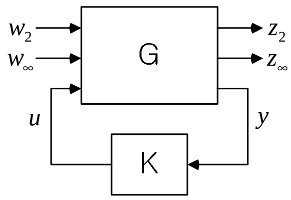

Fig. 2. Static H_2/H_∞ output controller.

4. PENALTY FUNCTION FORMULATION OF A RANK CONDITION

With abuse of notation, the problems in the previous sections have the form:

$$\begin{array}{ll} \min & c^T x\\ \text{subject to} & x \in \mathcal{C},\\ & \mathrm{rank}(X(x)) = r, \end{array} \tag{15}$$

where $\mathcal{C}$ is the convex set,

$$\mathcal{C} = \{x : X(x) \succeq 0, \quad L(x) \succ 0\}, \tag{16}$$

x is the decision vector, and $X(x),\ L(x)$ are matrices that are affine functions of x.

We briefly review the recently developed penalty function method for LMI optimization problems subject to a rank-condition (Kim *et al.*, 2004). In the penalty function method, we iteratively solve the following penalized optimization problem for obtaining a solution of (15),

$$\begin{array}{ll} \min & \rho\, c^T x + \mathrm{tr}(X) + \mu\, p(x; V)\\ \text{subject to} & x \in \mathcal{C}, \end{array} \tag{17}$$

where ρ is the optimization weight, μ is the penalty parameter. The penalty function $p(x; V)$ is defined by

$$p(x; V) = \mathrm{tr}(V^T X V), \tag{18}$$

where $V \in \mathbb{R}^{n\times(n-r)}$ consists of orthonormal columns. During the computation process, the parameters μ, ρ and the coefficient matrix V are successively updated.

If we assume that the eigenvalues of X are ordered $\lambda_1 \le \cdots \le \lambda_{n-r} \le \cdots \le \lambda_n$, then the following inequality holds for a given V such that $V^T V = I$,

$$\lambda_1 + \cdots + \lambda_{n-r} \le \mathrm{tr}\ (V^T X V). \tag{19}$$

(19) means that the penalty function denotes the upper bound of the sum of the $n - r$ smallest eigenvalues of X. Therefore, if the penalty function becomes zero, then $\mathrm{rank}(X) \le r$.

The weighting matrix V consisting of orthonormal columns can be determined from any feasible point

$X(x_0), x_0 \in \mathcal{C}$ by eigenvalue decomposition. Consequently, problem (15) reduces to a convex LMI optimization for given μ, ρ and V. With the notation

$$\varphi(x;\rho,\mu,V) = \rho\, c^T x + \mathrm{tr}(X) + \mu\, p(x;V), \quad (20)$$

consider the sequence of the solution for fixed μ, ρ and given V_0 in (17),

$$x_k = \min_x\{\varphi(x;\rho,\mu,V_{k-1}) : x \in \mathcal{C}\}, \quad k = 1,2,\ldots, \quad (21)$$

where V_k is computed from $x_k \in \mathcal{C}$. The following lemma states the convergence property of (21).

Lemma 5. Let $\mu > 0$, $\rho > 0$ in (21) be fixed. Then for a given V_0, the following inequality on the solution sequence holds:

$$\varphi(x_{k+1};\rho,\mu,V_k) \le \varphi(x_k;\rho,\mu,V_{k-1}). \quad (22)$$

Lemma 5 implies that for fixed μ and ρ, the sequence $\{\varphi(x_k;\rho,\mu,V_{k-1})\}$ is always non-increasing and convergent. When the limit value of the penalty function $p(x_k;V_{k-1})$ is not sufficiently small, we can get a new point that is closer to the feasible region by increasing μ. If we rewrite the objective function as

$$\varphi(x;\rho,\mu,V) = \rho\, c^T x + \sum_{i=1}^{n}\lambda_i + \mu\sum_{i=1}^{n-r}\lambda_i, \quad (23)$$

then we can understand that increasing μ makes the sum of the smallest $(n-r)$ eigenvalues decrease.

Remark 6. The convergence of the solution to (17) is guaranteed with an increasing sequence of μ. Also, note that the solution sequence to (17) always moves towards the region satisfying the rank constraint. However, like other local algorithms (Ghaoui *et al.*, 1997; Grigoriadis and Skelton, 1996), the global convergence of the penalty function method is not guaranteed; the convergence properties of the method are yet to be studied. Nevertheless, the proposed method did find solutions for many control problems reliably (Kim *et al.*, 2004).

Remark 7. From (18), we can see that the penalty function is linear, and that the value of the penalty function is always positive over $x \in \mathcal{C}$. These imply that the penalty function above can be regarded as an exact penalty function over the convex set $\mathcal{C}$. Hence, there exists a finite penalty parameter μ such that $p(x_{k+1};V_k) = 0$.

5. IMPLEMENTATION OF THE PENALTY FUNCTION METHOD

This section summarizes the penalty function method (PFM) for rank-constrained LMI optimization problems.

Algorithm 1. The PFM for rank-constrained LMI problems

(1) Initialization. Set the penalty parameter $\mu = 0, \rho_0 \gg 1$ and find an initial feasible point x_0 by solving the LMI optimization problem:

$$x_0 = \min_x\{\rho\, c^T x + \mathrm{tr}(X) : x \in \mathcal{C}\}.$$

Set $x_k = x_0$. Choose $\mu_k = \mu_0 > 1$, $\rho_k = \rho_0$, $\alpha \in (0,1)$, $\beta \ll 1$, $\tau > 1$, $\xi > 1$, $\varepsilon_1 \ll 1$, $\varepsilon_2 \ll 1$.

(2) Computation of V. Compute V_k from $X(x_k)$ by eigenvalue decomposition.

(3) Convex optimization. Compute x_{k+1} by solving the convex LMI optimization problem,

$$x_{k+1} = \min_x\{\varphi(x;\rho_k,\mu_k,V_k) : x \in C\}.$$

(4) Feasibility test. If $p(x_{k+1};V_k) \le \varepsilon_1$, then x_{k+1} is feasible and stop when computing a feasible solution.

(5) Optimality test. If x_{k+1} is feasible and $|c^T x_{k+1} - c^T x_k| \le \varepsilon_2$ then a locally optimal solution x_{k+1} is obtained. Stop.

(6) Penalty parameter update. If x_{k+1} is not feasible and $p(x_{k+1};V_k) > \alpha p(x_k;V_{k-1})$, then increase the penalty parameter by $\mu_{k+1} = \tau\mu_k$.

(7) Optimization weight update. If x_{k+1} is feasible and $|c^T x_{k+1} - c^T x_k| < \beta$, then increase the optimization weight by $\rho_{k+1} = \xi\rho_k$.

(8) Next step. Set $k = k+1$ and go to step (2).

The implementation code of the PFM is almost the same as that of the cone complementarity linearization algorithm (Ghaoui *et al.*, 1997) except for eigenvalue decomposition. Though the PFM is similar to the first-order method, it can be applied to optimization problems, and it shows good convergence characteristics attributed to the tuning factors μ and ρ.

6. NUMERICAL EXAMPLES

We selected some H_2/H_∞ static output feedback control examples to evaluate the performance of the proposed algorithm. Throughout the simulation, we have used the SeDuMi package as an LMI solver and the YALMIP for a SeDuMi interface (Sturm, 2001; Löfberg, 2004). The computation parameters used were

$$\mu_0 = 5000,\ \rho_0 = 1000,\ \alpha = 0.99,\ \tau = 1.05,$$

which were selected by a trial-and-error approach. Thus further work on initial values, computation parameters, and convergence properties is needed.

Example 1. This is a classical example taken from (Levine and Athans, 1970). The state-space matrices of the system are given by

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\ B_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix},\ B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$B_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix},\ C_0 = \begin{pmatrix} 0 & 1 \end{pmatrix},\ D_{00} = 0,$$

$$D_{02} = 0,\ C_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},\ D_{12} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$
$$C_2 = \begin{pmatrix} 0 & 1 \end{pmatrix},\ D_{20} = 0.$$

The analytical solution to the mixed H_2/H_∞ static control for this system is completely known (Arzelier and Peaucelle, 2002). Table 1 shows the computation results for the robustness condition $\gamma_\infty \leq 1.2$, and Fig. 3 shows the computational behavior of the PFM. In the table, γ_2-bound means the solution to Problem 4, and γ_2-actual is computed from the closed-loop system with the obtained static gain K. The optimal gain is -0.9458, and the computed gain by the PFM is -0.9735. As is shown in Fig. 3, the penalty function of the PFM is always decreasing and tends to zero in 20 iterations. We can see that the obtained solution is not overly conservative.

Table 1. Results for example 1

	γ_2-bound	γ_2-actual	γ_∞
Optimal	-	1.5735	1.2000
Arzelier(2002)	1.6825	1.5778	1.1706
PFM	1.5838	1.5772	1.1746

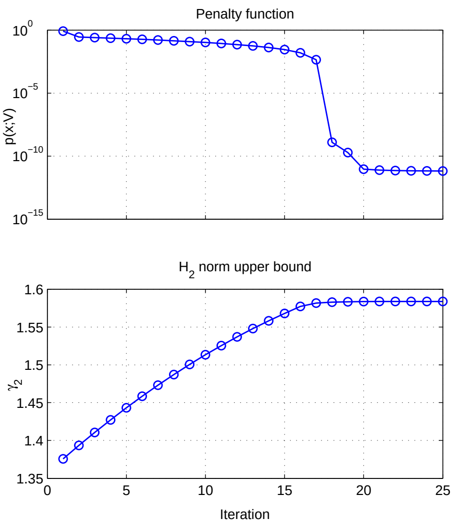

Fig. 3. Behavior of penalty function and H_2-norm upper bound for example 1.

Example 2. As a second example, we choose a mass-spring system described in (Shimomura and Fujii, 1999) with data matrices

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1.25 & 1.25 & 0 & 0 \\ 1.25 & -1.25 & 0 & 0 \end{pmatrix},\ B_0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -0.25 & 0.5 \\ 0.25 & 0 \end{pmatrix},$$
$$B_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5 \end{pmatrix},\ B_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix},\ C_0 = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$
$$C_1 = \begin{pmatrix} 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},\ D_{12} = \begin{pmatrix} 0 \\ 0.2 \end{pmatrix},$$
$$D_{02} = \begin{pmatrix} 0 \\ 0.2 \end{pmatrix},\ C_2 = I_{4\times 4}.$$

We design a static controller with the H_∞ specification, $\gamma_\infty \leq 1$. Computation results are displayed in Table 2, and Fig. 4. Calculated control gain is

$$K = \begin{pmatrix} -1.2127 & -0.2828 & -1.4208 & -0.6675 \end{pmatrix}.$$

In this numerical experiment, our result is less conservative.

Table 2. Results for example 2

	γ_2-bound	γ_2-actual	γ_∞
Shimomura(1999)	1.7827	1.7223	0.3979
PFM	1.5114	1.5111	0.9416

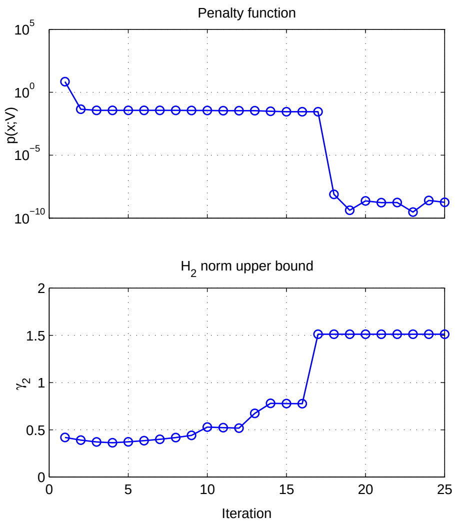

Fig. 4. Behavior of penalty function and H_2-norm upper bound for example 2.

Example 3. This is the longitudinal motion of a VTOL helicopter (Leibfritz, 2001). The system data matrices are given by

$$A = \begin{pmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.0100 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.7070 & 1.4200 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$
$$B_1 = \begin{pmatrix} 0.0468 & 0 \\ 0.0457 & 0.0099 \\ 0.0437 & 0.0011 \\ -0.0218 & 0 \end{pmatrix},\ C_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}^T$$
$$B_2 = \begin{pmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0 & 0 \end{pmatrix},\ C_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}^T$$
$$D_{20} = \begin{pmatrix} 0.00039 & 0.00174 \end{pmatrix}$$
$$D_{00} = 0_{2\times 2},\ D_{02} = D_{12} = I_{2\times 2}/\sqrt{2}$$
$$B_0 = B_1,\ C_0 = C_1.$$

Table 3 and Fig. 5 show the results with the constraint $\gamma_\infty \leq 0.423722$. The computed gain is

$$K = \begin{pmatrix} 1.0792 & 11.8505 \end{pmatrix}^T$$

Table 3. Results for example 3

	γ_2-bound	γ_2-actual	γ_∞
Leibfritz(2001)	0.4687	0.1033	0.2943
PFM	0.1050	0.1002	0.1778

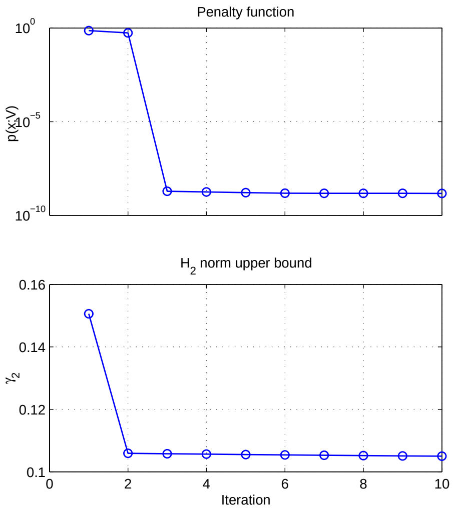

Fig. 5. Behavior of penalty function and H_2-norm upper bound for example 3.

From the results above, we can see that the PFM can efficiently solve mixed H_2/H_∞ static output control problems with a new rank-constrained LMI representation, and that our results are less conservative than those of the previous research.

7. CONCLUDING REMARKS

We have addressed a simple iterative algorithm for mixed H_2/H_∞ static output control problems. The mixed H_2/H_∞ problem was transformed to a new type of rank-constrained LMI optimization problem, which were solved iteratively by the recently developed penalty function method. Numerical experiments showed promising results.

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