LOW SPEED SENSORLESS VARIABLE STRUCTURE CONTROL OF INDUCTION MOTOR

Karel Jezernik¹, Gregor Edelbaher¹, Asif Šabanović²

¹University of Maribor, Faculty of Electrical Engineering and Computer Science, Smetanova ul. 17, SI-2000 Maribor, Slovenia E-mail: <u>karel.jezernik@uni-mb.si</u>, <u>gregor.edelbaher@uni-mb.si</u>

²Sabanci University, Faculty of Engineering and Natural Sciences Orhanli, 34956 Tuzla-Istanbul, Turkey E-mail: <u>asif@sabanciuniv.edu.tr</u>

Abstract: Torque and speed sensorless induction motor control is presented in this paper. The idea is realized using a sliding mode closed loop rotor flux observer for estimation of electromotive force of machine. This signal is then used in nonlinear stator flux and torque control of induction motor and in rotor flux observer for speed and flux estimation. The analysis of the proposed method is included. Proposed control scheme was implemented on DSP system extended with FPGA where PWM procedure with dead time compensation was realized. Experimental results demonstrated high efficiency of the proposed estimation and control method. *Copyright* © 2005 IFAC

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1. INTRODUCTION

Vector controlled induction motor drives without speed sensor have become an attractive and commercially expanding technology in the past few years (Rajashekara, 1996; Vas, 1998). However, the performance of the drive is still much inferior to that of the sensored drive. In particular, at low and zero speed stator frequency, the torque controllability of the drive is still far from satisfactory. At medium and high stator frequency, simple direct vector control method based on the integration of the stator terminal voltages satisfactory gives toraue control performance. Most of the sensorless drive algorithms are based on the assumption of d-q equivalent circuits of the induction machine and, hence, they are dependent on the machine parameters and measurements errors. At low stator frequency region,

signal-to-noise ratio of the stator voltage measurement is very poor and stator voltage drop is dominant. At zero stator frequency, even theoretically no motor dynamics can be recognized at the stator terminals (Holtz, 1996). For these reasons, the sensorless algorithm based on d-q circuits fails at low and zero stator frequency region, no matter how superior is the control algorithm. Another group of sensorless algorithms based on parasitic effects, such as rotor slot harmonics (Zinger et al., 1988), tracking of rotor saliencies (Holtz, 1998) or high frequency signal injection (Degner et al, 1997). These methods could solve the zero frequency problems in principle, but not without penalties. There is the requirement of additional hardware for signal acquisition, and/or the high computational load. Under these circumstances is the restriction of using the fundamental model of the induction machine extremely attractive. This

model considers only the fundamental spatial distribution of the flux density and the current density waves, thus describing the electromagnetic subsystem of machine as a dynamic system of second order.

In this paper by combining the variable structure system and Lyapunov design (Utkin, 1992) a novel sliding mode algorithm of controller/observer for induction motor is developed. This control method is based on estimation of an extended electromotive force (EMF) and is due to use of sliding mode principle robust against variation of load torque, machine parameters and external disturbances. For both, controller and observer, there are used nonlinear control principles, namely estimated EMF and machine terminal voltage built a nonlinear feedforward control, sliding mode principle based on state variable errors are used as feedback to guarantee stability of control system. The proposed method is investigated and verified experimentally.

2. DYNAMIC MODEL OF INDUCTION MOTOR

2. 1 Machine Dynamics

Control of induction motor (IM) is still a challenging problem due to its nonlinear dynamics, limited possibility to measure or estimate necessary state variables and presence of the switching converter with its own nonlinearity as a power modulator in control loop. The dynamics of IM consist of mechanical motion (1), dynamics of stator electromagnetic system (2) and the dynamics of the rotor electromagnetic system (3):

$$\frac{d\omega_r}{dt} = \frac{1}{J} \left(T_e - T_L \right), \tag{1}$$

$$\frac{d\boldsymbol{i}_{s}^{s}}{dt} = \frac{1}{\sigma L_{s}} \left(\boldsymbol{u}_{s}^{s} - R_{s} \boldsymbol{i}_{s}^{s} - \frac{L_{m}}{L_{r}} \frac{d\boldsymbol{\Psi}_{r}^{s}}{dt} \right), \qquad (2)$$

$$\frac{d\Psi_r^s}{dt} = R_r \frac{L_m}{L_r} \boldsymbol{i}_s^s + \begin{vmatrix} -\frac{R_r}{L_r} & -p\omega_r \\ p\omega_r & -\frac{R_r}{L_r} \end{vmatrix} \Psi_r^s, \quad (3)$$

$$T_e = \frac{2}{3} p \frac{L_m}{L_r} \left| \mathbf{\Psi}_r^s \times \mathbf{i}_s^s \right|,\tag{4}$$

where ω_r is mechanical rotor angle speed, the two dimensional complex space vectors $\boldsymbol{\Psi}_s^s = \begin{bmatrix} \boldsymbol{\Psi}_{sa}^s, \boldsymbol{\Psi}_{sb}^s \end{bmatrix}^T$, $\boldsymbol{\Psi}_r^s = \begin{bmatrix} \boldsymbol{\Psi}_{ra}^s, \boldsymbol{\Psi}_{rb}^s \end{bmatrix}^T$, $\boldsymbol{u}_s^s = \begin{bmatrix} u_{sa}^s, u_{sb}^s \end{bmatrix}^T$, $\boldsymbol{i}_s^s = \begin{bmatrix} i_{sa}^s, i_{sb}^s \end{bmatrix}^T$ are stator and rotor flux, stator voltage and current, respectively, T_e is motor torque, T_L is load torque, J is inertia of the rotor and p is the number of pole pairs. One of the most important issue in implementing direct torque control (DTC) or field oriented control (FOC) strategies for IM is to obtain real-time instantaneous flux level and position with sufficient accuracy for the entire speed range, from almost standstill to high speed level. The difficulty in flux estimation lies with the non-linear induction machine model, which is characterised by speed dependent and time varying parameters. In order to illustrate this non-linear behaviour of IM control let us express the derivation of developed electrical torque of IM from (4). This yields for torque variation:

$$\frac{dT_e}{dt} + \left(\frac{R_s}{\sigma L_s} + \frac{R_r}{\sigma L_r}\right) T_e =$$

$$= \frac{2}{3} \frac{p}{\sigma L_s} \frac{L_m}{L_r} \left(\mathbf{\Psi}_r^s \times \mathbf{u}_s^s - p \omega_r \mathbf{\Psi}_s^s \cdot \mathbf{\Psi}_r^s \right), \qquad (5)$$

where \times indicates cross product and • indicates dot product. It can be recognised from (5), that torque variation is the sum of two terms. The first term depends on the stator (R_s) and rotor (R_s) resistance and reduces the absolute value of the torque (T_e). The second term represents the effect of the applied control voltage vector (u_s^s) on the torque and is dependent on the operating condition of IM. It can be noted that some PWM voltage vectors may cause positive torque variation at low dynamic EMF value and negative torque variation at high value of back induced voltage.

The crucial point in control of IM is to make the electromagnetic torque and the flux of IM independently controllable. Similarly to torque variation, the rotor flux variation can be described from (2) and (3) as

$$\frac{d}{dt}\left|\boldsymbol{\Psi}_{r}^{s}\right| = \frac{L_{r}}{L_{m}}\frac{1}{\left|\boldsymbol{\Psi}_{r}^{s}\right|}\left(\boldsymbol{\Psi}_{r}^{s}\boldsymbol{\cdot}\boldsymbol{u}_{s}^{s}-R_{s}\boldsymbol{i}_{s}^{s}\boldsymbol{\cdot}\boldsymbol{\Psi}_{r}^{s}-\boldsymbol{\sigma}L_{s}\frac{d\boldsymbol{i}_{s}^{s}}{dt}\boldsymbol{\cdot}\boldsymbol{\Psi}_{s}^{s}\right).(6)$$

The variation of the rotor flux is determined with the dot product between rotor flux and applied input voltage vector and depends mostly on stator parameters variation R_s , σL_s . Both torque and flux variations are highly nonlinear in applied control voltage \boldsymbol{u}_{s}^{s} regarding IM rotor flux of machine $\boldsymbol{\Psi}_{r}^{s}$. The conventional control method of IM is based in case of FOC on simplification of rotor flux components $\boldsymbol{\Psi}_{r}^{s} = \left[\boldsymbol{\Psi}_{rd}, \boldsymbol{\Psi}_{rq}\right]^{T} = \left[\boldsymbol{\Psi}_{rd}, \boldsymbol{0}\right]^{T}$. The DTC method replace the IM coupling with hysteresis control. In real IM control rotor flux in q-axis will not be zero and FOC method is due variations of mostly parameters, inappropriate in sensorless drives applications. The DTC method is in principle speed sensorless, but due to use of voltage stator model of IM and approximation of stator resistance $R_s \sim 0$ in the flux model, current and torque variation by low

speed are slightly higher then in case of nominal speed (Buja et al., 2004; Leonhard, 2001).

2.2 Control Procedure

The main purpose of this research work is to apply the VSC combined with Lyapunov design approach to the torque and flux control of IM in sensorless operation. VSC are originally defined for dynamic systems defined by ordinary differential equation with discontinuous right hand side. In such a system so-called sliding mode motion can result. This motion is represented by the state trajectories in the sliding mode manifold and high frequency changes in the control. The fact that motion belongs to certain manifolds in state space with a dimension lower than that of the system results in the motion equation order reduction. This enables simplification and decoupling design procedure. For sliding mode application the equations of motion and the existence conditions are two basic questions to be defined.

The design of sliding mode system consists generally of two procedures: design of the switching surface and design of the sliding mode controller (Utkin, 1993a). The switching surface is designed to obtain a design performance for the system output variables. In VSS control, the goal is to keep the system motion on the manifold S, which is defined

$$\boldsymbol{S} = \left\{ \boldsymbol{y} : \boldsymbol{\sigma}(\boldsymbol{y}, t) = \boldsymbol{G} \, \boldsymbol{y} \right\} = \boldsymbol{\theta}; \quad \boldsymbol{\sigma} = \boldsymbol{y}^d - \boldsymbol{y} \,, \quad (7)$$

where y^d , y are state variables of desired and estimated value and σ is control error.

The "chattering free" sliding mode control (Šabanović et al., 1994) should be chosen such that the candidate Lyapunov function satisfies the Lyapunov stability criteria. This can be assured for

$$V = -\boldsymbol{\sigma}^T \, \boldsymbol{\sigma} / 2$$
 and $\dot{V} = \boldsymbol{\sigma}^T \, \dot{\boldsymbol{\sigma}}$ (8)

if the following inequality can be made true

$$\dot{V} = -\boldsymbol{\sigma}^T \boldsymbol{D} \boldsymbol{\sigma} < 0 , \qquad (9)$$

where **D** is positive definite matrix.

Therefore (8), (9) satisfy the Lyapunov conditions. From (8) and (9) reaching condition ($\dot{\sigma} = -D\sigma$) can be got. With the selected Lyapunov function the stability of the whole control system in the case of the initial conditions and parameter mismatch is guaranteed. The control function will satisfy ideal reaching conditions in the following form

$$\boldsymbol{u}(t) = \boldsymbol{u}_{eq} + (\boldsymbol{G}\boldsymbol{B})^{-1}\boldsymbol{D}\boldsymbol{\sigma}.$$
(10)

 u_{eq} can be expressed with

$$\boldsymbol{u}_{eq} = \boldsymbol{u}(t)^{-} + (\boldsymbol{G}\boldsymbol{B})^{-1} \dot{\boldsymbol{\sigma}} , \qquad (11)$$

and control input will be

$$\boldsymbol{u}(t) = \boldsymbol{u}(t)^{-} + (\boldsymbol{G}\boldsymbol{B})^{-1} (\boldsymbol{D}\boldsymbol{\sigma} + \dot{\boldsymbol{\sigma}}); t = t^{-} + \Delta, \Delta \rightarrow 0.(12)$$

Using the fact that u_{eq} is a continuous function, the approximation of control error $\sigma(y^d, y)$ can be written in discrete-time term after applying Euler's approximation

$$\frac{\boldsymbol{\sigma}((k+1)T) - \boldsymbol{\sigma}(k)}{T} \doteq \boldsymbol{G}\boldsymbol{B}\left(\boldsymbol{u}_{eq}(kT) - \boldsymbol{u}(kT)\right). (13)$$

Here *T* is the sampling time and $k = 2^+$. By discretizing (10) and combining it with (13) one can eliminate the $u_{eq}(kT)$ from (10) and arrive to the following discrete-time expression of the control (14)

$$\boldsymbol{u}_{s}(k+1) = \boldsymbol{u}_{s}(k) + \left(\boldsymbol{G}\boldsymbol{B}\boldsymbol{T}\right)^{-1} \left(\left(\boldsymbol{I} + T\boldsymbol{D}\right)\boldsymbol{\sigma}(k) - \boldsymbol{\sigma}(k-1) \right)^{(14)}$$

Proposed algorithm ensures the sliding mode existence in manifold (7) and thus ensures the robustness of the closed loop system behaviour against external disturbances and parameters' changes. The control algorithm (14) used the feedforward term expressed by $u_s(k)$, and feedback term determined with control error dynamics. The time delay which appears in discrete-time control algorithm due measurement and computation process will be compensated with rotational matrix $C(\Delta \theta_r)$ and :

$$\boldsymbol{u}_{s}(k+1) = \boldsymbol{C} \left(\Delta \boldsymbol{\theta}_{r} \right) \boldsymbol{u}_{s}(k) + \left(\boldsymbol{G} \, \boldsymbol{B} \, T \right)^{-1} \\ \left(\left(\boldsymbol{I} + T \boldsymbol{D} \right) \boldsymbol{C} \left(\Delta \boldsymbol{\theta}_{r} \right) \boldsymbol{\sigma}(k) - \boldsymbol{C} \left(2 \Delta \boldsymbol{\theta}_{r} \right) \boldsymbol{\sigma}(k-1) \right)^{-1} \right).$$
(15)

System is asymptotically stable and theoretically will reach the sliding manifold S in infinite time, but $\varepsilon = 0(\Delta)$ -vicinity of the manifold is reached in finite time.

3. PROPOSED VSC TORQUE AND FLUX CONTROL SCHEME

Specially, based on the state equations for torque (5) and rotor flux variation (6), a corresponding sliding mode torque controller to guarantee the asymptotic stability of both the sliding mode electromotive force (EMF) flux observer and the torque tracking controller, is to be designed. The additional goal of control of IM is to make the flux track the reference flux input. In this research, the "physical" coupling

between observer and controller is realised with the common signal of EMF in nonlinear control. From the above discussion and based on the state equations (5) and (6), sliding mode surfaces for torque and flux can be defined

$$\sigma_T = T_e^d - \hat{T}_e \,, \tag{16}$$

$$\sigma_{\Psi} = \left\| \boldsymbol{\Psi}_{r}^{d} \right\| - \left\| \hat{\boldsymbol{\Psi}}_{r}^{s} \right\|, \tag{17}$$

where T_e^d , Ψ_r^d are reference torque and rotor flux. The design task is reduced to enforcing sliding mode in the manifolds $\boldsymbol{\sigma} = \boldsymbol{\theta}$, $\boldsymbol{\sigma} = [\boldsymbol{\sigma}_T, \boldsymbol{\sigma}_{\Psi}]^T$ in d-q plane with control in stator fixed plane. Equations of the controller motion projection on the subspace $\boldsymbol{\sigma} = [\boldsymbol{\sigma}_T, \boldsymbol{\sigma}_{\Psi}]^T$ can be as

$$\dot{\sigma}_T = f_T - \frac{2}{3} \frac{L_m}{L_r} \frac{p}{\sigma L_s} \left(\boldsymbol{\Psi}_r^s \times \boldsymbol{u}_s^s \right), \qquad (18)$$

$$\dot{\boldsymbol{\sigma}}_{\boldsymbol{\Psi}} = f_{\boldsymbol{\Psi}} - \frac{L_r}{L_m} \frac{1}{\left\| \hat{\boldsymbol{\Psi}}_r^s \right\|} \left(\hat{\boldsymbol{\Psi}}_r^s \cdot \boldsymbol{u}_s^s \right), \tag{19}$$

where f_T, f_{Ψ} are continuous state functions.

Ideal control input u_{eq} could be computed from $\dot{\sigma}_T = 0$ and $\dot{\sigma}_{\psi} = 0$ (Utkin, 1993b). In proposed control of IM the u_{eq} is approximated with (11). The error dynamics is described with closed loop feedback control (12). In proposed discrete-time control scheme the feedforward term $u_s(k)$ (15) will be replaced with estimated EMF from rotor flux observer (\hat{e}_r). On this way there will be established observer/controller connection in the nonlinear feedforward/feedback robust control of IM.

The control input voltage vector (15) is evaluated in the following form

$$\boldsymbol{u}(k+1) = \boldsymbol{C} \left(\Delta \boldsymbol{\theta}_r \right) \hat{\boldsymbol{e}}_r(k) + \boldsymbol{K}_s \frac{\sigma L_s}{T}, \qquad (20)$$
$$\left(\left(\boldsymbol{I} + T \boldsymbol{D} \right) \boldsymbol{C} \left(\Delta \boldsymbol{\theta}_r \right) \boldsymbol{\sigma}(k) - \boldsymbol{C} \left(2\Delta \boldsymbol{\theta}_r \right) \boldsymbol{\sigma}(k-1) \right)$$

where $\boldsymbol{\sigma} = \begin{bmatrix} \sigma_T, \sigma_{\Psi} \end{bmatrix}^T$ are control torque / flux error and $\boldsymbol{K}_s = \begin{bmatrix} K_{sT}, K_{s\Psi} \end{bmatrix}^T$ is torque / flux feedback gain. In proposed nonlinear VSC scheme sliding mode can occur on manifolds $\sigma_T = 0$ and $\sigma_{\Psi} = 0$. The estimated torque and the magnitude of the rotor flux converge to the reference values: $\sigma_T = 0$ means $T_e^d = \hat{T}_e$ and $\sigma_{\Psi} = 0$ means $\Psi_r^d = |\hat{\Psi}_r|$, both tends to zero exponentially. Torque / flux error $(\sigma_T, \sigma_{\Psi})$ are then transformed from d-q to stator fixed a-b coordinate, where stator voltage input vector is computed in (20).

$$\begin{bmatrix} \sigma_{sa} \\ \sigma_{sb} \end{bmatrix} = \begin{bmatrix} \cos \hat{\theta}_r & -\sin \hat{\theta}_r \\ \sin \hat{\theta}_r & \cos \hat{\theta}_r \end{bmatrix} \begin{bmatrix} \sigma_T \\ \sigma_{\Psi} \end{bmatrix}.$$
(21)

The proposed control scheme is presented on Fig. 1.

4. SLIDING MODE FLUX AND SPEED OBSERVER

Design of an IM sensorless drive is still a challenge. The basic problem is speed and flux estimation especially at the low speed range and under light load conditions. In this section the sliding mode approach to rotor flux and speed estimation of an IM will be presented (Fig. 2).

Stator current and rotor flux observer is based on the voltage and current model of the IM. The derivation of the estimated stator current \hat{i}_{s}^{s} is based on the voltage model (2)

$$\frac{d\hat{\boldsymbol{i}}_{s}^{s}}{dt} + \frac{R_{s}}{\sigma L_{s}}\hat{\boldsymbol{i}}_{s}^{s} = \frac{1}{\sigma L_{s}}\boldsymbol{u}_{s}^{s} - \frac{1}{\sigma L_{s}}\frac{L_{m}}{L_{r}}\frac{d\hat{\boldsymbol{\Psi}}_{r}^{s}}{dt} \quad (22)$$



Fig.1. Proposed control scheme.



Fig.2. Closed-loop rotor flux observer.

and the derivation of the estimated rotor flux $\hat{\Psi}_r^s$ is based on the current model (3). To avoid problems connected with the use of open integrator in control scheme stator current expression with states Ψ_s^s and Ψ_r^s is introduced as

$$\boldsymbol{i}_{s}^{s} = \frac{1}{\sigma L_{s}} \left(\boldsymbol{\Psi}_{s}^{s} - \frac{L_{m}}{L_{r}} \boldsymbol{\Psi}_{r}^{s} \right).$$
(23)

The estimated rotor flux will be with (23) as

$$\frac{d\hat{\boldsymbol{\Psi}}_{r}^{s}}{dt} = R_{r} \frac{L_{r}}{L_{m}} \boldsymbol{\Psi}_{s}^{s} + \left(\frac{R_{r}}{\sigma L_{s}}\boldsymbol{I} + p\hat{\boldsymbol{\omega}}_{r}\boldsymbol{J}\right) \hat{\boldsymbol{\Psi}}_{r}^{s} + \boldsymbol{\xi}_{\boldsymbol{\Psi}},$$
(24)

with additional term ξ_{ψ}

$$\boldsymbol{\xi}_{\psi} = K_{\psi} \left((\boldsymbol{i}_{s}^{s} - \hat{\boldsymbol{i}}_{s}^{s}) \cdot \hat{\boldsymbol{\Psi}}_{r}^{s} + (\boldsymbol{i}_{s}^{s} - \hat{\boldsymbol{i}}_{s}^{s}) \times \hat{\boldsymbol{\Psi}}_{r}^{s} \right) \hat{\boldsymbol{\Psi}}_{r}^{s}, \quad (25)$$

which in observer theory will assure the asymptotic stability of rotor flux observer. The rotor flux model of IM in (24) is needed information about speed, so in speed sensorless control of IM rotor flux and mechanical speed have to be estimated. In this paper a novel method have been proposed for IM sensorless control based on estimation of electromotive force \hat{e}_s^s in which motors mechanical speed is included

$$\hat{\boldsymbol{e}}_{s}^{s} = \left(\frac{R_{r}}{\sigma L_{s}}\boldsymbol{I} + p\hat{\boldsymbol{\omega}}_{r}\boldsymbol{J}\right)\hat{\boldsymbol{\Psi}}_{r}^{s}.$$
(26)

Stator current estimation error $\boldsymbol{\varepsilon}_i = \boldsymbol{i}_s^s - \hat{\boldsymbol{i}}_s^s$ is expressed as

$$\frac{d\boldsymbol{\varepsilon}_{i}}{dt} = \frac{d(\boldsymbol{i}_{s}^{s} - \boldsymbol{\hat{i}}_{s}^{s})}{dt} = \frac{1}{\sigma L_{s}} \left(\frac{L_{m}}{L_{r}} \left(\boldsymbol{e}_{s}^{s} - \boldsymbol{\hat{e}}_{s}^{s} \right) - \left(R_{s} + R_{r} \left(\frac{L_{m}}{L_{r}} \right)^{2} \right) \boldsymbol{\varepsilon}_{i} \right)^{2}.$$
(27)

Sliding mode algorithm could be used to calculate control input, which has meaning of EMF

$$\hat{\boldsymbol{e}}_{s}^{s}(k+1) = \hat{\boldsymbol{e}}_{s}^{s}(k) + K_{e} \frac{\sigma L_{s}}{T} \frac{L_{m}}{L_{r}} \left(\left(\boldsymbol{I} + T\boldsymbol{D}_{e} \right) \boldsymbol{\varepsilon}_{i}(k) - \boldsymbol{\varepsilon}_{i}(k-1) \right)^{.}$$
(28)

The process of zeroing the current error utilizing torque regulated PWM is the essence of sliding mode flux and speed observer. The proposed rotor flux observer is based on principle of nonlinear feedforward Ψ_s^d and feedback \hat{e}_s^s control and is

presented in Fig. 2. The rotor flux estimation error is calculated as

$$\frac{d(\boldsymbol{\Psi}_{r}^{s}-\hat{\boldsymbol{\Psi}}_{r}^{s})}{dt} = \frac{R_{r}}{\sigma L_{s}} \frac{L_{m}}{L_{r}} (\boldsymbol{\Psi}_{s}^{s}-\boldsymbol{\Psi}_{s}^{d}) - (\boldsymbol{e}_{s}^{s}-\hat{\boldsymbol{e}}_{s}^{s}) - \boldsymbol{K}_{\boldsymbol{\Psi}}\boldsymbol{\xi}_{\boldsymbol{\Psi}} .$$
(29)

Design parameter K_{ψ} could be selected from (25) so that the estimated rotor flux tends to its real value. The full EMF $\hat{e}_r(k)$, which is effective in stator current control as feedforward term is now

$$\hat{\boldsymbol{e}}_{r}(k) = \frac{d\hat{\boldsymbol{\Psi}}_{r}^{s}}{dt} = \frac{L_{m}}{L_{s}} \frac{R_{r}}{\sigma L_{r}} \boldsymbol{\Psi}_{s}^{d} - \hat{\boldsymbol{e}}_{s}^{s} + \boldsymbol{K}_{\psi} \boldsymbol{\xi}_{\psi} \,. \quad (30)$$

The stator flux reference Ψ_s^d is calculated from the rotor flux reference Ψ_r^d and the torque reference as

$$\Psi_s^d = \sqrt{\left(\frac{L_s}{L_m}\Psi_r^d\right)^2 + \left(\frac{2}{3}\frac{\sigma L_s}{pL_m}\frac{T_e^d}{\Psi_r^d}\right)^2} .$$
 (31)

The dynamic behaviour of estimated rotor flux $\hat{\psi}_r(s)$ can be with transfer function

$$\hat{\Psi}_{r}^{s}(s) = \frac{\frac{R_{r}}{\sigma L_{s}} \frac{L_{m}}{L_{r}} \left(s + \frac{1}{\sigma L_{s}} \left(R_{s} + R_{r} \left(\frac{L_{m}}{L_{r}} \right)^{2} + \frac{L_{m}}{L_{r}} Ce(s) \right) \right)}{s^{2} + s \left(\frac{1}{\sigma L_{s}} \left(R_{s} + R_{r} \left(\frac{L_{m}}{L_{r}} \right)^{2} + \frac{L_{m}}{L_{r}} Ce(s) \right) \right) + \frac{R_{r}}{(\sigma L_{s})^{2}} \left(\frac{L_{m}}{L_{r}} \right)^{3} Ce(s)} \Psi_{s}^{s}(s) - \frac{Ce(s) \left(s + \frac{1}{\sigma L_{s}} \left(R_{s} + R_{r} \left(\frac{L_{m}}{L_{r}} \right)^{2} \right) \right) \right)}{s^{2} + s \left(\frac{1}{\sigma L_{s}} \left(R_{s} + R_{r} \left(\frac{L_{m}}{L_{r}} \right)^{2} + \frac{L_{m}}{L_{r}} Ce(s) \right) \right) + \frac{R_{r}}{(\sigma L_{s})^{2}} \left(\frac{L_{m}}{L_{r}} \right)^{3} Ce(s)} I_{s}^{s}(s) + \frac{1}{s^{2} + s \left(\frac{1}{\sigma L_{s}} \left(R_{s} + R_{r} \left(\frac{L_{m}}{L_{r}} \right)^{2} + \frac{L_{m}}{L_{r}} Ce(s) \right) \right) + \frac{R_{r}}{(\sigma L_{s})^{2}} \left(\frac{L_{m}}{L_{r}} \right)^{3} Ce(s)} U_{s}^{s}(s) + \frac{1}{s^{2} + s \left(\frac{1}{\sigma L_{s}} \left(R_{s} + R_{r} \left(\frac{L_{m}}{L_{r}} \right)^{2} + \frac{L_{m}}{L_{r}} Ce(s) \right) \right) + \frac{R_{r}}{(\sigma L_{s})^{2}} \left(\frac{L_{m}}{L_{r}} \right)^{3} Ce(s)} U_{s}^{s}(s)$$

Estimated synchronous speed will be now expressed with cross product

$$\hat{\omega}_{s} = \frac{1}{\left|\hat{\Psi}_{s}^{s}\right|^{2}} \frac{d\hat{\Psi}_{s}^{s}}{dt} \times \hat{\Psi}_{s}^{s}$$
(33)

and the mechanical rotor speed will be

$$\hat{\omega}_r = \frac{1}{p} \frac{1}{\left|\hat{\Psi}_r^s\right|^2} \hat{e}_r^s \times \hat{\Psi}_r^s \,. \tag{34}$$

6. EXPERIMENTAL RESULTS

Experiments were obtained using motor controller board based on floating point DSP TMS320C32 from Texas Instruments. All measured and controller internal variables are accessible through the serial link to the PC, where graphical data analysis software can be performed. The sampling time of the measurements and computation of control algorithm are both $200\mu s$. Brushless AC servomotor mechanical connected to the IM under test was used as the Load Machine. The control of both, speed and applied torque, is possible, thus hardware-in-the-loop operation can also be performed.

The performance of the proposed control system with closed loop rotor flux observer is presented with low speed no-load slow reversing experiment, shown on Fig. 3 and Fig. 4. Desired and estimated rotor speed at low speed reversing from -3rad/s to +3rad/s is shown on Fig. 3a, whereas on Fig. 3b speed estimation error is shown. As it can be seen from Fig. 3, transition through zero speed succeeds, some problems occur during speed reversal, but the system remains stable even in such conditions. Stator current in stator reference frame is featured in Fig. 4 and the sinusoidal behaviour can be assumed from the circular shapes. Experiments at low speed reversing were presented due to the fact that sensorless control presents small problems at high-speed operation, however no load low speed operation, due to the low values of EMF is very difficult.



Fig. 3. Slow reversing; (a) desired and estimated rotor speed, (b) speed estimation error.



Fig. 4. Stator current locus.

7. CONCLUSION

Sensorless sliding mode torque and flux control of induction motor is an emerging new technology, though in the early state of development. It allows the precise and quick control of IM flux and torque only with the use of stator voltages and currents. Combining VSS and Lyapunov design has developed a new discrete-time control algorithm. It possesses all the good properties of the sliding mode and eliminates the undesired chattering of control input. An algorithm for torque and flux tracking control is presented in the paper. The algorithm is aimed to solve practical problems of operation at low and high speed of machine including zero speed. This nonlinear control algorithm is suitable for the applications where desired torque and rotor flux are varying during the operation, for example when efficiency of the machine operation is an important issue. The performance of the proposed algorithm was verified by simulation and experiments.

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