FRICTION MODELLING AND ROBUST ADAPTIVE COMPENSATION

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Abstract: In this paper, a friction model is proposed which can easily be introduced in adaptive control algorithms. Based on the model, a robust adaptive friction compensation algorithm is developed for high precision positioning systems. It is shown that the proposed compensation scheme guarantees a prescribed accuracy for trajectory tracking. The paper also presents experimental results to demonstrate the applicability of the proposed control algorithm.

Keywords: Friction, Compensation, Adaptive Control, Lyapunov methods, Positioning Systems

1. INTRODUCTION

Friction is universally present in the motion of bodies in contact. In servo controlled machines friction has an impact in all regimes of operation. In high precision positioning systems it is inevitable to know the value of the friction force to assure good control characteristics and to avoid some undesired effects such as limit cycle and steady state error.

Many models were developed to explain the friction phenomenon. The introduced models are based on experimental results rather than analytical deductions. Tribological experiments showed that in the case of lubricated contacts the well-known static + kinetic + viscus friction model cannot explain some phenomena in low velocity regime, such as Striebeck effect (decreasing friction force with increasing velocities at low velocity regime), presliding displacement, friction lag (Armstrong-Hèlouvry, 1991). To explain these phenomena dynamic friction models were developed, such as the LuGre model (de Wit *et al.*, 1995). The contact of two rigid bodies is modelled as a set of elastic bristles. When a tangential force is applied, the bristles deflect like springs which gives rise to the friction. If the force is sufficiently large some of the bristles deflect so much that they will slip. Note the average deflection of the bristles with z and consider that its dynamics is modelled by:

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$$\frac{dz}{dt} = v - \sigma_0 \frac{|v|}{g(v)} z \quad (1)$$

v denotes the relative velocity between the two surfaces in contact, σ_0 is a constant parameter representing the stiffness, the function $g(v)$ is a positive continuous function which is meant to describe the Striebeck effect. It can be defined as an exponential function of velocity : $g(v) = (F_C + (F_S - F_C)e^{-|v|/v_S})$ where F_C represents the Coulombic friction term, F_S the static friction term, v_S is the Striebeck velocity.

The friction force generated from the bending of the bristles and from the viscous lubrication is described as:

$$F_f = \sigma_0 z + \sigma_1 \frac{dz}{dt} + F_V v \quad (2)$$

where F_V is the viscous friction coefficien and σ_1 is a damping coefficient.

The relations (1), (2) give the LuGre dynamic friction model. It was shown that if the function $g(v)$ is positive and bounded and the initial value of z is bounded, the dynamic state z remains bounded for any time instant (de Wit *et al.*, 1995).

In steady state ($dz/dt = 0$) the LuGre model can be written as a static mapping between the velocity and friction force (Tustin friction model):

$$\begin{aligned} F_f &= g(v)sign(v) + F_V v = \\ &(F_C + (F_S - F_C)e^{-|v|/v_S})sign(v) + F_V v \quad (3) \end{aligned}$$

In order to compensate the effect of the friction generally a feedforward term is introduced in the structure of the controller, which aim is the cancellation of the effect of friction force. In previous works that dealt with friction compensation, two trends can be separated: model-oriented friction compensation techniques and friction modelling using soft computing methods. Being a nonlinear mapping between the velocity and friction force, many papers try to model the friction phenomenon using universal approximators such as neural networks or fuzzy systems. Using feedforward type neural networks, a direct compensation of the friction force was proposed in (Kim and Lewis, 2000) for servo-systems with unknown dynamics. In (Du and Nair, 1999) the friction is modelled using RBF type networks. In (Garagic and Srinivasah, 2004) the nonlinear part of the friction model (3) is compensated using fuzzy system.

The model based friction compensation methods generally use the LuGre friction model. Both the states and the parameters of that type of dynamic models are unknown, moreover the parameters of the model are time varying. This is why the tractability and the direct usage of this model in the control algorithms are difficult. However, adaptive estimation of some parameters of the model and nonlinear state estimation techniques for the state (z) of this model were reported (Vedagarbha *et al.*, 1999). A comparative study of different compensation techniques was presented in (Ray *et al.*, 2001). Robust adaptive control techniques (Ioannou and Xu, 2001) are also very popular for friction compensation. Early results were presented in (Lee and Kim, 1995) using static friction model. Robust adaptive compensation of the dynamic friction effects were reported in (Misovec and Annaswamy, 1998) for positioning systems and in (Panteley *et al.*, 1998) for robotic arms.

2. FRICTION MODEL FOR ADAPTIVE COMPENSATION

The parameters of the friction model may change as a function of normal forces in contact, temperature variations, humidity, lubricant conditions, material proprieties and other factors that can hardly be controlled (Armstrong-Hèlouvry, 1991). This is why the parameters of the friction models should be considered as time varying and that adaptive control techniques using on-line parameter estimation methods are popular for friction compensation.

To apply the well known adaptive control schemes for friction compensation it is desirable that the friction force could be written in a linearly parameterized form, namely as a scalar product between a known regressor vector $\underline{\xi}_F(v)$ and an unknown parameter vector $\underline{\theta}_F$ ($F_f = \underline{\theta}_F^T \underline{\xi}_F(v)$).

In the other hand the friction parameters could change even in the function of the sign of velocity. Hence it is recommended to use different friction parameters in the positive and negative velocity regimes.

The previously introduced friction models should be rewritten or modified in order to obtain the previously presented requirements (linear parameterization, different parameters sets for positive and negative velocities) and in the same time to keep the qualitative characteristics of the original models. The model introduced in this paper, was developed based on Tustin model (3) and the dynamic behavior will be incorporated later on.

Recent works suggest that the piecewise continuous nonlinear functions can be approximated using linearly parameterized models, in which the regressor is discontinuous (Selmic and Lewis, 2002). The friction force always has discontinuous behaviour when crosses through $v = 0$ velocity. In order to introduce this discontinuity in the friction

model, define the following switching function:
$\mu(v) = \begin{cases} 1 & if\ v \geq 0 \\ 0 & otherwise \end{cases}$

For the simplicity only the positive velocity domain is considered, but similar study can be made for negative velocities. Let us consider that the mechanical system moves in $0 \dots v_{max}$ velocity domain. The model (3) is approximated using two lines: d_{1_+} which crosses through the $(0, F_f(0))$ point and it is tangent to curve and d_{2_+} which passes through the $(v_{max}, F_f(v_{max}))$ point and tangential to curve. These two lines meet each other at a v_{sw+} velocity. In the domain $0 \dots v_{sw+}$ the d_{1_+} can be used for the linearization of the curve and d_{2_+} is used in the domain $v_{sw+} \dots v_{max}$. The maximum approximation error occurs at the velocity v_{sw+} for both linearizations. The equations for d_{1_+} and d_{2_+}, using Taylor expansion, are:

$$\begin{aligned} d_{1_+}\ :\ F_{L1f_+}(v) &= F_S + \frac{\partial F_f(0)}{\partial v} v \\ &= F_S + (F_V - (F_S - F_C)/v_S)v \qquad (4) \\ d_{2_+}\ :\ F_{L2f_+}(v) &= F_f(v_{max}) + \frac{\partial F_f(v_{max})}{\partial v}(v - v_{max}) \\ &= F_f(v_{max}) + \\ &\quad (F_V - (F_S - F_C)/v_S)e^{-v_{max}/v_S}(v - v_{max}) \qquad (5) \end{aligned}$$

Thus the linearization of the Tustin friction model in the $0 \dots v_{max}$ velocity domain can be realized using two lines:

$$\begin{aligned} F_{L1f_+}(v) &= a_{1+} + b_{1+}v,\ if\ 0\ \leq v\ \leq\ v_{sw+} \qquad (6) \\ F_{L2f_+}(v) &= a_{2+} + b_{2+}v,\ if\ v_{sw+}\ < v\ \leq\ v_{max} \end{aligned}$$

Now consider two exponential membership functions parameterized in the following way:

$$\begin{aligned} \phi_{1_+}(v) &= (e^{-\beta(v - v_{sw+})})/(1 + e^{-\beta(v - v_{sw+})}) \\ \phi_{2_+}(v) &= 1/(1 + e^{-\beta(v - v_{sw+})}) \qquad (7) \end{aligned}$$

where β is a large positive constant and v_{sw+} is the switching velocity, where d_{1_+} and d_{2_+} meet each other. The value of v_{sw+} can easily be determined from linearization (6) $v_{sw+} = (a_{1+} - a_{2+})/(b_{2+} - b_{1+})$.

By applying the F_{L1f_+} from (6) on the membership function ϕ_{1+} from (7) and F_{L2f_+} on ϕ_{2+} a new model can be obtained that has the same behaviour as the Tustin friction model. Moreover it is linearly parameterized if the parameters of the lines are considered. For the positive velocity domain it can be written as:

$$\begin{aligned} F_{f_+}(v) &= a_{1+}\phi_{1_+}(v)\mu(v) + b_{1+}v\phi_{1_+}(v)\mu(v) \\ &\quad + a_{2+}\phi_{2_+}(v)\mu(v) + b_{2+}v\phi_{2_+}(v)\mu(v) \qquad (8) \end{aligned}$$

With same train of thoughts a similar model can be determined for the negative velocity domain. Combining the negative and positive velocity domains the obtained friction model reads as:

$$\begin{aligned} &F_f(v) = \underline{\theta}_f^T \underline{\xi}_f(v),\ where: \qquad (9) \\ &\underline{\theta}_f = (a_{1+}\ b_{1+}\ a_{2+}\ b_{2+}\ a_{1-}\ b_{1-}\ a_{2-}\ b_{2-})^T \\ &\underline{\xi}_f(v) = (\phi_{1_+}\mu(v)\ v\phi_{1_+}\mu(v)\ \phi_{2_+}\mu(v)\ v\phi_{2_+}\mu(v) \\ &\phi_{1_-}\mu(-v)\ v\phi_{1_-}\mu(-v)\ \phi_{2_-}\mu(-v)\ v\phi_{2_-}\mu(-v))^T \end{aligned}$$

For some applications it is important to determine the magnitude of the modelling error due to dynamic behaviour of friction. Denote the steady state value of z with z_{ss}. It can be expressed as: $z_{ss} = g(v)sign(v)/\sigma_0$. Since z is bounded the expression $z - z_{ss}$ is also bounded. From (2) yields:

$$\begin{aligned} F_f &= \sigma_0 z_{ss} + \sigma_0(z - z_{ss}) + \sigma_1 dz/dt + F_V v \\ &= g(v)sign(v) + F_V v + \\ &+\sigma_0(z - z_{ss}) + \left(\sigma_1 sign(v) + \frac{\sigma_0\sigma_1 z}{g(v)}\right)|v| \qquad (10) \end{aligned}$$

In the relation (10) the term $g(v)sign(v) + F_V v$ is the static Tustin friction model and the rest of the expression represents the dynamic behaviour of the friction.

The bound of the modelling error can be determined as it follows: since z, z_{ss}, $g(v)$ are bounded, there exist two positive constants a_D, b_D satisfying:

$$\begin{aligned} &\left|\sigma_0(z - z_{ss}) + \left(\sigma_1 sign(v) + \frac{\sigma_0\sigma_1 z}{g(v)}\right)|v|\right| \leq \\ &\leq a_D + b_D|v| \qquad (11) \end{aligned}$$

As it was mentioned before, the friction models could have separate parameters for negative and positive velocities. The parameters a_D and b_D could also be defined separately for positive and negative velocity regimes.

Consequently, the friction can be modelled by:

$$F_f = \underline{\theta}_f^T \underline{\xi}_f(v) + F_D(v),\ \ |F_D(v)| \leq \underline{\theta}_{fD}^T \underline{\xi}_{fD}(v) \qquad (12)$$

The static term $\underline{\theta}_f^T \underline{\xi}_f(v)$ is defined in (9). The bound of the dynamic term is defined as follows:

$$\begin{aligned} &|F_D(v)| \leq \underline{\theta}_{fD}^T \underline{\xi}_{fD}(v) \qquad (13) \\ &where\ \theta_{fD} = (a_{D+}\ b_{D+}\ a_{D-}\ b_{D-})^T \\ &\xi_{fD}(v) = (\mu(v)\ \mu(v)|v|\ \mu(-v)\ \mu(-v)|v|)^T \end{aligned}$$

The friction can be modelled as a sum of a static friction model and a dynamic term. The static term can be written in a linearly parameterized form. The regressor vector contains a discontinuous switching function which aim is to distinguish the positive and negative velocity regimes. The dynamic term is always bounded. Its bound can also be written in a linearly parameterized form with discontinuous regressor vector.

3. FRICTION COMPENSATION

To illustrate the applicability of the previously presented friction model for adaptive compensation a positioning system is considered driven by direct current (DC) servo motor. The dynamics of the system reads as:

$$\dot{\alpha} = \omega; \quad J\dot{\omega} + F_f(\omega) = \tau + d \tag{14}$$

α denotes the angular position, ω denotes the angular speed, τ is the command torque. The parameter $J > 0$ denotes the inertia and is also considered unknown. The friction term $F_f(\omega)$ is given by (12). In the friction model the angular velocity ω is used instead of v .

The term d represents a small bounded additive disturbance which incorporates external disturbances and unmodelled dynamics ($|d| \leq d_M$).

Note that the parameters w_{sw} and β which are incorporated in the regressor vectors should be determined *a-priori*. It is assumed that the variation of these parameters causes only small bounded modelling errors, which can be incorporated in d.

Define the tracking error $e(t) = \alpha_d(t) - \alpha(t)$ and the tracking error metric $S(t) = (\frac{d}{dt} + \lambda)e(t)$ with $\lambda > 0$. Here α_d is the prescribed trajectory, a smooth, twice differentiable function in time.

The control problem can be formulated as follows: design a control law τ so that the tracking error metric $S(t)$ satisfies $|S(t)| < \Phi$ for $t \to \infty$ where $\Phi > 0$ is the given precision.

In order to obtain a control law which satisfies this requirement, define S_Δ as $S_\Delta(t) = S(t) - sat(S(t)/\Phi)$ where $sat(\cdot)$ denotes the saturation function. The following proprieties can easily be verified:

$$\dot{S}_\Delta = \dot{S}, \ if \ \ |S| > \Phi; \ \dot{S}_\Delta = 0 \ otherwise \tag{15}$$

$$|S_\Delta| = |S| - \Phi = S_\Delta sat(S/\Phi), \ if \ \ |S| > \Phi \tag{16}$$

The parameters $\underline{\theta}_f, \ \underline{\theta}_{fD}, \ J$ are considered unknown, consequently the control law can be developed using estimated parameters. Denote the estimation errors and the estimated parameters as follows: $\underline{\widetilde{\theta}}_f = \underline{\theta}_f - \underline{\widehat{\theta}}_f$, $\underline{\widetilde{\theta}}_{fD} = \underline{\theta}_{fD} - \underline{\widehat{\theta}}_{fD}$, $\widetilde{J} = J - \widehat{J}$.

To solve the proposed control problem, let us define the following control law:

$$\begin{aligned}\tau = \widehat{J}(\ddot{\alpha}_d + \lambda\dot{e}(t)) + \underline{\widehat{\theta}}_f \underline{\xi}_f(\omega) + k_S S + \\ + \widehat{F}_D sat(S/\Phi))\end{aligned} \tag{17}$$

with $k_S > 0$ and $\widehat{F}_D(\omega) = \underline{\widehat{\theta}}_{fD}^T \underline{\xi}_{fD}(\omega)$. Assume that:

$$k_S \geq d_M/\Phi \tag{18}$$

The values of the friction and inertial parameters can be obtained using adaptive techniques. The adaptation rules are defined as follows:

$$\begin{aligned}\dot{\underline{\widehat{\theta}}}_f &= -S_\Delta(t)\Gamma_f \underline{\xi}_f(\omega) \\ \dot{\underline{\widehat{\theta}}}_{fD} &= -|S_\Delta(t)|\,\Gamma_{fD}\underline{\xi}_{fD}(\omega) \\ \dot{\widehat{J}} &= -S_\Delta(t)\gamma_J(\ddot{\alpha}_d + \lambda\dot{e}(t))\end{aligned} \tag{19}$$

with Γ_f, Γ_{fD} positive definite diagonal matrices and $\gamma_J > 0$.

To examine the behaviour of the closed loop system consider the following Lyapunov like cost function:

$$V(t) = JS_\Delta(t)^2 + \frac{1}{\gamma_J}\widetilde{J}^2 + \underline{\widetilde{\theta}}_f^T \Gamma_f^{-1} \underline{\widetilde{\theta}}_f + \underline{\widetilde{\theta}}_{fD}^T \Gamma_{fD}^{-1} \underline{\widetilde{\theta}}_{fD} \tag{20}$$

The time derivative of $V(t)$ is given by:

$$\begin{aligned}\dot{V}(t) = J\dot{S}_\Delta(t)S_\Delta(t) - \frac{1}{\gamma_J}\widetilde{J}\dot{\widehat{J}} - \underline{\widetilde{\theta}}_f^T \Gamma_f^{-1} \dot{\underline{\widehat{\theta}}}_f - \\ - \underline{\widetilde{\theta}}_{fD}^T \Gamma_{fD}^{-1} \dot{\underline{\widehat{\theta}}}_{fD}\end{aligned} \tag{21}$$

Due to the definition of S_Δ and the adaptation laws (19), $\dot{V}(t) = 0$ for $|S(t)| \leq \Phi$.

Outside the boundary layer Φ the tracking error dynamics can be written as:

$$\begin{aligned}J\dot{S}_\Delta(t) &= J\dot{S}(t) = J(\ddot{\alpha}_d + \lambda\dot{e}(t)) + F_f - \tau - d \\ &= \widetilde{J}(\ddot{\alpha}_d + \lambda\dot{e}(t)) + \underline{\widetilde{\theta}}_f^T \underline{\xi}_f(\omega) - k_S S(t) + \\ &+ F_D - \underline{\widehat{\theta}}_{fD}^T \underline{\xi}_{fD}(\omega) sat(S/\Phi) - d\end{aligned} \tag{22}$$

By substituting (22) and the adaptation laws (19) into (21), yields:

$$\begin{aligned}\dot{V}(t) = -k_S S_\Delta(t)S(t) - dS_\Delta(t) + F_D S_\Delta(t) - \\ - \underline{\widehat{\theta}}_{fD}^T \underline{\xi}_{fD}(\omega) sat(S/\Phi) S_\Delta(t) - \underline{\widetilde{\theta}}_{fD}^T \underline{\xi}_{fD}(\omega)\,|S_\Delta(t)|\end{aligned} \tag{23}$$

According to (13), $F_D S_\Delta(t) \leq |F_D||S_\Delta(t)| \leq \theta_{fD}^T \xi_{fD}\,|S_\Delta|$. Using this inequality and the propriety (16) the time derivative of the Lyapunov function simplifies:

$$\dot{V}(t) \leq -k_S S_\Delta(t)S(t) - dS_\Delta(t) \tag{24}$$

Outside the boundary layer (Φ) $sign(S) = sign(S_\Delta)$, hence $SS_\Delta = |S|\,|S_\Delta|$. Because the disturbance d is bounded $|d| \leq d_M$ the following relation holds: $-dS_\Delta(t) \leq d_M |S_\Delta(t)|$. Using the propriety (16), it results:

$$\dot{V}(t) \leq -k_S |S_\Delta(t)| |S_\Delta(t)| + (-k_S\Phi + d_M) |S_\Delta(t)| \tag{25}$$

According to assumption (18) the second term in the inequality (25) is always negative. Hence, it yields:

$$\dot{V}(t) \leq -k_S S_\Delta(t)^2 \tag{26}$$

Notice that (26) is also valid for $|S(t)| \leq \Phi$.

Since $V(t)$ is a positive and non-increasing function, therefore $V(\infty)$ is finite. It is assumed that the initial values of the estimated parameters and the initial value of the tracking error metric are finite. Thus, if $S_\Delta(0)$, $\widetilde{J}(0)$, $\underline{\widetilde{\theta}}_{fD}(0)$ and $\widetilde{\underline{\theta}_f}(0)$ are finite $\Rightarrow$ $S_\Delta(t)$, $\widetilde{J}(t)$, $\underline{\widetilde{\theta}}_{fD}(t)$ and $\widetilde{\underline{\theta}_f}(t) \in L_\infty$ $\forall$ $t > 0$. Since $S_\Delta(t) \in L_\infty$, it implies that $S(t) \in L_\infty$

If $S_\Delta(t) \in L_\infty$, $e(0)$ and $\dot{e}(0)$ are finite $\Rightarrow e(t)$ and $\dot{e}(t) \in L_\infty$.

If $e(t)$, $\dot{e}(t)$, $\alpha_d(t)$ and $\omega_d(t) \in L_\infty \Rightarrow \alpha(t)$ and $\omega(t) \in L_\infty$.

From:

$$\int_0^\infty S_\Delta(t)^2 dt \leq \frac{-1}{k_S} \int_0^\infty \dot{V}(t) = \frac{V(0) - V(\infty)}{k_S} < \infty \tag{27}$$

it follows that $S_\Delta(t) \in L_2$.

Since the elements of the parameter $\underline{\theta}_{fD}$ are finite and $\underline{\widetilde{\theta}}_{fD}(t) \in L_\infty \Rightarrow \underline{\widehat{\theta}}_{fD}(t) \in L_\infty$.

From (22) results that if $S(t)$, $\widetilde{J}(t)$, $\ddot{\alpha}_d(t)$, $\dot{e}(t)$, $\underline{\widetilde{\theta}}_f(t)$, F_D, $\underline{\widehat{\theta}}_{fD}$, $\omega(t)$, $d(t) \in L_\infty \Rightarrow \dot{S}_\Delta(t) \in L_\infty$.

Because $S_\Delta(t)$ and $\dot{S}_\Delta(t) \in L_\infty$ and the relation (27) holds, by Barbalat's lemma $S_\Delta(t) \to 0$ when $t \to \infty$, consequently the inequality $|S(t)| \leq \Phi$ is obtained asymptotically. Thus the control law (17) with the adaptation law (19) solve the formulated control problem.

4. EXPERIMENTAL RESULTS

In this section the effectiveness of the proposed control scheme is illustrated by an experiment using a 1 DOF positioning system. The performance of the proposed scheme is compared with that of a PID controller.

The experimental setup consists of a permanent magnet $24V$ DC servo motor, which drives a metal disc with known inertia ($J = 0.015\, kgm^2$) through a $1:66$ gearhead. Friction is introduced via a metal surface, which is held against the disc (see Fig. 2). The contact between the disc and the metal surface is lubricated with grease. The control algorithm is implemented on a PIC-18 type microcontroller which C compiler allows floating point representation. The Euler's approximation is used for the integration with a sampling period of 5 ms. The angular position and velocity of the mechanical system are measured using a 5000 PPT rotational encoder. The impulses of the encoder are counted using the embedded 16 bit timers of the controller. The microcontroller is interfaced to the current servo amplifier through a 11 bit DAC.

The reference trajectory was chosen in such way to have acceleration, deceleration and constant speed regimes for both positive and negative velocity domain. The algorithm was tested in ± 0.5 $[rad/sec]$ velocity domain. The duration of an acceleration-constant speed-deceleration cycle was 5 $[sec]$. For the controller, the following parameters were chosen: $\Phi = 0.25$, $\lambda = 10$, $K_S = 40$, $d_M = 10$.

The adaptation of the parameters determining the behavior of friction force for the positive velocity domain are presented in Fig. 3. It can be observed that the parameters are tuned only when the plant is in the corresponding velocity regime. The performance of the control algorithm is compared with the response of a well tuned PID controller. As it can be seen in Fig. 1 the robust adaptive control law guarantees the convergence inside the boundary layer Φ and after the convergence the error metric is smaller than in the case of PID controller.

For the numerical evaluation of the control performances, the average of the absolute values of the tracking error metrics ($S_A = \frac{1}{N}\sum_{i=1}^{N} |S|$) was calculated during the first $N = 250$ sampling periods for the PID controller and the adaptive friction compensator controller. It was found that these controllers guarantees same performances ($S_A(PID) = 0.2449$, $S_A(adapt) = 0.2487$). It is because the parameters are not adapted yet and the the adaptive controller does not work properly. However in the second 250 sampling period, when the adaptive laws already tuned the parameters the adaptive controller clearly overperforms the PID controller ($S_A(PID) = 0.2573$, $S_A(adapt) = 0.1106$).

5. CONCLUSIONS

The use of a novel friction model in adaptive trajectory tracking control for positioning systems is explored in this paper. The model incorporates both the Striebeck effect and dynamic frictional

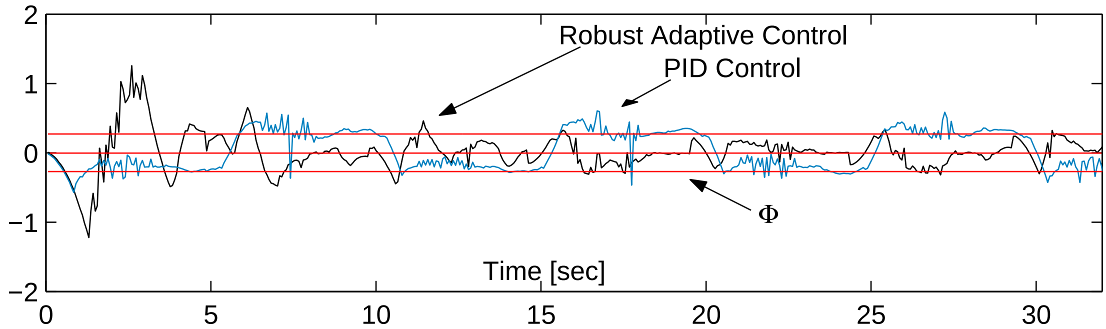

Fig. 1. Tracking error metric (S)

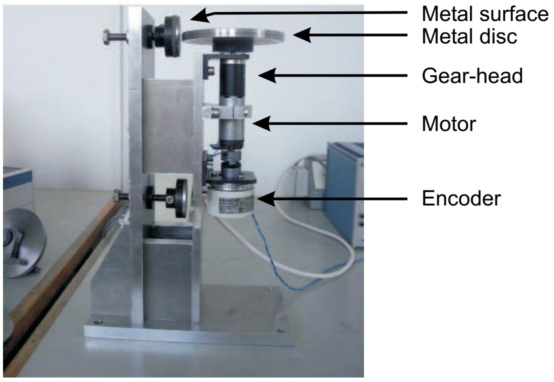

Fig. 2. Experimental apparatus for friction compensation

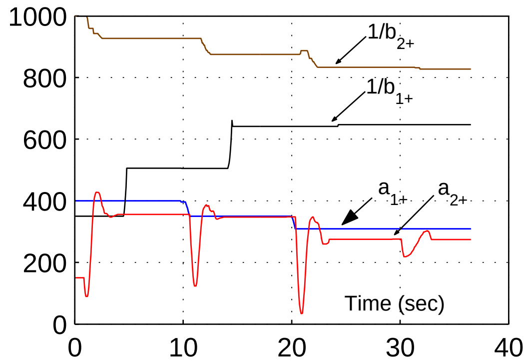

Fig. 3. Friction parameter convergence ($\omega > 0$)

behaviour and it can easily be incorporated in adaptive control algorithms. Using the model a robust adaptive control law was developed for positioning systems. The behaviour of the closed loop system with the proposed control algorithm is studied using Lyapunov techniques. It was shown that the algorithm which incorporates the introduced friction model guarantees that the tracking error remains bounded with known bound even in the presence of bounded external disturbances and modelling errors. Experimental results also were presented to illustrate the effectiveness of the control algorithm. To implement the algorithm, a microcontroller based architecture was used. Experimental results show good convergence proprieties of the tracking error metric.

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