INVENTORY OUTSOURCING AND RISK MANAGEMENT

Charles S. Tapiero* and Alberto Grando**

** ESSEC, B.P. 91, Avenue Bernard Hirsch, 95021 Cergy Pontoise, France*

*** SDA Bocconi, Milan Via Bocconi, 8, 20144 Milano, Italy*

Abstract: This paper considers the "inventory outsourcing problem" as a VaR (Value at Risk) problem providing a mechanism for managing outsourcing firms when the supplier is a leader having full information of the outsourcing firm's demand distributions and cost parameters. This leads to a Stackleberg game which is solved under a number of assumptions. Both demand dependent and independent models are considered, the latter resulting from (statistical) risk aggregation. A number of examples are solved as well to highlight essential issues underlying the practice of inventory outsourcing-price and supply priorities. These solutions can be be expressed as nonlinear optimization problems which can be solved by standard numerical routines.

Keywords: Production systems, Inventory control, Uncertainty.

1. INTRODUCTION

Outsourcing consists in the transfer of previously in-house activities to a third party (Gattorna, 1988; La Londe and Cooper, 1989; Razzaque and Sheng, 1998). Inventory outsourcing in a supply chain, in particular, arises from three essential motivations: economies of scale, risk and focus (Rao and Young, 1994; van Damme and Ploos van Amstel, 1996; McIvor, 2000). These motivations presume that economic advantage arises from collaboration and exchange between firms, leading to firm's restructuring along supply chains organizations and operations (Bowersox, 1990; Skjoett-Larsen, 2000; van Laarhoven, Berglund and Peters, 2000).

A typical example would be the practice to focus on a JIT (Just in Time) manufacturing strategy and outsource the management of inventories to a carefully selected supplier (for example, see Crawford et al. 1988, Grout and Christy, 1992, Celley et al. 1987).

This practice raises a number of issues however, spanning: A bounded rationality due to limits in information available to the supplier that may lead to excess shortages; A potential for opportunistic behavior by the supplier and the firm alike, as well as problems associated to small numbers bargaining and information asymmetry leading to moral hazard and to adverse selection.

In some cases, several firms acting on the same market might outsource to a common supplier, augmenting significantly the demand volatility faced by the supplier (and thereby augmenting costs). As a result the inventory outsourcing decision as well as the management of supplies are not always worthwhile, involving risks to the outsourcing firm. In order for such decisions to be economic, they require a careful synchronization of orders and supplies as well as a clear statement regarding the supply contracts (such as minimum supply guarantees), emergency supply options and setting priorities by the supplier to minimize clients supply risks and maximize profits as well.

In practice, outsourcing risks includes the risks of outsourcing critical inventories activities and ex-ante and ex-post dependency on suppliers. Additional factors such as supply delays and preferential supplier-firms relationships that cannot be met for one reason or another compound the risks of outsourcing.

The purpose of this paper is to consider the problem of "inventory outsourcing" from a risk perspective using a Value at Risk approach widely applied in finance. Such an approach has the advantage of expressing outsourcing firms risk specifications in a unified and fiancial-money manner and still be coherent with traditional approaches to inventory management (based on cost minimization).

The problem of inventory outsourcing is then set as an inventory game where the firm or the supplier can act as a leader or follower (Stackleberg 1952, see also Tapiero, 2000, 2004). When the supplier is a leader and outsourcing firms followers, the supplier selects a supply policy which is consistent with (outsourcing) the outsourcing firms objectives optimization. And vice versa, when the outsourcing firm is a leader it uses the supplier objective optimization to reach its decision to outsource. For simplicity however only the case of a supplier "leader" is considered and some implications for the outsourcing process are derived.

The practical value of the approach presented in this paper arises however from our transforming the risk associated to the inventory policy into a specific financial-risk which can be optimized conjointly with firms (suppliers and outsourcing) objective function. Such an approach can contribute to a better communication between financial and operational managers in selecting strategic inventory outsourcing and supply policies.

2. INVENTORY OUTSOURCING: MODEL

Consider first and for simplicity individual firms managing inventories independently and ordering the quantities R_j incurring inventory and shortage costs given by c_{1j} and c_{2j} respectively where $\tilde{D}_j$ are the random demand for these quantities faced by firms. The inventory costs for each of the firms j are thus random and defined by:

$$\tilde{C}_j = p_m R_j + c_{1j}\left(R_j - \tilde{D}_j\right)^+ + c_{2j}\left(R_j - \tilde{D}_j\right)^-$$

where p_m is the current market price of buying the product (or a part that might be needed in a production process),

$$\left(x\right)^+ = \max\left(x,0\right) \text{ and } \left(x\right)^- = -\min\left(x,0\right).$$

An optimal ordering policy based on expected costs minimization for each of the firms yields an optimal quantile risk that each firm will use in its deterining the order quantity if it were to self-manage inventories:

$$1 - F_j\left(R_j^*\right) = 1 - \alpha_j = \frac{c_{1j} + p_m}{c_{1j} + c_{2j}}$$

Note that $F_j(\tilde{D}_j)$ is the cumulative density function of the jth firm's demand. The corresponding firm's inventory cost is thus a random variable given by $\tilde{C}_j\left(R_j^*\right)$. In this sense, there is an equivalence between the minimization of expected costs and the risk specification of an inventory shortage—one implies the other. When a firm outsources inventories it benefits from the supplier economies scale resulting in a potentially lower selling price $p_{ms} \le p_m$. Thus, if the firm outsources inventories, its costs are given by:

$$\begin{aligned}\tilde{C}_j^{(o)} &= p_{ms}\left(\tilde{X}_j - \tilde{D}_j, 0\right)^- \\ &\quad + \left(p_m + \Delta p\right)\left(\tilde{D}_j - \tilde{X}_j\right)^+\end{aligned}$$

where $\tilde{X}_j$ is the supply that the firm receives, a function of a number of factors we shall see subsequently. The missing supply quantity is bought through an "emergency" supplier at a premium price. The quantity bought is $\left(\tilde{D}_j - \tilde{X}_j\right)^+$ while Δp is a premium paid for just in time delivery by the "emergency" supplier. The cost differences are thus:

$$\begin{aligned}\Delta\tilde{C}_j &= \tilde{C}_j^{(o)} - \tilde{C}_j = \\ &= p_{ms}\left(\tilde{X}_j - \tilde{D}_j\right)^- + \left(p_m + \Delta p\right)\left(\tilde{D}_j - \tilde{X}_j\right)^+ \\ &\quad - p_m R_j^* - c_{1j}\left(R_j^* - \tilde{D}_j\right)^+ - c_{2j}\left(R_j^* - \tilde{D}_j, 0\right)^+\end{aligned}$$

Where firms' optimal order quantities are:

$$R_j^* = F_j^{-1}\left(\frac{c_{2j} - p_m}{c_{1j} + c_{2j}}\right)$$

The decision to outsource inventories is thus based on the supply risk that managers are willing to sustain (necessarily smaller than the supply risk sustained if they were to self managed their inventories). If this risk is stated as a cost growth percentage risk β in case the firm does decided not to outsource, then:

$$P\left(\frac{\Delta\tilde{C}_j}{\tilde{C}_j} \geq \beta\right) \leq \xi_j$$

The term $\beta\tilde{C}_j$ is therefore cost for risk exposure" that the inventory manager is willing to sustain. While, ξ_j, is the risk probability the firm is willing to assume for costs overruns. Ex-ante these costs are not known however. Thus, an appropriate alternative is to consider the Value of Outsourcing Risk ($VoR_j = \beta E\left(\tilde{C}_j\right)$) given by:

$$P\left(\Delta\tilde{C}_j \geq \beta E\left(\tilde{C}_j\right)\right) \leq \xi_j$$

Where $E\left(\tilde{C}_j\right)$ is the optimal cost of the inventory manager with an optimal inventory order policy $F_j\left(R_j^*\right) = \alpha_j$ calculated earlier. The expression VoR is then the money quantity that a firm will be willing to risk in an outsourcing decision. This expression can be calculated easily by calculating the mean and the variance of $\Delta\tilde{C}_j$ and approximating it by a Normal distribution. Note however that the alternative costs $\left(\tilde{C}_j^{(o)}, \tilde{C}_j\right)$ are correlated and therefore their variance will tend to be larger. As a result, in expectation, there must be benefits to outsourcing to compensate the increased risk.

By the same token, a supplier, a manager of inventories, may be faced with the same types of costs and an aggregate demand. However, due to focusing and economies of scale, the supplier may acquire the goods at a lower price which we denote by $p_{ms} \leq p_m$ which he might partly transfer to firms while reducing the holding and shortage costs given by parameters $\left(c_{1s}, c_{2s}\right)$. In this case, if the supplier adopts an optimal order policy, it would be given by:

$$F_N\left(R^*\right) = \alpha_S$$

where

$$1 - \alpha_s = \frac{c_{1s} + p_{ms}}{c_{1s} + c_{2s}}$$

while $F_N(.)$ is the cumulative distribution function of the aggregate demand $\Im = \sum_{j=1}^{n} \tilde{D}_j$ for all firms.

Due to the statistical aggregation of firms demands and the law of large numbers, it is can be assumed in fact that $F_N(.)$ has a normal probability distribution with mean and variance given by: $\mu = E(\Im)$; $\sigma^2 = \mathrm{var}(\Im)$. Of course, when demands are independent or demands are dependent and negatively correlated, the variance will be smaller. However, when demands are dependent and positively correlated, the demand variance faced by the supplier can be much greater, incurring important and additional costs associated to the outsourcing decision by firms. In such situations, the outsourcing firm might sustain too large a risk which it may have been ill prepared (since it did could not the the fupplier's firms hav dependent demands).

When firms outsource to a supplier, the quantities supplied equal either their state needs (demands) or they incur shortages. Thus, supplies are given by $V_j \leq \tilde{D}_j$, while shortage are $\left(\tilde{D}_j - V_j\right)^+$, potentially supplied by an "emergence" and costly supplier. Let R be the supplier's order from some manufacturer (alternatively, the supplier can be a manufacturer and R is the period production quantity). Then, if we let the supplier be a leader and the outsourcing firms followers, the supplier's problem can be stated as a Stackleberg game where the supplier minimizes expected costs subject to outsourcing firms' optimal ordering policies (implied in their inventory cost minimization).

$$P_S: \begin{cases} \underset{(R,V_i)\geq 0}{Min}\ \Phi = E\left\{\text{Supplier exp. costs}\right\} \\ \qquad \text{Subject to}: \\ F(V_j) \geq \dfrac{c_{2j} - p_m}{c_{1j} + c_{2j}}, \\ F_N\left(R\right) \geq \dfrac{c_{2s} - p_{ms}}{c_{1s} + c_{2s}}; \sum_{j=1}^{n} V_j \leq R \\ p_{ms} \leq p_s \leq p_m, j = 1,2,3,....,n; \end{cases}$$

The supplier's expected costs can be measured in a number of manners. Below we consider an example which is reduced to a quadratic programming problem.

An Example:
Let the unit price of an order R be p_{ms} while the selling price is p_s, $p_{ms} \leq p_s \leq p_m$. The supplier's revenue is $(p_s - p_{ms})\sum_{j=1}^{n} V_j$, the inventory cost is , $c_{1S}\left(R - \sum_{j=1}^{n} V_j\right)$, $c_{1s} \leq c_{1j}$ and the penalty (potentially the contracted cost) in case of supply failure is $\sum_{j=1}^{n} C_{2Sj} E\left(\tilde{D}_j - V_j\right)^+$. The expected cost of the supplier is thus:

$$\underset{(R,V_1,V_2,\dots V_n)\geq 0}{Minimize}\ \Phi = -(p_s - p_{ms})\sum_{j=1}^{n} V_j + $$
$$+C_{1S}\left(R - \sum_{j=1}^{n} V_j\right) +$$
$$+\sum_{j=1}^{n} C_{2Sj} E\left(\tilde{D}_j - V_j\right)^+$$

where, $E\left(\tilde{D}_j - V_j\right)^+ = \int_{V_j}^{\infty}\left(\tilde{D}_j - V_j\right) dF_j\left(\tilde{D}_j\right)$. Since $F_N(R)$ is a normal distribution, we have: $F_N(R) \geq \frac{c_{2s} - p_{ms}}{c_{1s} + c_{2s}}$ or the linear constraint:

$$R \geq \mu + \sigma Z_{\alpha_S}\ ;\ \alpha_s = \frac{c_{2s} - p_{ms}}{c_{1s} + c_{2s}} \quad .$$

Explicitly, say that outsourcing firms have uniform and independent demands in the intervals $\left[0, a_j\right]$. In this case, there is the following quadratic programming problem:

$$\underset{(R,p_s,V_1,V_2,\dots V_n)\geq 0}{Minimize}\ \Phi = C_{1S} R +$$
$$+\left(p_{ms} - p_s - C_{1S}\right)\sum_{j=1}^{n} V_j + \sum_{j=1}^{n} C_{2Sj} \frac{\left[a_j - V_j\right]^2}{2a_j}$$

Subject to :

$$\sum_{j=1}^{n} V_j - R \leq 0$$
$$a_j\left[\frac{c_{2j} - p_m}{c_{1j} + c_{2j}}\right] \leq V_j \leq a_j,$$
$$j = 1,2,3,\dots.,n;$$

and

$$R - \left(\mu + \sigma Z_{\alpha_S}\right) \leq 0;$$
$$\alpha_s = \frac{c_{2s} - p_{ms}}{c_{1s} + c_{2s}}$$
$$p_s - p_m \leq 0$$
$$p_{ms} - p_s \leq 0$$

Say that prices are fixed, the firms are homogenous, then we have:

$$\underset{(R,V)\geq 0}{Min}\ \Phi = c_{1S} R +$$
$$+n\left(p_{ms} - p_s - c_{1S}\right)V + nc_2 \frac{\left[a - V\right]^2}{2a}$$

Subject to :

$$nV \leq R \leq \left(\mu + \sigma Z_{\alpha_S}\right)$$

and $a\left[\frac{c_2 - p_m}{c_1 + c_2}\right] \leq V \leq a$

which can be solved by the usual techniques. Assume that there is an interior solution for V and boundary solutions for R. Consider first the boundary solution $V = R/n$, in which case:

$$V = \max\left\{\begin{matrix} \frac{c_2}{\left(c_2/a - \left[p_s - p_{ms}\right]\right)}; \\ a\left[\frac{c_2 - p_m}{c_1 + c_2}\right] \end{matrix}\right\} \leq a$$

In other words, the larger the quantity delivered, the larger the profit per unit and the higher the shortage cost of the outsourcing firm. Such a decision may be appropriate when demands are independent. However, when demands are dependent, the supplier is likely to violate the risk constraints. In this case, the solution is the following:

$$R = \left(\mu + \sigma Z_{\alpha_S}\right);\ nV < R$$

and therefore the supplier will have on the average excess inventories. The optimal solution is then:

$$V = \max\left\{\begin{matrix} \frac{c_2}{\left(c_2/a - \left[p_s + c_{1s} - p_{ms}\right]\right)}; \\ a\left[\frac{c_2 - p_m}{c_1 + c_2}\right] \end{matrix}\right\},$$

$V \leq a$ and the expected supplier cost of inventory is:

$$c_{1S}\left(\mu + \sigma Z_{\alpha_S}\right) -$$
$$-c_{1S} \max\left\{\begin{matrix} \frac{c_2}{\left(\frac{c_2}{a} - \left(p_s + c_{1s} - p_{ms}\right)\right)}; \\ a\frac{c_2 - p_m}{c_1 + c_2} \end{matrix}\right\}$$

In other words, aggregate supplier inventory costs can only be reduced by augmenting the selling price to outsourcing firms. In the case of demand dependence, predicted excess inventories will have to be compensated by a higher selling price. When this price exceeds the market price, a strong case for supply chain inventory outsourcing is needed which must be based on arguments other than inventory costs.

Alternatively, a supplier can also use priorities to assure some clients-firms a preferential treatment contracted in terms of minimal deliveries in case supply failure. In such cases, the inventory outsourcing model is slightly more involved and can be solved equally numerically. For example, if firm j has priority over firm k, denoted by a binary variable $\left(y_j, y_k\right) = 1,0$, then we can replace the statement that "firm j has priority over firm k" by: $y_j \geq y_k$ and $0 \leq V_i \leq My_i$; $i = j,k$ where M is a large number. Similarly, if in addition, the supplier guarantees to firm j a least supply d_j, the constraint set is: $y_j \geq y_k$ and $\min\left(d_i, \tilde{D}_i\right) \leq V_i \leq My_i$, $i = j,k$. Of course, other preferential agreements and priorities can be worked out and integrated in a supplier's schedule of firms' demands. These priorities might be needed to reduce the costs derived by demand dependence across outsourcing firms. Their analyses will require the application of integer optimization or simulation techniques however.

3. CONCLUSION

This paper has considered an "inventory outsourcing problem" when the supplier is a leader having full information of the outsourcing firm's demand distributions and parameters. Cases of dependent and independent demands were considered, providing a practical framework for handling such problems based on both pricing and priority ordering. Extensions and further developments of this paper can be considered emphasizing both the economic risks in supply outsourcing as well as imperfect information related issues between the supplier and the supplied firms, coordination and collaboration.

REFERENCES

Ansari, A. and B. Modaress (1986). Just in Time purchasing: Problems and solutions. *Journal of Purchasing and materials Management*, Summer, 11-15.

Ansari, A. and B. Modaress (1988). Just in Time purchasing as a quality and productivity centre, *International Journal of Production Research*, **25**, 19-26.

Bowersox, D.J., (1990). The strategic benefits of logistics alliances, *Harvard Business Review*, July-august, **68**, 36-45.

Celley, A.F., H. Clegg, A.W. Smith and M.A. Vonderembse, (1987). Implementation of JIT in the United States, *Journal of Purchasing and Materials Management*, 9-15

Crawford, K., J. Blackstone and J. Cox, (1988). A study of JIT implementation and operating problems, *International Journal of Production Research*, **26**, 1561-1568

Gattorna J. (Ed.), (1998). *Strategic Supply Chain Alignment*, Chap. 27, Gower, Aldershot.

Grout, R. and D.P. Christy, (1992). Incentive contracts for Just in Time delivery, *Working Paper,* Department of Management Information Sciences, Southern Methodist University, Dallas, Texas, 75275

La Londe, B. and M. Cooper (1989). *Partnership in providing customer service:a third-party perspective*, Council of Logistics Management, Oak Brook, IL.

McIvor, R., (2000). A practical framework for understanding the outsourcing process, *Supply Chain Management: an International Journal*, **5**, 22-36.

Newman, R., (1988). The buyer-supplier relationship under just in time, *Journal of Production and Inventory Management*, 45-50

Rao, K., and R.R. Young (1994). Global Supply Chain: factors influencing Outsourcing of logistics functions, *International Journal of Physical Distribution & Logistics Management*, **24**, 11-19.

Razzaque, M.A., ans C.C. Sheng, (1998). Outsourcing of logistics functions: a literature survey. *International Journal of Physical Distribution & Logistics Management*, **28**, 89-107.

Reyniers, Diane J. and C.S. Tapiero (1995). The supply and the control of quality in supplier-producer contracts, *Management Science*, **41**, 1581-1589

Ritchken, P. and C.S. Tapiero, (1986). Contingent Claim Contracts and Inventory Control. *Operations Research*, **34**, 864-870.

Rubin, P.A. and J.R. Carter (1990). Joint optimality in buyer –supplier negotiations. *Journal of Purchasing and Materials Management,* **26**, 54-68.

Stackleberg, H.V., (1952). *The Theory of the Market Economy*, Translated by Peacock A.T., William Hodge and CO., London.

Sulem, A. and C.S. Tapiero, (1993). Supply delays and the inventory stocking policy. *International Journal of Production Economics,* **32**, 83-90.

Skjoett-Larsen T., (2000). Third party Logistics – from an interorganizational point of view, *International Journal of Physical Distribution & Logistics Management*, **30**, 112-127.

Tapiero, C.S., (2000). Ex-Post Inventory Control. *Int. J. of Production Research,* **38**, 1397-1406.

Tapiero, C.S., (2004). Value at Risk and Inventory Control. *European Journal of Operations Research,* Forthcoming.

Van Damme, D.A., and M.J. Ploos van Amstel (1996). Outsourcing Logistics management Activities. *The International Journal of Logistics management*, **7**, 85-94.

Van Laarhoven P., M. Berglund and M. Peters, (2000), Third-party logistics in Europe – five years later. *International Journal of Physical Distribution & Logistics Management*, **30**, 425-442.