OPTIMAL INPUTS FOR GUARANTEED SYSTEM IDENTIFICATION

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Abstract: The problem of choice of optimal inputs for control system parameters identification is studied. The uncertain items are assumed to be unknown but bounded with no statistical information whatever. The problem is treated in the framework of guaranteed (set membership) approach. An integral of information function is considered as a criterion of optimality.

Keywords: Identification, Uncertain dynamic systems, Bounded disturbances, Inputs

1. INTRODUCTION

Optimal input choice for control system parameter estimation is a typical example of the experiment design problem for dynamic systems. The goal of input design is to get maximal information about system parameters from available observations. The conventional approach to the experiments design is based on stochastic models for uncertain parameters and measurement errors and uses statistical criteria as the measures for amount of extracted information (Mehra, 1974). Within the framework of guaranteed (set-membership) approach a quality of estimation may be characterized by a quantity of some functional defined on information (feasible) sets of parameters, consistent with the system equations, measurements, and a priori constraints on the uncertain items. The design of optimal inputs turns here into an optimization problem for information sets (Kurzhanski *et al.*, 1991, Pokotilo, 1991, Gusev, 1992).

The information sets may be described as the level sets for so-called information function (Baras and Kurzhanski, 1995). An information function is defined as a value function for a certain auxiliary optimal control problem. This permits to implement the optimal control methods, especially the dynamic programming methods, for description of information sets (Kurzhanski and Valyi, 1997).

In this article the problem of choice of optimal inputs for set-membership nonlinear system identification is considered. The integral of information function over the set of a priori constraints on parameters is considered as a criterion of optimality. It is shown that considered problem may be reduced to an optimal control problem with functional depending on the integral over parameters. The necessary conditions of optimality for this problem are given, and the results of numerical simulation are described.

2. PROBLEM STATEMENT

Consider the following control system

$$\dot{x} = f(t, q, x, u(t)), \quad t \in [t_0, t_1], \; x(t_0) = x^0, \quad (1)$$

($x \in R^n, \; u \in R^r$) with the right-hand side f depending on unknown parameter $q \in R^m$. We assume that all a priori information on q is given by the inclusion

$$q \in Q, \tag{2}$$

where Q is a compact set in R^m. As an admissible control (input) we will consider a Lebesque-measurable function $u : [t_0, t_1] \to U$, where $U \subset R^r$.

The function $f(t, q, x, u)$ is assumed to be continuous together with its derivatives in x on the set $[t_0, t_1] \times Q \times R^n \times U$. Let for every $q \in Q$ and admissible control $u(t)$ there exists an absolutely continuous vector-function $x(t)$ such that equality (1) is fulfilled a.e. $t \in [t_0, t_1]$; it will be denoted as $x(t, q)$ (or $x(t, q, u(\cdot))$.

Consider the measurement equation on $[t_0, t_1]$ to be specified by the equality

$$y(t) = g(t, x(t)) + \xi(t), \quad t \in [t_0, t_1], \tag{3}$$

where $g : [t_0, t_1] \times R^n \times U \to R^k$ is a continuous function, $\xi(t)$ is a measurement error. A priori information on $\xi(t)$ is assumed to be given by the inclusion

$$\xi(\cdot) \in \Xi, \tag{4}$$

where Ξ is a bounded set in the space $L_2^k[t_0, t_1]$. Suppose that

$$\Xi = \{\xi(\cdot) : W(\xi(\cdot)) \le 1\},$$

with W to be a nonnegative functional on $L_2^k[t_0, t_1]$. The case of

$$W(\xi(\cdot)) = \langle \xi(\cdot), \xi(\cdot) \rangle := \int_{t_0}^{t_1} \xi^\top(t) R \xi(t) dt$$

corresponds to integral (soft) constraints, in the case

$$W(\xi(\cdot)) = \alpha \operatorname{esssup}\{\|\xi(t)\|, \quad t \in [t_0, t_1]\},$$

we have the problem with magnitude (hard) constraints. Here R is a given positively defined matrix, α is a positive number. Let $y(t)$ be the result of measurements, generated by unknown "true" value of $q^* \in Q$, input $u(t)$, and measurement error $\xi(t)$. The function $q \to V(q, y(\cdot), u(\cdot))$, defined by the equality

$$V(q, y(\cdot), u(\cdot)) = W(y(\cdot) - g(\cdot, x(\cdot, q)))$$

is said to be an information function of the problem (1),(3). The set $Q(y(\cdot), u(\cdot))$ of all parameters $q \in Q$ that are consistent with (1), (3) and a priori constraints is referred to as the information set relative to measurement $y(t)$ (Kurzhanski, 1977). It follows directly from definitions that

$$Q(y(\cdot), u(\cdot)) = Q \bigcap \{q : V(q, y(\cdot), u(\cdot)) \le 1\}.$$

Unknown q^* belongs to the information set.

The error of estimation of q^* may be characterized by the values of some functionals of $Q(y(\cdot), u(\cdot))$, such as radius, diameter, volume etc. Note that construction of information sets and calculating theirs characteristics constitute a difficult problem, especially in the case of nonlinear systems (see, e.g., Veres and Norton, 1996, Kumkov *et.al*, 2000). Instead, we shall consider an integral of information function as a functional of the problem

$$I(y(\cdot), u(\cdot)) = \int_Q V(q, y(\cdot), u(\cdot)) d\mu(q). \tag{5}$$

Here μ is a nonnegative measure defined on Lebesque subsets of Q such that $\mu(Q) = 1$. If Q has nonzero Lebesque measure, then μ may be taken as follows

$$\mu(E) = \int_E \alpha(q) dq,$$

where α is a nonnegative function such that $\mu(Q) = 1$. Another example is a measure μ with a finite support set $S \subset Q$.

The functional I is nonnegative, the most value of I corresponds to a more accurate estimate of unknown quantity of parameter q.

The information function is defined not uniquely. If $\phi(x)$ is a monotonically increasing nonnegative function of $x \ge 0$ such that $\phi(1) = 1$, then $V_1(q, y(\cdot), u(\cdot)) = \phi(V(q, y(\cdot), u(\cdot)))$ may also be considered as an information function. The function $\phi(x)$ may be chosen so that maximization of the integral of V_1 will be approximately equivalent to minimization of the measure of the information set.

The integral (5) depends on $u(\cdot)$ and the result of measurements $y(\cdot)$. In turn, $y(t) = y^*(t) + \xi(t)$, where $y^*(t) = g(t, x(t, q^*, u(\cdot)))$ and $\xi(t)$ is the measurement error. In the worst case, the value of I is equal to

$$J(u(\cdot)) = \inf_{W(\xi(\cdot)) \le 1} \int_Q V(q, y(\cdot) + \xi(\cdot), u(\cdot)) d\mu(q).$$

For the case of integral constrains the last minimum may be specified directly. At fact, define the inner product in $L_2^k[t_0, t_1]$, assuming

$$\langle \xi(\cdot), \eta(\cdot) \rangle = \int_{t_0}^{t_1} \xi^\top(t) R \eta(t) dt = \int_{t_0}^{t_1} (\xi(t), \eta(t))_R dt,$$

then

$$J(u(\cdot)) = \inf_{\langle \xi(\cdot), \xi(\cdot) \rangle \le 1} \int_Q \int_{t_0}^{t_1} \Phi(t, \xi(t), q) dt d\mu(q),$$

where

$$\Phi(t, \xi, q) := (y^*(t) + \xi - \bar{g}(t, q), y^*(t) + \xi - \bar{g}(t, q))_R,$$

$\bar{g}(t, q) := g(t, x(t, q, u(\cdot)))$, $(x, y)_R := x^\top R y$, $x, y \in R^k$. Changing the order of integrating in t and q, we get

$$J(u(\cdot)) = \inf_{\langle \xi(\cdot), \xi(\cdot) \rangle \le 1} S(\xi(\cdot)).$$

Here

$$S(\xi(\cdot)) := c + 2\langle \xi(\cdot), b(\cdot) \rangle + \langle \xi(\cdot), \xi(\cdot) \rangle,$$

$$c = \int_{t_0}^{t_1} c(t)dt,$$

$$c(t) = \int_Q (y^*(t) - \bar{g}(t,q), y^*(t) - \bar{g}(t,q))_R d\mu(q),$$

$$b(t) = \int_Q (y^*(t) - \bar{g}(t,q))d\mu(q), \quad t \in [t_0, t_1].$$

Thus,

$$J(u(\cdot)) = \min_{0\le\alpha\le 1} \min_{\langle \xi(\cdot),\xi(\cdot)\rangle=\alpha^2} S(\xi(\cdot))$$

$$= \min_{0\le\alpha\le 1} \{c - 2\alpha\|b(\cdot)\| + \alpha^2\} = c + \phi(\|b(\cdot)\|^2),$$

where $\|b(\cdot)\| = \langle b(\cdot), b(\cdot)\rangle^{1/2}$, and continuously differentiable function $\phi(x)$ is defined as follows

$$\phi(x) = \begin{cases} -x & 0 \le x \le 1 \\ 1 - 2\sqrt{x} & x \ge 1. \end{cases}$$

Finally, we have

$$J(u(\cdot) = \Psi(I_1(u(\cdot)), I_2(u(\cdot))),$$

where $\Psi(z_1, z_2) = z_1 + \phi(z_2)$, and

$$I_1(u(\cdot))) = \int_{t_0}^{t_1}\int_Q r(t,q)^\top R r(t,q) d\mu(q)dt,$$

$$I_2(u(\cdot))) = \int_{t_0}^{t_1}\int_Q r(t,q)^\top d\mu(q) R \int_Q r(t,q)d\mu(q)dt,$$

$$r(t,q) = g(t, x(t,q^*)) - g(t, x(t,q)).$$

3. OPTIMAL CONTROL PROBLEM WITH FUNCTIONAL DEPENDING ON INTEGRAL OVER PARAMETERS

Consider the optimal control problem for the system (1) under above specified restrictions on q and $u(t)$. Let $F_i(t, x, y^i, u)$, $i = 1, ..., p$, be continuous functions with continuous derivatives in x, y^i on the set $[t_0, t_1] \times R^n \times R^{k_i} \times U$, $h_i : R^n \to R^{k_i}$, $i = 1, ..., p$ be continuously differential mappings. Assume the functional of the problem to be as follows

$$J(u(\cdot)) = \Psi(I_1, ..., I_p), \tag{6}$$

where $\Psi(z_1, ..., z_p)$ is a given function of p variables, and functionals $I_i = I_i(u(\cdot))$, $i = 1, ..., p$, are defined by the equalities

$$I_i = \int_{t_0}^{t_1} F_i(t, x(t,q^*), \int_Q h_i(t, x(t,q))d\mu(q), u(t))dt,$$

$q^* \in Q$ is given. Let $u(\cdot)$ be an admissible control, $x(t,q)$ be the solution of the system (1) corresponding to this control and a parameter q. Define by usual way a variation of the control function in a neighborhood of a regular point $\tau \in (t_0, t_1)$, assuming

$$u_\varepsilon(t) = \begin{cases} u(t) & t \notin (\tau - \varepsilon, \tau) \\ v & t \in (\tau - \varepsilon, \tau). \end{cases}$$

Here v is an arbitrary vector from U, ε is a sufficiently small positive number. By $x_\varepsilon(t,q)$ denote the solution of (1), corresponding to $u_\varepsilon(t)$. With q being fixed the trajectory variation under transition from $u(\cdot)$ to u_ε may be described by known equality

$$x_\varepsilon(t,q) = x(t,q) + \varepsilon y(t,q) + o_q(\varepsilon), \ t \ge \tau,$$

where $y(t,q)$ is a solution of the linear differential equation

$$\dot{y} = \frac{\partial f}{\partial x}(t, q, x(t,q), u(t))y$$

with boundary condition

$$y(\tau, q) = f(\tau, q, x(\tau,q), v) \\ -f(\tau, q, x(\tau, q), u(\tau)),$$

and $o_q(\varepsilon)/\varepsilon \to 0$ as $\varepsilon \to 0$, uniformly in $q \in Q$.

Then

$$\Delta I_i = I_i(u_\varepsilon(\cdot)) - I_i(u(\cdot))$$

$$= \varepsilon \left(w^i + \int_\tau^{t_1} \eta^{i\top}(t) y(t,q^*)dt \right.$$

$$\left. + \int_\tau^{t_1}\int_Q \zeta^{i\top}(t,q)y(t,q)d\mu(q)dt) + o_q(\varepsilon) \right), \tag{7}$$

where

$$w^i = F_i(\tau, x(\tau,q^*), p^i_\tau, v) - F_i(\tau, x(\tau,q^*), p^i_\tau, u(\tau)),$$

$$\zeta^i = \frac{\partial h_i^\top}{\partial x}(t, x(t,q)) \frac{\partial F_i}{\partial y^i}(t, x(t,q^*), p^i_t, u(t)), \tag{8}$$

$$\eta^i(t) = \frac{\partial F_i}{\partial x}(t, x(t,q^*), p^i_t, u(t)), \tag{9}$$

$$p^i_\tau = \int_Q h_i(\tau, x(\tau, q))d\mu(q).$$

Let $\psi(t, q, \theta(\cdot, q))$ be a solution of the following differential equation with the right hand side depending on parameter $q \in Q$, with the boundary condition $\psi(t_1, q) = 0$

$$\frac{d\psi}{dt} = -\frac{\partial f^\top}{\partial x}(t, q, x(t,q), u(t))\psi + \theta(t,q), \tag{10}$$

where $\theta(t,q)$ is a given vector-function, measurable and integrable in t and continuous in q . Then

$$\frac{d\psi^\top}{dt}(t, q, \theta(\cdot, q))y(t,q) = \theta^\top(t,q)y(t,q),$$

and taking into account the equality $\psi(t_1, q) = 0$ we get

$$\psi^\top(\tau, q, \theta(\cdot, q))y(\tau, q) = \int_\tau^{t_1} \theta^\top(t,q)y(t,q)dt. \tag{11}$$

Suppose that $u(\cdot)$ gives a minimum to J, then

$$\lim_{\varepsilon\to 0}\frac{J(u_\varepsilon(\cdot))-J(u(\cdot))}{\varepsilon}$$

$$=\sum_{i=1}^{p}\frac{\partial\Psi}{\partial z_i}\Bigg[w^i+\int_{\tau}^{t_1}\eta^{i\top}(t)y(t,q^*)dt$$

$$+\int_{\tau}^{t_1}\int_{Q}\zeta^{i\top}(t,q)y(t,q)d\mu(q)dt\Bigg]\geq 0. \qquad (12)$$

Using the equality (11) and the formula for w^i rewrite the last inequality as follows

$$\sum_{i=1}^{p}\frac{\partial\Psi}{\partial z_i}\big[F_i(\tau,x(\tau,q^*),p_t^i,v))-F_i(\tau,x(\tau,q^*),p_t^i,u(\tau))$$

$$-\psi^\top(\tau,q^*,\eta^i(\cdot))y(\tau,q^*)$$

$$-\int_{Q}\psi^\top(\tau,q,\zeta^i(\cdot,q))y(\tau,q)d\mu(q))\Bigg]\geq 0. \qquad (13)$$

Here derivatives $\partial\Psi/\partial z_i$ are calculated in the point $\bar z=(\bar z_1,...,\bar z_p)$ with coordinates

$$\bar z_i=\int_{t_0}^{t_1}F_i(t,x(t,q^*),p_t^i,u(t))dt,\ i=1,...,p.$$

Consider the following functional (a generalized Hamiltonian of the problem)

$$H(\tau,q,x,x(\cdot),\psi(\cdot),u)=-\sum_{i=1}^{p}\frac{\partial\Psi}{\partial z_i}(\bar z)F_i(\tau,x,p_\tau^i,u)$$

$$+\sum_{i=1}^{p}\frac{\partial\Psi}{\partial z_i}(\bar z)\psi^\top(\tau,q,\eta^i(\cdot))f(\tau,q,x,u)+\sum_{i=1}^{p}\frac{\partial\Psi}{\partial z_i}(\bar z)$$

$$\times\int_{Q}\psi^\top(\tau,q,\zeta^i(\cdot,q))f(\tau,q,x(\tau,q),u)d\mu(q).$$

In view of introduced designation, (13) implies the following equality for the optimal control of the problem (1),(6)

$$H(\tau,q^*,x(\tau,q^*),x(\cdot),\psi(\cdot),u(\tau))$$

$$=\max_{v\in U}H(\tau,q^*,x(\tau,q^*),x(\cdot),\psi(\cdot),v). \qquad (14)$$

Assuming functions f,F_i to be continuously differentiable in u, we get an expression for the gradient of the functional J. Let $\delta u(t)$ be a measurable bounded function on $[t_0,t_1]$. Then

$$\Delta J=J(u(\cdot)+\delta u(\cdot))-J(u(\cdot))$$

$$=\int_{t_0}^{t_1}\Bigg[\sum_{i=1}^{p}\frac{\partial\Psi}{\partial z_i}(\bar z)\frac{\partial F_i}{\partial u}(t,x(t,q^*),p_t^i,u(t))$$

$$-\sum_{i=1}^{p}\frac{\partial\Psi}{\partial z_i}(\bar z)\psi^\top(t,q^*,\eta^i(\cdot))\frac{\partial f}{\partial u}(t,q^*,x(t,q^*),u(t))$$

$$-\sum_{i=1}^{p}\frac{\partial\Psi}{\partial z_i}(\bar z)\int_{Q}\psi^\top(t,q,\zeta^i(\cdot,q))$$

$$\times\frac{\partial f}{\partial u}(t,q,x(t,q),u(t))d\mu(q)\Bigg]\delta u(t)dt+o(\|\delta u(\cdot)\|).$$

The last equality gives an analog of known formula,

$$\Delta J=-\int_{t_0}^{t_1}\frac{\partial H}{\partial u}(t,x(t),\psi(t),u(t))\delta u(t)dt+o(\|\delta u(\cdot)\|),$$

for the gradient of the functional of optimal control problem.

The considered input design problem may be written in the form (6), where $\Psi(z_1,z_2)=-z_1-\phi(z_2)$, $k_1=k+1$, $k_2=k$, $h_1^\top(t,x)=(g^\top(t,x),g^\top(t,x)Rg(t,x))$, $h_2(t,x)=g(t,x)$,

$$F_1(t,x,y,u)=g^\top(t,x)Rg(t,x)-2g^\top(t,x)Ry(1:k)$$

$$+y_{k+1},\quad y\in R^{k+1},\ y(1:k)=(y_1,...,y_k)^\top,$$

$$F_2(t,x,y,u)=g^\top(t,x)Rg(t,x)-2g^\top(t,x)Ry+y^\top Ry.$$

Functions F_1,F_2 do not depend on u. Direct calculations lead to the following formulas for $\zeta^i,\eta^i,\ i=1,2$,

$$\zeta^1(t,q)=\frac{\partial g}{\partial x}^\top(t,x(t,q))\frac{\partial F_1}{\partial y}$$

$$+\frac{\partial}{\partial x}(g^\top(t,x(t,q))Rg(t,x(t,q)))^\top$$

$$=-2\frac{\partial g}{\partial x}^\top(t,x(t,q))Rg(t,x(t,q))$$

$$+\frac{\partial}{\partial x}(g^\top(t,x(t,q))Rg(t,x(t,q)))^\top=0,$$

$$\eta^1(t)=\frac{\partial F_1}{\partial x}=2\frac{\partial g}{\partial x}^\top(t,x(t,q^*))R(g(t,x(t,q^*))$$

$$-\int_{Q}g(t,x(t,q))d\mu(q)),$$

$$\zeta^2(t,q)=\frac{\partial g}{\partial x}^\top(t,x(t,q))\frac{\partial F_2}{\partial y}$$

$$=-2\frac{\partial g}{\partial x}^\top(t,x(t,q))Rg(t,x(t,q)+\int_{Q}g(t,x(t,q))d\mu(q)),$$

$$\eta^2(t)=-\zeta^2(t,q^*).$$

The Hamiltonian takes here the following form

$$H=F_1+\frac{\partial\phi}{\partial z}(z_2)F_2+(\psi(\tau,q,\eta^1(\cdot))$$

$$+\frac{\partial\phi}{\partial z}(z_2)\psi(\tau,q,\eta^2(\cdot)))^\top f(\tau,q,x,u)$$

$$+\int_{Q}\frac{\partial\phi}{\partial z}(z_2)\psi(\tau,q,\zeta^2(\cdot))^\top f(\tau,q,x(\tau,q),u)d\mu(q).$$

Under the assumption of differentiability of $f(t, q, x, u)$ with respect to u the gradient of J may be represented by the formula

$$\delta J = \int_{t_0}^{t_1} \Big[\psi(\tau, q^*, \eta^1(\cdot) + \frac{\partial \phi}{\partial z}(z_2)\eta^2(\cdot))^\top \times \frac{\partial f}{\partial u}(\tau, q^*, x(\tau, q^*), u(\tau)) + \int_Q \frac{\partial \phi}{\partial z}(z_2)\psi(\tau, q, \zeta^2(\cdot))^\top \times \frac{\partial f}{\partial u}(\tau, q, x(\tau, q), v)d\mu(q) \Big] \delta u(\tau) d\tau. \quad (15)$$

The expressions (14), (15) are the basis for constructing the numerical algorithms for solution of the problem. In the next chapter the solutions of two examples, based on the conditional gradient method, are given. In these examples q is a two-dimensional vector, the set Q is a square. The measure μ is assumed to take equal values on the finite set of random vectors uniformly distributed on Q. In this case the optimal control problem has sufficiently large dimension proportional to nk, where n is the dimension of system (1), and k is the number of points in the measure support set. Note that under large k the value of $\phi(I_2(u(\cdot)))$ may be neglected, and it is possible to consider $I_1(u(\cdot))$ as a functional instead of $J(u(\cdot))$, which simplifies the problem.

Another way of simplification of the problem consists in considering the measure μ concentrated at the vertices of Q. This considerably reduces the dimension of the problem, but does not imply in the examined examples the significant change in the structure of optimal inputs.

4. EXAMPLES

E x a m p l e 1. Consider the problem of optimal elevator deflection inputs for identifying parameters in the longitudinal shot period equations of aircraft (Mehra, 1974). The second order control system is considered on the interval $[0, 4]$. The system equations are as follows

$$\dot{x}_1 = q_1 x_1 + q_2 x_2 - 1.66u,$$
$$\dot{x}_2 = x_1 + 0.737 x_2 + 0.005u, \quad (16)$$

$x_1(0) = 0$, $x_2(0) = 0$. Here x_1 is the angle of attack, x_2 is the pitch rate, u is the elevator command, $|u| \le 10$, t is a time. The coefficients q_1, q_2 are unknown and should be specified on the base of observations of $y(t) = x_1(t)$ on the interval $[0, 4]$. A priori constraints on the measurement error ξ and parameters $q = (q_1, q_2)$ are given by the relations $\int_0^4 \xi^2(t)dt \le 1$, $q \in Q$, where Q is the square

$$Q = \{q : \max_i |q_i - \hat{q}_i| \le 0.4\}$$

with center $\hat{q} = (-1.25, -0.75)$. Let unknown true value of a vector q equals to $q^* = (-1.588; -0.562)$.

For μ we take a measure, with equal values at the vertices of Q.

The optimal input $u(t) = 10$ on the interval $[0, 2.9]$ and -10 on $[2.9, 4]$. The maximal value of the functional equals 242.75. Note that optimal input for this system with the error of estimation of a linear functional of q as a criterion has two switching points (Gusev, 1992). In figure 1 the information sets are shown for the optimal input (the boundary is a thick line) and for comparison for $u(t)$, which is equal to -10 on $[0, 1]$ and 10 on $(1, 4]$ (the boundary is a thin line). The value of functional for the last control function is equal to 73.64. The straight portion of the boundary corresponds to a priori constraints Q.

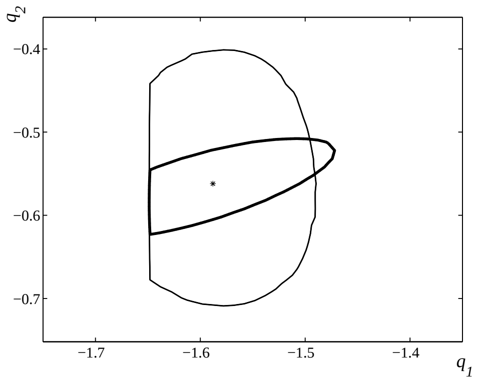

Fig. 1. Information sets for Example 1

For this example the case of a measure μ taking equal values on the set of 20 (100) random vectors uniformly distributed on Q is also considered. The optimal input in this case is close to the previous one.

E x a m p l e 2. Consider the Duffing equation (Lee and Markus, 1967)

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = q_1 x_1 + q_2 x_1^3 + u, \quad (17)$$

which describes the motion of nonlinear stiff spring on impact of an external force u, $|u| \le 3$. Let $x_1(0) = 0$, $x_2(0) = 0$.

The value of $y(t) = x_2(t)$ is available for measurements on time interval $[0, 4]$. The measurement error is restricted by the inequality $\int_0^4 \xi^2(t)dt \le \frac{1}{10}$. The coefficients q_1, q_2 are assumed to be unknown with a priori information given by the inclusion $q \in Q$; here $Q = \{q : \max_i |q_i - \hat{q}_i| \le 0.5\}$, $\hat{q} = (-1, -2)$. Let unknown true value $q^* = \hat{q}$. The measure μ takes equal values at the vertices of Q. As a criterion in this example we consider $I_1(u(\cdot))$ instead of $J(u(\cdot))$. The optimal input has

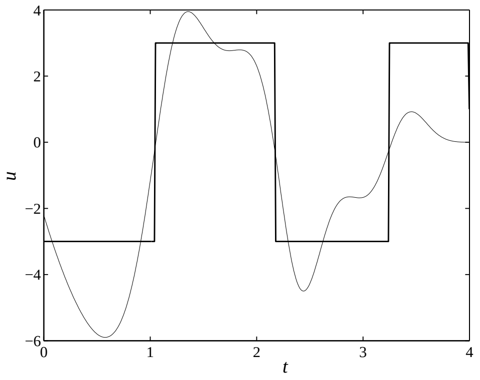

Fig. 2. Optimal input and a gradient of functional for Example 2

here three switching points as is shown in figure 2. The marking of axis of ordinate in the figure is given for u.

In figure 3 the information sets are shown for the optimal input (the boundary is a thick line) and for the input $u(t) \equiv 3$ (the boundary is a thin line). The straight portions of the boundary

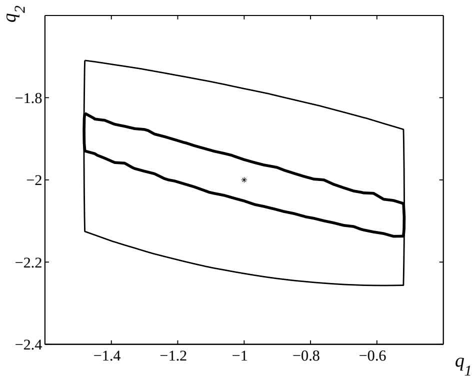

Fig. 3. Information sets for Example 2

corresponds to a priori constraints Q.

5. CONCLUSION

An approach to the problem of optimal input design for guaranteed identification of the parameters of control system is presented. It is based on the use of an integral of an information function over the set of a priori constraints on uncertain items Q as a criterion of optimality. This allows one to represent linear and nonlinear problems in the unified form as the optimal control problem with functional depending on integral over parameter. The optimality conditions are derived for the problem with integral restrictions on measurement errors. An integration measure is assumed to be concentrated on a dense grid of the set Q or on the set of extremal points of Q. The last case is characterized by the smaller dimension of the optimization problem. The results of numerical simulation show a similar structure of optimal inputs in both these cases.

6. ACKNOWLEDGMENTS

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