SUBSPACE METHOD AIDED DATA-DRIVEN DESIGN OF OBSERVER BASED FAULT DETECTION SYSTEMS

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Abstract: This paper deals with the *data-driven* design of observer based FDI systems. The basic idea is to identify parity space and the related matrices, instead of a state space model of the process under consideration, *directly from test data.* The proposed method can be used for the data-driven design of parity space, observer and kalman filter based FDI systems.

Keywords: Fault detection; Parity space methods; Observer based FDI; Subspace methods; Kalman filter based FDI

1. INTRODUCTION

This paper addresses the data-driven design of fault detection and isolation (FDI) systems. It is assumed that the processes under consideration can be modelled as an LTI system described by

$$x(k+1) = Ax(k) + Bu(k) + w(k) \tag{1}$$

$$y(k) = Cx(k) + Du(k) + v(k) \tag{2}$$

where $x(k) \in \mathbf{R}^n, u(k) \in \mathbf{R}^l, y(k) \in \mathbf{R}^m$ denote the vectors of the state variable, process inputs and outputs, respectively. $w(k) \in \mathbf{R}^n, v(k) \in \mathbf{R}^m$ are assumed to be discrete time, zero-mean, white noise satisfying

$$\mathbf{E}w(k) = 0, \mathbf{E}v(k) = 0$$

$$\mathbf{E}\left(\begin{bmatrix} w(i) \\ v(i) \end{bmatrix} \begin{bmatrix} w^T(j) & v^T(j) \end{bmatrix}\right) = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \delta_{ij}$$

and statistically independent of the input vector $u(k)$. It is assumed that system matrices A, B, C, D and system order n as well as Q, R, S **are unknown** *a priori.*

During the last two decades, observer based FDI methodology for LTI systems has been well established and a great number of standard methods are available for designing an observer based FDI system if the system matrices A, B, C, D are given (Chen and Patton, 1999; Gertler, 1998; Patton *et al.*, 2000). It is a reasonable and convincing argument that there exist powerful tools for identifying the system matrices, especially thanks to the significant development of the subspace methods in the last decade (Favoreel *et al.*, 2000; Van Overschee and De Moor, 1996). From the viewpoint of application, the procedure from (test) data to the design of an FDI system consists of two steps: (a) modelling, i.e. identifying A, B, C, D and n (b) FDI system design based on A, B, C, D and n.

[1] supported by the EU grant IST-2001-32122 IFATIS

In this paper, a procedure from the (test) data directly to the FDI system design is proposed. In this procedure, **the residual generator**, instead of A, B, C, D and n, **will be directly identified from the test data**. The major advantage of this design method is that the design procedure is simplified and no special knowledge of control theory is needed for the design of observer or parity space based residual generators.

This study is strongly motivated by the successful application of the so-called PCA (Principal Component Analysis) in process industry (Dunia *et al.*, 1996), where FDI is achieved based on the identification of the process principal components directly from the test data. Some recent results demonstrate that FDI in dynamic systems can also be realised by means of Dynamic PCA (Li and Qin, 2001). A further motivation of this study is the intimate relationship between the subspace methods and parity vectors which was pointed out, for instance, by Basseville in (Basseville, 1998) and more recently in (Basseville, 2003).

2. NOTATIONS AND PRELIMINARIES

Throughout this paper, the following data structure will be used:

$$u_p(k) = \begin{bmatrix} u(k-s_p) \\ \vdots \\ u(k-1) \end{bmatrix}, u_f(k) = \begin{bmatrix} u(k) \\ \vdots \\ u(k+s_f-1) \end{bmatrix}$$
$$y_p(k) = \begin{bmatrix} y(k-s_p) \\ \vdots \\ y(k-1) \end{bmatrix}, y_f(k) = \begin{bmatrix} y(k) \\ \vdots \\ y(k+s_f-1) \end{bmatrix}$$
$$Y_p(k) = \begin{bmatrix} y_p(k) \ y_p(k+1) \ \cdots \ y_p(k+N-1) \end{bmatrix}$$
$$U_p(k) = \begin{bmatrix} u_p(k) \ u_p(k+1) \ \cdots \ u_p(k+N-1) \end{bmatrix}$$
$$Y_f(k) = \begin{bmatrix} y_f(k) \ y_f(k+1) \ \cdots \ y_f(k+N-1) \end{bmatrix}$$
$$U_f(k) = \begin{bmatrix} u_f(k) \ u_f(k+1) \ \cdots \ u_f(k+N-1) \end{bmatrix}$$
$$Z_p(k) = \begin{bmatrix} Y_p(k) \\ U_p(k) \end{bmatrix}, Z_f(k) = \begin{bmatrix} Y_f(k) \\ U_f(k) \end{bmatrix} \quad (3)$$

where the subscripts p, f denote, respectively, the past and the future (data), and s_p, s_f stand for some integers with $s_p, s_f > n$.

A straightforward calculation based on system model (1)-(2) leads to the following input-output matrix equation which is standard in the framework of subspace identification (Favoreel *et al.*, 2000; Van Overschee and De Moor, 1996):

$$Y_f(k) = \Gamma_{s_f} X(k) + H_{s_f} U_f(k) + G_{s_f} W_f(k) + V_f(k)$$
$$X(k) = \begin{bmatrix} x(k) \ \cdots \ x(k+N-1) \end{bmatrix} \quad (4)$$
$$V_f(k) = \begin{bmatrix} v_f(k) \ \cdots \ v_f(k+N-1) \end{bmatrix}$$
$$W_f(k) = \begin{bmatrix} w_f(k) \ \cdots \ w_f(k+N-1) \end{bmatrix}$$
$$v_f(k) = \begin{bmatrix} v(k) \\ \vdots \\ v(k+s_f-1) \end{bmatrix}, w_f = \begin{bmatrix} w(k) \\ \vdots \\ w(k+s_f-1) \end{bmatrix}$$
$$G_{s_f} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ CA^{s_f-2} & \cdots & C & 0 \end{bmatrix}, \Gamma_{s_f} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{s_f-1} \end{bmatrix}$$
$$H_{s_f} = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ CA^{s_f-2}B & \cdots & CB & D \end{bmatrix}$$

(4) is the model used in the sequent sections.

Parity space and observer based FDI schemes (Chen and Patton, 1999; Gertler, 1998; Patton *et al.*, 2000) are two well-established techniques. On the assumption that A, B, C, D are known, the design of a parity space based residual generator consists in the solution of equation

$$\alpha_s \Gamma_s = 0, \alpha_s = \begin{bmatrix} \alpha_{s,0} \ \alpha_{s,1} \ \cdots \ \alpha_{s,s-1} \end{bmatrix} \quad (5)$$

for the so-called parity vector $\alpha_s \in \mathbf{R}^{1\times ms}$, where Γ_s is identical with Γ_{s_f} (replacing subscript s_f by s), while the design of an observer based residual generator is achieved by solving the so-called Luenberger equations,

$$TA - A_z T = LC, B_z = TB - LD \quad (6)$$
$$c_z T + gC = 0, d_z + gD = 0 \quad (7)$$

for A_z (should be stable), B_z, c_z, d_z, g, L together with the transformation matrix T. It follows then the construction of residual generators

$$r(k) = \alpha_s \left(y_s(k) - H_s u_s(k) \right) \quad (8)$$
$$y_s(k) = \begin{bmatrix} y(k-s+1) \\ \vdots \\ y(k) \end{bmatrix}, u_s(k) = \begin{bmatrix} u(k-s+1) \\ \vdots \\ u(k) \end{bmatrix}$$

for the parity space based residual generation and

$$z(k+1) = A_z z(k) + B_z u(k) + Ly(k) \quad (9)$$
$$r(k) = c_z z(k) + gy(k) + d_z u(k) \quad (10)$$

for the observer based residual generation, where H_s has the same definition like H_{s_f} and $r(k)$ is called residual signal.

3. IDENTIFICATION OF PARITY SPACE

As shown in (8), for the construction of a parity space based residual generator $\alpha_s, \alpha_s H_s$ are needed. Note that for $s = s_f$, $\alpha_s \in \Gamma_{s_f}^{\perp}$ and $\alpha_s H_s \in \Gamma_{s_f}^{\perp} H_{s_f}$ with $\Gamma_{s_f}^{\perp}$ denoting the null space of Γ_{s_f}. Thus, identification of $\Gamma_{s_f}^{\perp}$, $\Gamma_{s_f}^{\perp} H_{s_f}$ would allow a direct construction of a parity space based residual generator. In this section, an algorithm is proposed for the identification of $\Gamma_{s_f}^{\perp}, \Gamma_{s_f}^{\perp} H_{s_f}$ *directly*. It is similar to the subspace method reported in (Wang and Qin, 2002).

It follows from (4) that

$$Z_f(k)Z_p^T(k) = \begin{bmatrix} \Gamma_{s_f} & H_{s_f} \\ 0 & I \end{bmatrix} \begin{bmatrix} X(k) \\ U_f \end{bmatrix} Z_p^T(k) + \begin{bmatrix} G_{s_f} W_f(k) + V_f(k) \\ 0 \end{bmatrix} Z_p^T(k)$$

Recall that $\lim_{N\to\infty} \frac{1}{N} \left(G_{s_f} W_f(k) + V_f(k) \right) Z_p^T(k) = 0$. Thus, for large N

$$Z_f(k)Z_p^T(k) \approx \begin{bmatrix} \Gamma_{s_f} & H_{s_f} \\ 0 & I \end{bmatrix} \begin{bmatrix} X(k) \\ U_f \end{bmatrix} Z_p^T(k)$$

By selecting $U_f(k), U_p(k)$ in such a way that

$$rank\left(Z_f Z_p^T\right) = rank\left(Z_f\right)$$

$$rank\left(Z_f(k)\right) = rank\left(\begin{bmatrix} \Gamma_{s_f} & H_{s_f} \\ 0 & I \end{bmatrix} \begin{bmatrix} X(k) \\ U_f \end{bmatrix}\right) = rank \begin{bmatrix} \Gamma_{s_f} & H_{s_f} \\ 0 & I \end{bmatrix} = n + s_f l$$

the following relations hold

$$rank\left(Z_f^{\perp}\right) = rank\left(\Gamma_{s_f}^{\perp}\right) = s_f m - n$$

$$\Longrightarrow Z_f^{\perp} \begin{bmatrix} \Gamma_{s_f} & H_{s_f} \\ 0 & I \end{bmatrix} = 0 \qquad (11)$$

Now, do an SVD on $Z_f(k)Z_p^T(k)$,

$$Z_f Z_p^T = U_z \begin{bmatrix} \Sigma_{z,1} & 0 \\ 0 & \Sigma_{z,2} \end{bmatrix} V_z^T$$

with unitary matrices $U_z \in \mathbf{R}^{(l+m)s_f \times (l+m)s_f}, V_z^T \in \mathbf{R}^{(m+l)s_p \times (m+l)s_p}$. Suppose that $\Sigma_{z,2} = 0$ and thus

$$rank\left(\Sigma_{z,1}\right) = rank \begin{bmatrix} \Gamma_{s_f} & H_{s_f} \\ 0 & I \end{bmatrix} = n + s_f l$$

It leads to

$$Z_f^{\perp} = \begin{bmatrix} 0 & P \end{bmatrix} U_z^{-1}$$

where $P \in \mathbf{R}^{\eta\times\eta}, \eta = s_f m - n$, is an arbitrary regular matrix. Let

$$U_z = \begin{bmatrix} U_{z,11} & U_{z,12} \\ U_{z,21} & U_{z,22} \end{bmatrix}, U_{z,11} \in \mathbf{R}^{ms_f \times (s_f l + n)}$$

$$U_{z,22} \in \mathbf{R}^{ls_f \times \eta}, U_{z,12}^T \in \mathbf{R}^{\eta \times ms_f}$$

It then turns out

$$Z_f^{\perp} = P \begin{bmatrix} U_{z,12}^T & U_{z,22}^T \end{bmatrix} \in \mathbf{R}^{\eta\times(l+m)s_f}$$

$$\Longrightarrow \Gamma_{s_f}^{\perp} = PU_{z,12}^T, \Gamma_{s_f}^{\perp} H_{s_f} = -PU_{z,22}^T \qquad (12)$$

The following algorithm summarises the major steps to the identification of $\Gamma_{s_f}^{\perp}, \Gamma_{s_f}^{\perp} H_{s_f}$:

S1: Generate data sets $Z_f(k), Z_p(k)$;

S2: Construct $Z_f(k)Z_p^T(k)$ and moreover do an SVD on $Z_f(k)Z_p^T(k)$;

S3: Find $U_{z,12}^T, U_{z,22}^T$ and select P;

S4: Compute $\Gamma_{s_f}^{\perp}, \Gamma_{s_f}^{\perp} H_{s_f}$ according to (12).

4. RESIDUAL GENERATOR DESIGN BASED ON $\Gamma_{S_F}^{\perp}, \Gamma_{S_F}^{\perp} H_{S_F}$

In this section, residual generator design based on $\Gamma_{s_f}^{\perp}, \Gamma_{s_f}^{\perp} H_{s_f}$ will be studied. Since the design of parity space based residual generator is straightforward, the major focus is on the design of observer based residual generators.

4.1 Design of parity space based residual generators

Having identified $\Gamma_{s_f}^{\perp}, \Gamma_{s_f}^{\perp} H_{s_f}$, the construction of a parity space based residual generator is straightforward. Suppose that $\alpha_{s_f} \in \Gamma_{s_f}^{\perp}, \alpha_{s_f} H_{s_f} \in \Gamma_{s_f}^{\perp} H_{s_f}$, then following (8) the corresponding residual generator can be built as follows:

$$r(k) = \alpha_{s_f} y_{s_f}(k) - \alpha_{s_f} H_{s_f} u_{s_f}(k) \qquad (13)$$

It is worth emphasising that *residual generator (13) is designed only based on the parameter matrices identified using test data. No knowledge of the process model is needed.* Remember that for the identification purpose s_f could be selected significantly larger than n, the order of the process under consideration. For the purpose of achieving a reduced order residual generator, one can use an algorithm similar to the well known Gaussian (elimination) algorithm (Gantmacher, 1986). First, find a regular matrix P_1 so that in matrix

$$P_1 \Gamma_{s_f}^{\perp} := \begin{bmatrix} \bar{\alpha}_{1,1} & \cdots & \bar{\alpha}_{1,s_f m} \\ \vdots & \vdots & \vdots \\ \bar{\alpha}_{\eta,1} & \cdots & \bar{\alpha}_{\eta,s_f m} \end{bmatrix}$$

$\bar{\alpha}_{\eta,s_f m} \neq 0$. Now, let

$$P_2 = \begin{bmatrix} 1 & 0 & & -\frac{\bar{\alpha}_{1,s_f m}}{\bar{\alpha}_{\eta,s_f m}} \\ 0 & \ddots & & \vdots \\ \vdots & \ddots & 1 & -\frac{\bar{\alpha}_{\eta-1,s_f m}}{\bar{\alpha}_{\eta,s_f m}} \\ 0 & \cdots & 0 & 1 \end{bmatrix}$$

It turns out that $\begin{bmatrix} 0 & \cdots & 0 & \bar{\alpha}_{\eta,s_f m} \end{bmatrix}^T$ is the last column of $P_2 P_1 \Gamma_{s_f}^{\perp}$. Repeating this procedure will lead to

$$P\Gamma_{s_f}^{\perp} = \begin{bmatrix} \tilde{\alpha}_{1,1} & \cdots & \tilde{\alpha}_{1,n}^1 & 0 & \cdots \\ \vdots & & & \ddots & \ddots \\ \tilde{\alpha}_{\eta,1} & & \cdots & & \tilde{\alpha}_{\eta,s_f m} \end{bmatrix}$$

where P is the product of all regular matrices $P_i, i = 1, \cdots$. Let $\tilde{\alpha}_1 = \begin{bmatrix} \tilde{\alpha}_{1,1} & \cdots & \tilde{\alpha}_{1,n}^1 \end{bmatrix}$, then

$$r(k) = \tilde{\alpha}_1 y_n(k) - \tilde{\alpha}_1 H_n u_n(k)$$

is a residual generator of the n-th order, where $u_n(k), y_n(k), H_n$ are equal to $u_{s_f}(k), y_{s_f}(k), H_{s_f}$, respectively, for $s_f = n$. It is worth pointing out that the minimum order of the parity vectors is the minimum observability index (Ding *et al.*, 1999), which can be much smaller than n.

4.2 Design of observer based residual generators

In this subsection, the problem of designing an observer based residual generator based on $\Gamma_{s_f}^{\perp}, \Gamma_{s_f}^{\perp} H_{s_f}$ will be addressed. To this end, the following theorem is first introduced.

Theorem 1. Given system model (1)-(2), parity vector $\alpha_s = \left[\alpha_{s,0}\ \alpha_{s,1}\ \cdots\ \alpha_{s,s-1}\right]$, then

$$A_z = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}, L = -\begin{bmatrix} \alpha_{s,0} \\ \alpha_{s,1} \\ \vdots \\ \alpha_{s,s-2} \end{bmatrix} \quad (14)$$

$$c_z = \left[0 \cdots 0\ 1\right] \in \mathbf{R}^{s-1}, g = -\alpha_{s,s-1}$$

$$T = \begin{bmatrix} \alpha_{s,1} & \alpha_{s,2} & \cdots & \alpha_{s,s-1} & \alpha_{s,s-1} \\ \alpha_{s,2} & \cdots & \cdots & \alpha_{s,s-1} & 0 \\ \vdots & \cdots & \cdots & \vdots & \vdots \\ \alpha_{s,s-1} & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{s-2} \end{bmatrix}$$

$$B_z = TB - LD, d_z = -gD \quad (15)$$

solve Luenberger equations (6)-(7).

The proof is straightforward and is, due to the space limitation, omitted here. Note that

$$TB - LD = \begin{bmatrix} \alpha_{s,0} & \alpha_{s,1} & \alpha_{s,2} & \cdots & \alpha_{s,s-1} \\ \alpha_{s,1} & \alpha_{s,2} & \cdots & \alpha_{s,s-1} & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{s,s-2} & \alpha_{s,s-1} & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} D \\ CB \\ \vdots \\ CA^{s-2}B \end{bmatrix}$$

Moreover,

$$\left[\alpha_{s,i}\ \alpha_{s,i+1}\ \cdots\ \alpha_{s,s-1}\ 0 \cdots 0\right] \begin{bmatrix} D \\ CB \\ \vdots \\ CA^{s-2}B \end{bmatrix}$$

can be re-written into $\alpha_s H_{s,i+1}$ with

$$H_{s,i} = \left[0 \cdots 0\ D^T\ (CB)^T\ \cdots\ \left(CA^{s-1-i}B\right)^T\right]^T$$

As a result,

$$B_z = \begin{bmatrix} \alpha_s H_{s,1} \\ \alpha_s H_{s,2} \\ \vdots \\ \alpha_s H_{s,s-1} \end{bmatrix}, d_z = \alpha_s H_{s,s}. \quad (16)$$

Thus, it follows from Theorem 1 and (16) that a residual generator of form (9)-(10) with A_z, c_z defined in Theorem 1 can be constructed based on $\alpha_{s_f} \in \Gamma_{s_f}^{\perp}, \alpha_s H_{s_f} \in \Gamma_{s_f}^{\perp} H_{s_f}$. Note that this residual generator is open-loop structured, although it is given in a recursive form, and moreover its poles are fixed at the original. To achieve additional design freedom and a closed-loop structure, residual generator (9)-(10) is now modified by feeding back the residual signal $r(k)$ to state equation (9), i.e.

$$\begin{aligned} z(k+1) &= A_z z(k) + B_z u(k) + Ly(k) - L_0 r(k) \\ r(k) &= c_z z(k) + gy(k) + d_z u(k) \end{aligned} \quad (17)$$

where L_0 is the so-called observer gain and provides designers with additional degree of design freedom. Suppose that the process model (1)-(2) is extended by two additional terms, f_c, f_s, to represent component and sensor faults as follows

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + w(k) + f_c(k) \\ y(k) &= Cx(k) + Du(k) + v(k) + f_s(k) \end{aligned}$$

then the dynamics of residual generator (17) is governed by

$$\begin{aligned} e(k+1) &= (A_z - L_0 c_z)\, e(k) + \bar{w}(k) + \bar{f}_c(k) \\ r(k) &= -c_z e(k) + gv(k) + gf_s(k) \quad (18) \\ \bar{w}(k) &= Tw(k) - (L - L_0 g)\, v(k) \\ \bar{f}_c(k) &= Tf_c(k) - (L - L_0 g) f_s(k) \end{aligned}$$

Note that

$$\text{for } L_0 = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_{s-1} \end{bmatrix}, A_z - L_0 c_z = \begin{bmatrix} 0 & 0 & \cdots & -l_1 \\ 1 & 0 & \cdots & -l_2 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & -l_{s-1} \end{bmatrix}$$

Thus, the eigenvalues of $A_z - L_0 c_z$ are arbitrarily assignable.

In practice, in order to achieve a successful FDI a bank of residual generators will be used for the purpose of residual generation. To this end, a set of vectors selected from $\Gamma_{s_f}^{\perp}, \Gamma_{s_f}^{\perp} H_{s_f}$ are needed. For instance, for $f_c = 0$ an isolation of sensor faults can be achieved by using a bank of m residual generators constructed as follows. Suppose that

$$\Gamma_{s_f}^{\perp} = \left[\Gamma_{s_f,0}^{\perp}\ \Gamma_{s_f,1}^{\perp}\ \cdots\ \Gamma_{s_f,s_f-1}^{\perp}\right]$$

Solve equations

$$\begin{aligned} \bar{P}\Gamma_{s_f,j}^{\perp} &= diag(\alpha_{n,j}^1, \alpha_{n,j}^2, \cdots, \alpha_{n,j}^m), j = 0, \cdots, n-1 \\ \bar{P}\Gamma_{s_f,j}^{\perp} &= 0, j = n, \cdots, s_f - 1 \end{aligned} \quad (19)$$

for $\bar{P} \in \mathbf{R}^{m \times \eta}$, where $\alpha_{n,j}^i, i = 1, \cdots, m$ are some nonzero constants. Let

$$\bar{P} = \begin{bmatrix} \bar{p}_1 \\ \vdots \\ \bar{p}_m \end{bmatrix}, \alpha_n^i = \bar{p}_i \Gamma_{s_f,i}^{\perp}, i = 1, \cdots, m$$

Then, a bank of m residual generators in form of (17) can be designed by using Theorem 1. The order of these residual generators will not exceed n, and each of them can be described by

$$
\begin{aligned}
z^i(k+1) &= A_z^i z^i(k) + B_z^i u(k) + L^i y_i(k) - L_0^i r^i(k) \\
r^i(k) &= c_z^i z^i(k) + g^i y_i(k) + d_z^i u(k) \\
y_i(k) &= C_i x(k) + D_i u(k) + v_i(k) + f_{s,i}(k) \\
B_z^i &= \begin{bmatrix} \alpha_n^i H_{s_f,1} \\ \vdots \\ \alpha_n^i H_{s_f,s_f-1} \end{bmatrix}, L^i = -\begin{bmatrix} \alpha_{n,0}^i \\ \vdots \\ \alpha_{n,n-2}^i \end{bmatrix} \\
g^i &= -\alpha_{n,n-1}^i, d_z^i = -g^i D_i, i = 1, \cdots, m
\end{aligned}
$$

whose dynamics is governed by

$$
\begin{aligned}
e^i(k+1) &= \left(A_z^i - L_0^i c_z^i\right) e^i(k) + T^i w(k) - \\
&\quad (L^i - L_0^i g) v_i(k) - (L^i - L_0^i g) f_{s,i}(k) \\
r^i(k) &= -c_z^i e^i(k) + g^i v_i(k) + g^i f_{s,i}(k)
\end{aligned}
$$

It is evident that each residual generator is only influenced by one sensor fault. Indeed, the basic idea behind this scheme is that each residual generator is driven by only one output signal and thus the generated residual signal will only be influenced by one sensor fault. This also proves the solvability of eq.(19).

In order to isolate component faults, which can be generally formulated as $f_c = E\bar{f}_c$, an identification of matrix $\Gamma_{s_f}^{\perp} H_{f_c,s_f}$ is needed, where

$$
H_{f_c,s_f} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ CE & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ CA^{s_f-2}E & \cdots & CE & 0 \end{bmatrix}
$$

This can be achieved using the same procedure like the one of identifying $\Gamma_{s_f}^{\perp} H_{s_f}$.

5. DESIGN OF KALMAN FILTER BASED RESIDUAL GENERATORS

In the former section, an approach to the design of observer based residual generators has been presented without considering the influence of process and measurement noises. In this section, an approach to the design of Kalman filter based residual generators will be developed, which are knowingly powerful to solve FDI problems for processes with strong process and measurement noises.

Recall that given $\alpha_s \in \Gamma_{s_f}^{\perp}$ a residual generator of form (9)-(10) can be constructed, whose dynamics is governed by

$$
\begin{aligned}
e(k+1) &= A_z e(k) + T w(k) + L v(k) \quad (20) \\
r(k) &= -c_z e(k) + g v(k) \quad (21)
\end{aligned}
$$

Thus, if

$$
\mathbf{E}\left(\begin{bmatrix} \bar{w}(i) \\ \bar{v}(i) \end{bmatrix} \begin{bmatrix} \bar{w}^T(j) & \bar{v}^T(j) \end{bmatrix}\right) = \begin{bmatrix} V_{\bar{w}\bar{w}} & V_{\bar{w}\bar{v}} \\ V_{\bar{w}\bar{v}}^T & V_{\bar{v}\bar{v}} \end{bmatrix} \delta_{ij}
$$

is known with

$$
\begin{aligned}
\bar{w}(k) &= T w(k) + L v(k), \bar{v}(k) = g v(k) \\
V_{\bar{w}\bar{w}} &= TQT^T + LRL^T + TSL^T + LS^T T^T \\
V_{\bar{w}\bar{v}} &= TSg^T + LRg^T, V_{\bar{v}\bar{v}} = gRg^T
\end{aligned}
$$

then the following residual generator

$$
\begin{aligned}
z(k+1) &= A_z z(k) + B_z u(k) + L y(k) - L_0 r(k) \quad (22) \\
r(k) &= c_z z(k) + g y(k) + d_z u(k) \quad (23) \\
L_0 &= -P\tilde{R}^{-1}, \Sigma = A_z \Sigma A_z^T - P\tilde{R}^{-1} P^T + V_{\bar{w}\bar{w}} \\
P &= V_{\bar{w}\bar{v}} - A_z \Sigma c_z^T, \tilde{R} = c_z \Sigma c_z^T + V_{\bar{v}\bar{v}}
\end{aligned}
$$

would deliver a white residual signal $r(k)$ (innovations sequence) with

$$
\mathbf{E}\left(r(i) r^T(j)\right) = \left(c_z \Sigma c_z^T + gRg^T\right) \delta_{ij}
$$

For this reason, the identification of $V_{\bar{v}\bar{v}}, V_{\bar{w}\bar{v}}, V_{\bar{w}\bar{w}}$ is the major focus of this section.

For the purpose of real application but without loss of generality (see also the discussion in Subsection 4.1), it is assumed that there exists $\alpha_s = \begin{bmatrix} \alpha_{s,0} & \cdots & \alpha_{s,s-1} \end{bmatrix} \in \mathbf{R}^s$ so that

$$
\bar{\alpha}_s = \begin{bmatrix} \alpha_s & 0 & \cdots & 0 \end{bmatrix} \in \Gamma_{s_f}^{\perp}
$$

with $s << s_f, s_p$. Below, the problem of identifying $V_{\bar{v}\bar{v}}, V_{\bar{w}\bar{v}}, V_{\bar{w}\bar{w}}$ for given $\alpha_s, \alpha_s H_s$ as well as U_f, Y_f, U_p, Y_p will be outlined in the form of an algorithm.

Step1: Generate data set:

$$
\Psi(k+i) = \frac{1}{N} \begin{bmatrix} \alpha_s & -\alpha_s H_s \end{bmatrix} Z_{f,s}(k+i) Z_p^T(k)
$$

for $i = 0, \cdots, s-1$, where $Z_{f,s}(k+i)$ denotes the first s rows of $Z_f(k+i)$. It leads to, according to Theorem 1 and eqs.(20)-(21),

$$
\Gamma_z = \begin{bmatrix} c_z \\ \vdots \\ c_z A_z^{s-2} \end{bmatrix} = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ \vdots & & & 0 \\ 0 & 1 & & \vdots \\ 1 & 0 & \cdots & 0 \end{bmatrix}
$$

$$
\begin{aligned}
\Psi_k &= \frac{1}{N}\left(\Gamma_z E(k) + G_{s-1} W_{f,s-1}(k) + V_{f,s-1}(k)\right) Z_p^T(k) \\
\Psi_k &= \begin{bmatrix} \Psi^T(k) & \cdots & \Psi^T(k+s-2) \end{bmatrix}^T \\
E(k) &= \begin{bmatrix} e(k) & \cdots & e(k+N-1) \end{bmatrix}
\end{aligned}
$$

$W_{f,s-1}(k), V_{f,s-1}(k)$ denote the first $s-1$ rows of $W_f(k), V_f(k)$ respectively. Note that

$$
\begin{aligned}
&\frac{1}{N}\left(G_{s-1} W_{f,s-1}(k) + V_{f,s-1}(k)\right) Z_p^T(k) \simeq 0 \\
&\Longrightarrow \Psi_k \simeq \frac{1}{N} \Gamma_z E(k) Z_p^T(k)
\end{aligned}
$$

Step 2: Compute $E(k), E(k+1)$ as follows

$$E(k) = \Gamma_z^{-1}\Psi_k\Phi_k, E(k+1) = \Gamma_z^{-1}\Psi_{k+1}\Phi_{k+1}$$

where Φ_k, Φ_{k+1} are the pseudo-inverse of $Z_p^T(k)/N$, $Z_p^T(k+1)/N$ respectively;

Step 3: Compute

$$\bar{W} = E(k+1) - A_z E(k)$$
$$\bar{V} = \left[\, \alpha_s \; -\alpha_s H_s \,\right] Z_{f,s}(k) + c_z E(k)$$

Step 4: Compute

$$V_{\bar{v}\bar{v}} = \bar{V}\bar{V}^T/N, V_{\bar{w}\bar{v}} = \bar{W}\bar{V}^T/N, V_{\bar{w}\bar{w}} = \bar{W}\bar{W}^T/N$$

It is worth pointing out that some of subspace methods also deliver a sequence of state estimates that can be interpreted as the solution of a bank of Kalman filters (Favoreel *et al.*, 2000). The major differences between the Kalman filters implicated in the subspace methods and the Kalman filter based residual generator (22)-(23) lie in: (a) in (22)-(23), $z(k) = Tx(k)$, instead of $x(k)$, is estimated. The order of the Kalman filter (22) s can be much smaller than n (Ding *et al.*, 1999). (b) The estimator (22) will be computed on-line and recursively aiming at generating the residual signal.

6. CONCLUDING REMARKS

In this paper, an approach to data-driven design of parity space, observer and Kalman filter based residual generators has been introduced. The basic idea behind this approach is the identification of parity space using the test data. This allows a direct design of parity space based residual generators. Moreover, based on a relationship between a parity vector and an observer based residual generator, data-driven design of observer and Kalman filter based residual generators is also realised.

An FDI system consists of two parts: residual generation and residual evaluation. Due to the limited space, only the part concerning residual generation has been presented in this paper. In the context of residual evaluation study, both model uncertainties and noises have to be taken into account. For the case that only noises are present, the residual evaluation is trivial. One can use, for instance, standard methods for the evaluation of the innovations sequence generated by the Kalman filter based residual generator (22)-(23) (Basseville and Nikiforov, 1993). For the evaluation of residual signals generated by parity space based residual generator (13), $E\left(r(k)r^T(k)\right)$ will be additionally computed using standard statistical methods. The residual evaluation follows, for instance, methods introduced in (Basseville and Nikiforov, 1993; Hagenblad *et al.*, 2002).

The developed approach has been successfully applied to some academic examples. Due to the long data sets, they cannot be included in this paper. It is also planned to test this approach on some well-defined benchmark processes.

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