A HARMONIC CONTROLLER OF ENGINE SPEED OSCILLATIONS FOR HYBRID VEHICLES

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Abstract: In conventional internal combustion engines, the periodic fuel combustion in cylinders and oscillating masses result in pulsating engine torque. In hybrid powertrains, the electrical motor can be used to control such pulsations. The main originality of this paper is to present an efficient harmonic controller synchronized with the engine speed: the feedback law controls the Fourier parameters of the command. For time-variation of the engine speed, the adaptation of the harmonic controller ensures the convergence of the command. Conditions of stability are given, and the robustness of the approach is presented. Simulation shows that the undesirable harmonics of engine speed oscillations are perfectly rejected in spite of engine speed variation.

Keywords: hybrid vehicles, harmonic, oscillation, adaptation, engine control, permanent magnet motor, rejection.

1. INTRODUCTION

In conventional internal combustion engines, the periodic fuel combustion in cylinders and oscillating masses result in pulsing engine torque which affects the smoothness and quality of the vehicle's ride, causes increased noise, vibration and harshness and reduces fuel economy (Chauvin et al, 2003). Usually, the mean value of the instantaneous torque over an engine cycle is controlled and the instantaneous torque waveform produced by each cylinder is imposed. Usually, only passive solutions are implemented to attenuate the torque pulsations as the flywheel for example. In hybrid powertrains, an electric motor is coupled to the thermal engine from the crankshaft in order to improve its energy efficiency. But, the electrical motor can also be used to control the instantaneous torque produced by the thermal engine. Consequently, it can control their undesirable effects such the vibrations.

One possible application of the instantaneous torque control is the implementation of a virtual flywheel (Gusev, 1997). The impact is to reduce the mass flywheel or to control torsional vibrations affecting the driveline. Previous experimental studies have shown that open loop control of the electrical motor can lead to reduce the engine speed oscillations (Nakajima et al., 2000). Such active flywheel must be efficient over an engine speed range, hence the controller must be able to actively control a periodic disturbance of time-varying frequency. Simulations show that a harmonic activation neural network can be used to compensate the combustional torque pulsation (Beuschel and Schroder, 1999). The neural network gives interesting results but it does not physical insight about the parameters to adapt. Also, simulations show that a learning control provides adequate active damping and high robustness with respect to system parameter variations (Zaremba, 1998); but, the effects of actuator dynamics and sensor noise on learning control performance need to be evaluated. Finally, a mixed H_2/H_∞ synthesis can be applied to design a parameter-dependent state feedback gain, *i.e.* a gain scheduling controller (Tnami et al, 2004).

On the other hand, a time-frequency controller is well adapted to reject time-varying tonal disturbances (Micheau and Coirault, 2000). For this problem where the frequency can be perfectly measured, the time-frequency controller can be implemented as a harmonic controller: it will measure the Fourier parameters of the engine speed in order to adapt the Fourier parameters of the command signal. Such approach provides a narrowband filtering of the disturbance which reduces the noise effects, and it increases the robustness because the dynamic of the physical system can be reduced to a gain. Section 2 presents the engine dynamics and the synchronous machine modelling with this approach. Section 3 presents the signal processing tools, the MIMO harmonic model, and the design of the gain scheduling harmonic controller. The robustness of the approach is presented in Section 3.5. Finally, simulations presented in section 4.

2. MODEL

2.1 Model of the thermal engine

The torque balance on the crankshaft of an internal combustion engine is given by (Kiencke and Nielsen, 2000) :

$$J\frac{d\omega}{dt}+b\omega=T_{osc}+T_{comb}+T_m-T_{load} \tag{1}$$

With J: lumped inertia, b: effective damping coefficient, ω: crankshaft angular velocity, T_{osc}: torque generated by oscillating masses and connecting rods, T_{comb}: combustion torque, also referred the indicated torque, T_{load}: exogenous load torque, T_m: torque of the synchronous machine coupled to the internal combustion engine from the crankshaft.

For a simple two-mass model for the rod, the oscillating torque results of the sum each oscillating mass indexed *k*, due to the motion of each piston and each rod:

$$T_{osc}=\sum_{k=1}^{CYL}(m_a+m_p)\omega^2 r\big(\cos(\theta_k)+\lambda\cos(2\theta_k)\big)\left(r\cos(\theta_k)+l\sqrt{1-\lambda^2\sin^2(\theta_k)}\right)\tan(\varphi_k) \tag{2}$$

with $\lambda=\frac{r}{l}$, $\theta_k=\theta+k\pi$: angle associated to the kth piston, θ: crankshaft angle, $\sin(\varphi_k)=-\lambda\sin(\theta_k)$, CYL: number of cylinder, l: length of the connecting rods, r: course of the stroke, $m_a=m_{rod}2/3$: mass of the connecting rod, m_p: mass of the piston.

The combustion torque results of the sum of the each cylinder indexed *k*:

$$T_{comb}=\sum_{k=1}^{CYL}P(\theta_k)\left(r\cos(\theta_k)+l\sqrt{1-\lambda^2\sin^2(\theta_k)}\right)\tan(\varphi_k) \tag{3}$$

where $P(\theta_k)$ is the upward thrust on the kth stroke.

For the electric motor coupled to the crankshaft, Equations (2) and (3) model a disturbance torque due to the thermal engine, $T_d=T_{osc}+T_{comb}$. This torque can be split in a constant mean torque, $\overline{T}_d$, and a periodic torque pulsation perfectly synchronized with the crankshaft angle, $\widetilde{T}_d$: $T_d=\overline{T}_d+\widetilde{T}_d$. Usually, the energy management in hybrid powertrains implies the driving of the mean electromagnetic torque $\overline{T}_m$ in order to assist the thermal engine. For $\overline{T}_m>0$, the electric motor delivers supplementary power to the powertrain. For $\overline{T}_m<0$, the electric motor is used as a generator (it brakes the thermal engine) in order to produce electrical energy stored in batteries or super-capacities (Westbrook, 2001). But, the purpose of this paper is to reject the periodic disturbance torque $\widetilde{T}_d$. For this purpose, we assume that T_{load} is a constant value and that the electric torque is composed of a constant mean torque, $\overline{T}_m$, plus a periodic anti-torque pulsation, $\widetilde{T}_m$: $T_m=\overline{T}_m+\widetilde{T}_m$. Hence, according to Equation (1), the crankshaft angular velocity can be split in two terms, $\omega(t)=\overline{\omega}+\widetilde{\omega}(t)$, a constant mean engine speed, $\overline{\omega}$, plus a periodic engine speed pulsation, $\widetilde{\omega}$. The torque balance for pulsating terms is given by :

$$J\frac{d\widetilde{\omega}}{dt}+b\widetilde{\omega}=\widetilde{T}_d+\widetilde{T}_m \tag{4}$$

In the sequel, only the pulsating terms of the engine speed will be considered.

2.2 Model of the Permanent Magnet Synchronous Motor

Using the *d-q* transformation, the permanent magnetic synchronous machine is described in the rotor reference frame as follows (Boldea and Nasar, 1999):

$$v_q=Ri_q+\frac{d\lambda_q}{dt}+p\omega\lambda_d \tag{5}$$

$$v_d=Ri_d+\frac{d\lambda_d}{dt}-p\omega\lambda_q \tag{6}$$

where R : stator resistance, v_d and v_q: stator voltages, i_d and i_q : armature currents, ω : rotor speed, *p*: number of pole pairs. The stator flux linkages are written as:

$$\lambda_d=L_d i_d+\lambda_a \tag{7}$$

$$\lambda_q = L_q i_q \tag{8}$$

with L_d and L_q are the stator inductances, and λ_a is the flux linkage per phase due to the permanent magnet. The torque of the electric motor is given by:

$$T_m = \frac{3}{2} p\left((L_d - L_q) i_d i_q + \lambda_a i_q\right) \tag{9}$$

The *d*-*q* stator currents in the rotor reference frames can be defined with the stator current amplitude, $i_s = \sqrt{i_q^2 + i_d^2}$, and the torque angle δ (the angle between the rotor field and stator current phasor) :

$$\begin{bmatrix} i_q \\ i_d \end{bmatrix} = i_s \begin{bmatrix} \sin\delta \\ \cos\delta \end{bmatrix} \tag{10}$$

Substituting Eqs. (10) into (9), yields

$$T_m = \frac{3}{2} p\left(\frac{1}{2}(L_d - L_q) i_s^2 \sin 2\delta + \lambda_a i_s \sin\delta\right) \tag{11}$$

The first term of Eq. (11) represents the reluctance torque (maximum at $\delta = \pi/4$) and the second term is the electromagnetic torque produced by the permanent magnet flux (maximum at $\delta = \pi/2$). In the PMSM drive system, the commonly used strategy is the constant torque-angle control with $\delta = \pi/2$ (R. Krishnan, 2001). This mode of operation is for speeds lower than the base speed. The *d*-axis reference current is made to be zero, $i_d = 0$, and the electromagnetic torque (11) is controlled by the *q*-axis current:

$$T_m = K_T i_q \tag{12}$$

with $K_T = \frac{3}{2} p\lambda_a$: torque constant. Consequently, to generate the pulsating anti-torque, $\widetilde{T}_m$, it is necessary to drive a pulsating *q*-axis current.

2.3 *Indirect voltage and current vector control*

The control problem of the synchronous machine is to reach zero *d*-axis current, $i_d \to 0$ and that *q*-axis current converges to a reference one, $i_q \to i_q^*$, in spite of the coupling effects, $\omega\lambda_d$ in Equation (5) and $-\omega\lambda_q$ in Equation (6). This problem may be solved by using stator voltage equation for voltage decoupling (Boldea and Nasar, 1999):

$$v_d = -p\omega\lambda_q \tag{13}$$

$$v_q = +p\omega\lambda_d + u \tag{14}$$

where u is the new input of the synchronous machine drive. Such approach is equivalent to a feedback linearization of the non-linear system described by the equations (5) and (6) (Slotine, 1991). In other words, the non-linear feedback (13) and (14) allow to linearize Equations (5) and (6). Equations (5), (8), (12) and (14) describe a stable linear system of input, u , and output T_m (by assuming a perfectly estimated inductance and no saturation) :

$$T_m(s) = \frac{3p\lambda_a/2}{R + L_q s} u(s) \tag{15}$$

Equation (15) in Equation (4) with $\bar{u} = 0$ (or $u = \widetilde{u}$) leads to write:

$$\widetilde{\omega}(s) = H(s)u(s) + G(s)\widetilde{T}_d(s) \tag{16}$$

with the transfer functions $G(s) = \frac{1}{Js + b}$ and $H(s) = \frac{3p\lambda_a/2}{(Js + b)(L_q s + R)}$. The mechanical time constant is $\tau_m = J/b$ and the electric time constant is $\tau_e = L_q/R$.

3. HARMONIC CONTROLLER

3.1 *Harmonic analysis of the engine speed*

The oscillating terms, $\widetilde{\omega}$, coming from the disturbance pulsating torque $\widetilde{T}_d$, can be written as a Fourier series of *N* harmonics to reject:

$$\widetilde{\omega}(\theta) = 2\,\mathrm{Re}\left(\mathbf{Y}^t \mathbf{X}(\theta)\right) \tag{17}$$

where the Fourier series is written with θ the crankshaft angle, $\mathbf{Y}^t = \begin{bmatrix} A_1 & \dots & A_N \end{bmatrix}$ and $\mathbf{X}^t(\theta) = \begin{bmatrix} e^{j\theta} & \dots & e^{jN\theta} \end{bmatrix}$. By applying Equation (16) in Equation (4), it could be possible to compute theoretical values of the Fourier coefficients with the balance harmonic method. However, for a practical issue, the Fourier coefficients must be obtained from the measurement of the instantaneous engine speed. Moreover, their values can not be assumed constant: they are time-varying due to the time-varying engine speed, the time-varying load torque and the active control action of the synchronous machine. Hence, the Fourier coefficients must be periodically actualized. If we consider an actualization at every ½ turn of the crankshaft, the vector $\mathbf{Y}[k]$ is obtained by harmonic analysis of the crankshaft angle captured for ½ turn:

$$\mathbf{Y}[k] = \frac{2}{\pi}\int_{S_k} \omega(\theta)\mathbf{X}(-\theta)d\theta \tag{18}$$

where $\mathbf{X}^t(-\theta) = \lfloor e^{-j\theta} \quad e^{-j2\theta} \quad \dots \quad e^{-jN\theta} \rfloor$ and $S_k = [k\pi/2; (k+1)\pi/2]$ is the angle support of integration at the iteration *k*. For example, the actualization may be synchronized at the top dead center of the piston movements.

3.3 Harmonic Model

The considered command of the synchronous machine is the fluctuating *q*-axis voltage. This is a periodic signal written as a Fourier series:

$$u(\theta) = 2\,\mathrm{Re}\left(\mathbf{U}^t[k]\mathbf{X}(\theta)\right) \text{ for } \theta \in S_k . \tag{19}$$

where the Fourier coefficients are written under the compact form: $\mathbf{U}^t = [U_1 \quad U_2 \quad \dots \quad U_N]$.

By applying Fourier series in Equations (4), (17) and (19), we obtained the following discrete time system:

$$\mathbf{Y}[k] = \mathbf{H}(\varpi)\mathbf{U}[k] + \mathbf{D}(\varpi) \tag{20}$$

where $\mathbf{H}(\varpi) = diag\{H(jn\varpi)\}$ a diagonal matrix with $H(jn\varpi) = \dfrac{3p\lambda_a/2}{(Jjn\omega + b)(L_q jn\omega + R)}$. The vector of disturbances, $\mathbf{D}$, is the harmonic analysis of the engine speed pulsation without active control: $\mathbf{D} = \mathbf{Y}$ when $\mathbf{U} = \mathbf{0}$. According to Equations (2), (3) and (4), the values in this vector are time-varying for mean engine speed variation.

3.4 Harmonic Controller

With harmonic controller, an iterative algorithm is designed to minimize a quadratic error criterion : $J(\mathbf{U}) = \|\mathbf{Y}\|_2^2$. According to the Parseval's theorem, this equivalent to minimize $\int_S \widetilde{\omega}^2 dt$. One of the most useful learning laws is the Newton algorithm which is used to iteratively adjust the vector $\mathbf{U}$ according to the law: $\mathbf{U}[k+1] = \mathbf{U}[k] - \mu \mathbf{\Delta}_{\mathbf{U}}^{-1} \nabla_{\mathbf{U}} J$ where $\nabla_{\mathbf{U}} J$ and $\mathbf{\Delta}_{\mathbf{U}} J$ are respectively the gradient and the hessien of the criterion, and μ the adaptation coefficient. According to Equation (20), the gradient is $\nabla_{\mathbf{U}} J = 2\mathbf{H}(\varpi)\mathbf{Y}$ and the hessien $\mathbf{\Delta}_{\mathbf{U}} J = 2\mathbf{H}^h(\varpi)\mathbf{H}(\varpi)$ (*h* denotes the hermitian), consequently, the learning law is:

$$\mathbf{U}[k+1] = \mathbf{U}[k] - \mu \hat{\mathbf{H}}^{-1}(\varpi)\mathbf{Y}[k] \tag{21}$$

where $\hat{\mathbf{H}}(\varpi)$ is a biased estimation of $\mathbf{H}(\varpi)$ due to the parameter variations. Such harmonic controller is equivalent to *N* independent resonant controllers tuned on each harmonic (Sievers and Flotow, 1992).

3.5 Stability analysis of the harmonic controller

For a given ϖ , Equations (20) and (21) give

$$\mathbf{X}[k+1] = \mathbf{A}\mathbf{X}[k] \tag{22}$$

where the evolution matrix is $\mathbf{A} = \mathbf{I} - \mu\mathbf{\Delta} \in C^{N \times N}$, $\mathbf{X} = \mathbf{U} - \mathbf{U}_{opt} \in C^N$ and $\mathbf{\Delta}$ is a multiplicative uncertainty: $\mathbf{H} = \hat{\mathbf{H}}\mathbf{\Delta}$. Hence, the closed loop system is stable if and only if $|\lambda_n(\mathbf{A})| < 1$ for any *n*, which is equivalent to say that the necessary and sufficient condition of stability is :

$$\mu < 2\,\mathrm{Re}(\Delta_n) \text{ for any } n \tag{23}$$

When the system is perfectly estimated, $\Delta_n = 1$, the upper bound on the adaptation coefficient is 2, but the tuning $\mu = 1$ ensures a theoretical convergence in one step. On the other hand, in case of biased estimation, $\Delta_n \neq 1$.

A necessary condition of stability is $\mathbf{\Delta}$ to be positive definite: $\mathbf{\Delta}^h + \mathbf{\Delta} > 0$ where $\mathbf{\Delta}^h$ denotes the hermitian of $\mathbf{\Delta}$. In our case, with diagonal matrix, this implies that $\mathrm{Re}(\Delta_n) > 0$ for any *n*. Such condition is verified if the phase-shift errors, the angle of each Δ_n, are bounded between $-\pi/2$ and $\pi/2$. Hence, a slow but robust control can be achieved with a low adaptation coefficient. However, if the phase-shift errors is superior than $\pi/2$, or inferior than $-\pi/2$, robust harmonic control may not be achieved. For our application, the next section explains that robust stability may be achieved.

3.6 Robust Stability of the harmonic controller

When the frequencies to control are superior than the natural cut-off frequencies of the electrical system and mechanical system, $n\varpi >> 1/\tau_e$ and $n\varpi >> 1/\tau_m$, an approximation of $H(s)$ is $H(s) \approx 3p\lambda_a / 2JL_q s^2$. In other words, the complex gains are approximated by $\dfrac{1}{H(jn\varpi)} \approx -k_2 n^2 \varpi^2$ with $k_2 = 2L_q J / 3p\lambda_a$. This means that the phase shift of $\mathbf{H}(\varpi)$ is not a function of k_2. Consequently, the necessary condition of stability is verified for any values of inertia or inductance and most critical cases are for low values of inertia of inductance. Hence, a practical issue to ensure the stability of the harmonic controller consists to estimate the lower bound on k_2 (associated to the lower values of inertia and inductance): $k_2 > k_{2,\min}$; then, the adaptation coefficient can be chosen such that

$$\mu < \frac{\hat{k}_2}{k_{2,\min}} \qquad (24)$$

In other words, an adaptation coefficient sufficient small according to Equation (24) can ensure the stability of the harmonic controller in spite of slow time variation of the inertia of inductance.

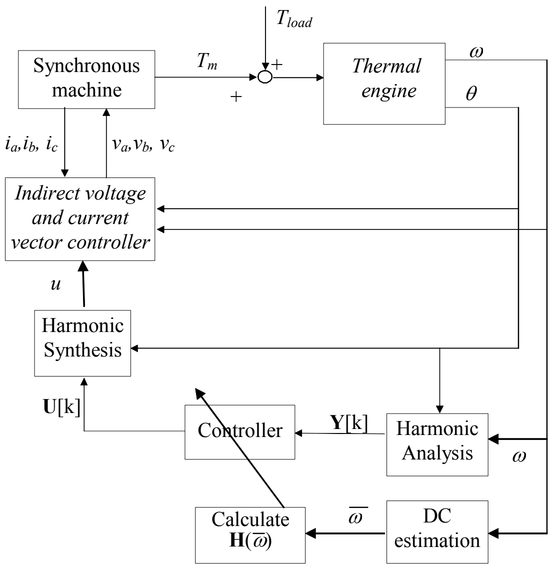

Fig. 1. diagram of the adaptive harmonic controller.

4. SIMULATION

4.1 Simulator

The simulation is carried out using Matlab/Simulink. Table 1 presents the values used for the parameters. Figure 2 shows the pressure in the cylinder versus the crankshaft angle. The internal combustion engine speed is 3000 rpm.

Table 1 values used for the simulation.

damping	b	2	N.m/s
Mass of the rod	ma	0.335	Kg
Mass of the piston	mp	0.840	Kg
Rayon of the rod	r	49×10^{-3}	m
Length of the rod	l	0.186	m
Rod Centre of mass	l_{osc}	0.093	m
Diameter of the piston	d	0.08	m
Mass of the crankshaft	m_{crank}	1	Kg
Direct self	L_d	8.5×10^{-3}	H
Quadrature self	L_q	8.5×10^{-3}	H
Resitance	R	0.2875	Ω
Number of pole pairs	p	4	
Rotor flux linkage	λ_a	0.3	

4.2 Simulations

To demonstrate the usefulness of the controller in counteracting periodic load disturbances, Figures 3 and 4 present the responses without active control from 0 to 0.1 s, and with active control after 0.1s. The order 2, 4, 6 and 8 are considered by the controller. The complex gains used by the controller are approximated by $\frac{1}{H(jn\overline{\omega})} \approx -k_2 n^2 \overline{\omega}^2$ and the adaptation coefficient is set to μ=1. From 0.1 s to 0.25 s the speed fluctuations associated to the orders converge to zero.

The simulations were done for different mean engine load torque. Figure 5 is the plot of the mean engine speed versus time. Figure 6 shows the active control of the engine oscillation after each transient in spite of quick variation of the mean engine speed in the range 2500 rpm to 3200 rpm. This is a critical case. In fact, the harmonic control is not the first priority during acceleration or deceleration of the engine: the PMSM will be used to assist the thermal engine during these phases instead to control the harmonic. Hence, the most important is to ensure the convergence of the controller after the transient, and that is what the Figure 6 shows.

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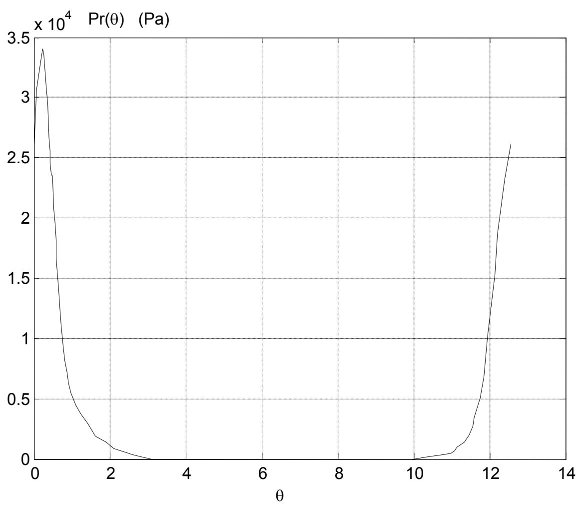

Fig. 2. upward thrust in the cylinder versus the crankshaft angle.

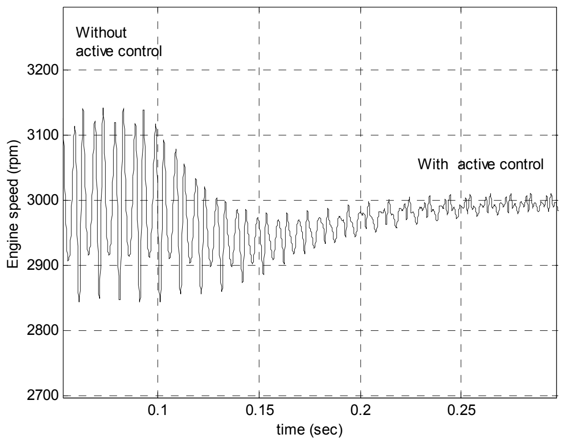

Fig. 3. Transient of the instantaneous engine speed versus the time when the active control is activated: without active control before t=0.1 s, and with active control after t=0.1s .

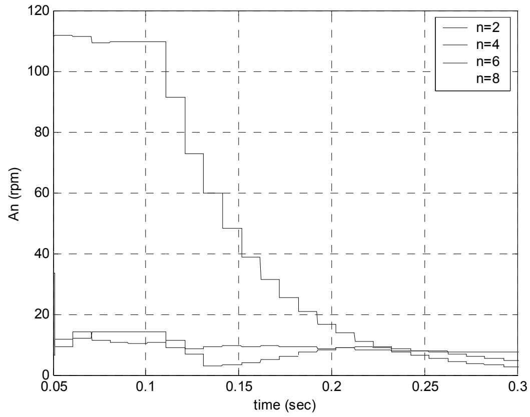

Figure 4: Transient of Fourier coefficient versus time when the active control is activated at time *t*=0.1s.

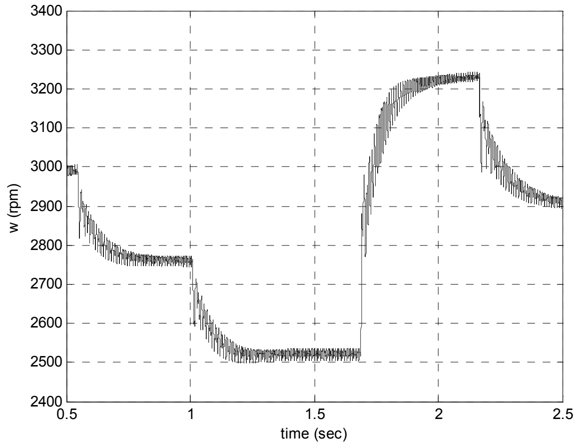

Fig. 5. instantaneous engine speed versus the time for different mean engine speed.

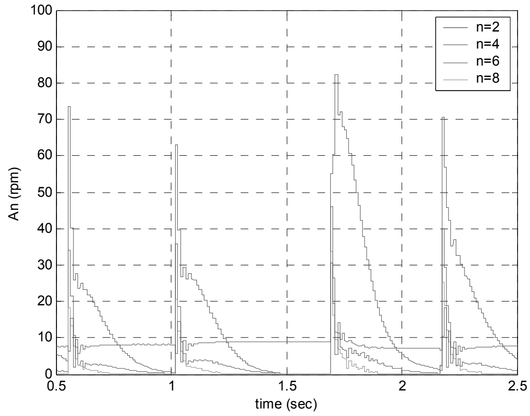

Figure 6: the coefficients versus time when the mean engine speed is time-varying.

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