PLANNED LEAD-TIMES DESIGN IN STOCHASTIC MULTISTAGE ASSEMBLY SYSTEMS

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Abstract: This paper considers a general multistage assembly system operating on a make to order basis with stochastic manufacturing lead-times. The objective is to design optimal planned lead-times by minimising the expected sum of inventory holding costs and tardiness cost. A perturbation analysis based stochastic approximation (PASA) procedure and a Simulated Annealing (SA) method are developed to solve this problem. Compared with the deterministic backward scheduling, the PASA and SA can reduce the costs significantly. The PASA achieves the similar cost to the SA with substantially less CPU time. Case studies using industrial data are given to demonstrate the results.

Keywords: Manufacturing, assembly, stochastic, optimisation, perturbation analysis, simulated annealing, lead-time.

1. INTRODUCTION

Setting appropriate lead-times is an important problem for the control of manufacturing systems with uncertainties, particularly for multistage assembly systems, where late arrival of an item may delay the production of subsequent assembly while early arrival may incur holding cost. This effect is cumulative and interacting along the product structure. Manufacturing lead-time is defined as 'the time taken from the time production is authorised, to the time it is completed and the material is available for use to fill demand by the customer or the next stage' (Karmarkar, 1993). In multistage production environments, the manufacturing lead-time can be separated by stages. This paper assumes that the manufacturing lead-time is assigned to each operation activity and the lead-time of an operation has the same mean of this operation's processing time.

Two types of contingencies, i.e. safety lead-time and safety stock, are often used to offset the effects of uncertainty. The term safety lead-time refers to the difference between the planned lead-time and the average lead-time. It has been identified that safety lead-time is preferable to safety stock when timing is uncertain (Whybark and Williams, 1976). Especially, in make to order systems, where final products are highly customised, safety stock is always unrealistic.

The problem of setting safety lead-times to buffer against the uncertainty in material procurement was studied by Hopp and Spearman (1993), Chu, *et al*. (1993) and Shore (1995). They considered multiple component assembly systems, where the component purchased times were stochastic and the assembly production starting time was fixed. The objective was to find the optimal safety lead-times for each component (which implied the optimal order instants

of the components) so that the total cost was minimised. Their models were equivalent to a simplified two-stage assembly system where no decision was made for the assembly stage.

Setting planned lead-times or activity start times in the situations with stochastic processing times was reported in Yano (1987a,b), Gong *et al*. (1994), Matsuura *et al*. (1996), Molinder (1997), Hasan and Spearman (1999), Song *et al*. (2001) and Elmaghraby *et al*. (2000). However, the majority of the above research is limited to relative simple environments such as one or two stage assembly systems (Yano, 1987a), serial production line (Yano, 1987b, Gong *et al*., 1994), non-assembly job shop (Matsuura and Tsubone, 1993, Matsuura *et al*., 1996, Hasan and Spearman, 1999, Emaghraby *et al*., 2000).

This paper considers general multistage assembly systems operating on a make to order basis with stochastic processing times. The objective is to design optimal planned lead-times for activities by minimising the expected sum of work-in-progress holding costs, product earliness and tardiness costs. Different from Song *et al*. (2001), which focused on allocation of resources over time on a given sequence of operations, here the resource constraints are not considered due to the fact that the resource capacity requirements are often derived based on the planned lead-times. A Perturbation Analysis Stochastic Approximation (PASA) procedure and a Simulated Annealing (SA) method are described to solve this problem. Ten case studies using industrial data from a make-to-order company are performed. Compared with backward scheduling method based on mean data (i.e. zero safety lead-times at each stage), both PASA and SA can reduce the costs significantly. The PASA can achieve very similar costs at substantially less CPU time compared with the SA method.

The rest of the paper is organised as follows. In the next section, the problem is formulated and notation is defined. In section 3, a Perturbation Analysis algorithm is developed to estimate the gradient of the system performance measure with respect to the planned start lead-times and this estimator is shown to be unbiased. In section 4, a Stochastic Approximation procedure based on PA gradient estimator is described. In section 5, a Simulated Annealing algorithm is presented. Case studies are given in section 6 and conclusions are made in section 7.

2. PROBLEM FORMULATION AND NOTATION DEFINITIONS

The model used here is a generic multistage production environment operating on a make to order basis. The production starts after receiving a customer order. The customer order is assumed to be composed of a single product type and a specified due date. There is no initial work-in-progress in the system. To produce a product, it involves multiple stages of manufacturing and assembling. This can be represented by a tree-type product structure, where the root node represents the final product and the leaf nodes represents the components. The final product is denoted by code 1. For example, figure 1 shows a system with five components (code 6, 7, 8, 9, 10) and five assemblies (code 5, 4, 3, 2, 1). To simplify the narrative, each node is termed a part.

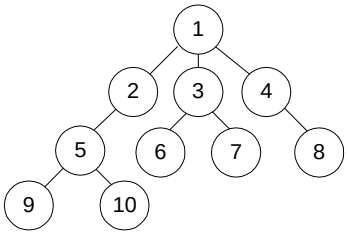

Fig. 1. A multistage assembly system

It is assumed that each part has only one operation activity. The operation processing times are stochastic and can be described by independent continuous random variables. The operation lead-time and processing time are used interchangeably in this paper. It is assumed that machines are available after the corresponding parts' planned start lead-times (i.e. planed processing start times). This treatment is reasonable since resource requirements are often derived based on the planned lead-times.

The part dispatching scheme can be described as follows. A part is dispatched at its planned start lead-time if all its subassemblies have been finished before the planned start lead-time. Otherwise, it is dispatched immediately at the latest completion time of its subassemblies. There are several reasons for setting planned start lead-times. Firstly, they provide a rough plan for the operation activities and form a basis for resource requirement planning. Secondly, starting operation activities as soon as possible may result in earliness (holding) cost at downstream stages which is much more expensive than those at upstream stages. Thirdly, setting a planned start lead-time for each operation activity may reduce the variability of the total production lead-time and the system variability. Notation is defined as follows.

d – the due date of the product;
h_i – holding cost per time unit for part i;
h^- – tardiness penalty per time unit for the product;
$\rho(i)$ – the immediately successive part of part i in the product structure;
X_i – planned lead-time for part i;
x_i – actual lead-time for part i; it is a random variable;
s_i – planned start lead-time for part i;
a_i – actual start lead-time for part i;
c_i – actual processing completion time for part i;
N – total number of parts in the product structure.

Throughout this paper, define $\rho(1):=0$ and $s_0:=d$. Note that $X_i=s_{\rho(i)} - s_i$ for $i=N, N\text{-}1, ..., 1$, therefore, designing the planned lead-times is equivalent to designing the planned start lead-times. To describe the detailed relationships between an assembly part and its subassemblies, additional notations are required. For part i, let n_i be the number of its subassemblies and $i(1)$, $i(2)$, …, $i(n_i)$ denote its subassemblies respectively. Clearly, if part i is a component, then $n_i = 0$ because each component has no subassembly. For the system shown in figure 1, $n_1=3$, $1(1)=2$, $1(2)=3$, $1(3)=4$; $n_2 =1$, $2(1)=5$ and $\rho(2) =1$.

The actual processing start time of a part is the maximum of its planned start lead-time and all the actual completion times of its subassemblies. Hence, the relationships of planned start lead-times, actual start lead-times and actual completion times can be described by

$$a_i = \max(s_i, c_{i(1)}, c_{i(2)}, \ldots, c_{i(n_i)}) \text{ for } i=N, \ldots, 1, 0 \quad (1)$$

$$c_i = a_i + x_i \text{ for } i=N, N\text{-}1, \ldots, 1. \quad (2)$$

The objective is to find the optimal production plan $\{s_i, i=1,2, \ldots, N\}$ by minimising the following expected cost function

$$J(\mathbf{s}) = \mathrm{E}\Big[\sum_{i=2}^{N} h_i\,(a_{\rho(i)} - c_i) + h_1\max(0, s_0 - c_1) + h^-\max(0, c_1 - s_0)\Big] \quad (3)$$

where $\mathbf{s}:=(s_N, s_{N-1}, ..., s_1)^{\mathrm{T}}$. The first term of the right hand side (RHS) of (3) represents the holding costs of work-in-progress, the second and the last terms represent the product earliness and tardiness costs respectively.

This is a standard stochastic optimisation problem and could be solved by either stochastic approximation or random search method. The key step in using stochastic approximation is to find the effective gradient estimate of the cost function with respect to decision parameters. This can be fulfilled by a Perturbation Analysis algorithm which is developed in the next section.

3. PERTURBATION ANALYSIS GRADIENT ESTIMATOR AND ITS UNBIASEDNESS

Perturbation analysis (PA) technique was well addressed in Ho and Cao(1991) and Glasserman(1991). The idea is to take advantage of the structure and real variable nature of the problem, from a single simulation run or experiment to derive as much information as possible, in terms of the gradient information of the objective function with respect to each parameter. However, the availability of PA technique depends on the unbiasedness of the gradient estimator.

Let ω denote a sample process of the system with certain predetermined parameters $\{s_N, s_{N-1}, ..., s_1\}$. Let $L(\mathbf{s}, \omega)$ denote the sample cost function, that is

$$L(\mathbf{s}, \omega) = \sum_{i=2}^{N} h_i\,(a_{\rho(i)} - c_i) + h_1\max(0, s_0 - c_1) + h^-\max(0, c_1 - s_0) \quad (4)$$

Clearly, $J(\mathbf{s}) = \mathrm{E}L(\mathbf{s}, \omega)$. The sample realisation under $\{s_N, s_{N-1}, ..., s_1\}$ is called nominal path (NP); and the sample realisation under $\{s_N, s_{N-1}, ..., s_{i+1}, s_i+\Delta, s_{i-1}, \ldots, s_1\}$ is called perturbed path (PP), where Δ is a sufficiently small positive number. Let $\{a_j\}$ and $\{c_j\}$ represent the nominal path and $\{a_j^i\}$ and $\{c_j^i\}$ represent the perturbed path with $s_i+\Delta$.

Let ζ_i denote the part code string of the path from part i to final product 1. For example, $\zeta_i =\{i, l, m, \ldots, 1\}$ represents that $l =\rho(i)$, $m =\rho(l)$, and so on. Because the product structure is of tree-type, the last part must be 1 in every part-path string. Define

- $\kappa(i)$ = the first part code in string ζ_i whose completion time is less than its successive part's actual processing start time. Define $\kappa(i) = 0$, if every part in string ζ_i satisfies that its completion time is equal to its successive part's actual processing start time and $c_1=a_0$.

The perturbation rules can be summarised by presenting the relationships of $\{a_j, c_j\}$ with $\{a_j^i, c_j^i\}$.

Proposition 1. If there is a sufficient small perturbation Δ on part i with $a_i = s_i$, and $\zeta_i =\{i, l, m, \ldots, r, \kappa(i), \ldots, 1\}$, then: $a_j^i= a_j+\Delta$, $c_j^i = c_j+\Delta$ for $j\in \{i, l, m, \ldots, r, \kappa(i)\}$ and $a_j^i= a_j$, $c_j^i = c_j$ for $j\notin \{i, l, m, \ldots, r, \kappa(i)\}$. On the other hand, if $a_i > s_i$, then the NP and PP overlap.

Theorem 1. Suppose $\zeta_i =\{i, l, m, \ldots, r, \kappa(i), \ldots, 1\}$. The gradient of the sample cost function is given by

(i) if i is a component, then $\partial L(\mathbf{s}, \omega)/\partial s_i= (h_{l(1)} + h_{l(2)} + \ldots + h_{l(n_l)} - h_i) + (h_{m(1)} + h_{m(2)} + \ldots + h_{m(n_m)} - h_l) + \ldots + (h_{\kappa(i)(1)} + h_{\kappa(i)(2)} + \ldots + h_{\kappa(i)(n_{\kappa(i)})} - h_r) - h_{\kappa(i)}\cdot I\{\kappa(i)>0\} + h^-\cdot I\{\kappa(i)=0\}$;

(ii) if i is not a component and $a_i = s_i$, then $\partial L(\mathbf{s}, \omega)/\partial s_i = (h_{i(1)} + h_{i(2)} + \ldots + h_{i(n_i)}) + (h_{l(1)} + h_{l(2)} + \ldots + h_{l(n_l)} - h_i) + (h_{m(1)} + h_{m(2)} + \ldots + h_{m(n_m)} - h_l) + \ldots + (h_{\kappa(i)(1)} + h_{\kappa(i)(2)} + \ldots + h_{\kappa(i)(n_{\kappa(i)})} - h_r) - h_{\kappa(i)}\cdot I\{\kappa(i)>0\} + h^-\cdot I\{\kappa(i)=0\}$;

(iii) if i is not a component and $a_i > s_i$, then $\partial L(\mathbf{s}, \omega)/\partial s_i = 0$.

where $\kappa(i)(n_{\kappa(i)})$ is the $(n_{\kappa(i)})th$ subassembly of part $\kappa(i)$, and $I\{.\}$ is the indicator function, which takes 1 if $\{.\}$ is true, otherwise takes 0.

The gradient information of the sample cost function given in theorem 1 is an unbiased estimator of the gradient of the cost function (3). That is,

Theorem 2. Let $J(\mathbf{s})$ be defined in (3) and $\partial L(\mathbf{s}, \omega)/\partial s_i$ be given in theorem 1, then: $E\partial L(\mathbf{s}, \omega)/\partial s_i = \partial J(\mathbf{s})/\partial s_i$ and $E\,|\partial L(\mathbf{s}, \omega)/\partial s_i| < +\infty$, for $i=N, N\text{-}1, \ldots, 1$.

Proof: Define $\Omega_0=\{\omega \mid \exists\, i\in\{N, N\text{-}1, \ldots, 1\}\ s.t.\ s_i = \max(c_{i(1)}, c_{i(2)}, \ldots, c_{i(n_i)})\}$. That is, Ω_0 is a set of all sample processes in which there exists at least one part whose planned start lead-time equals the latest completion time of its subassemblies. Since all processing times are independent continuous random variables, it yields $\text{Prob}\{\omega \mid \omega\in\Omega_0\}=0$. Then with the similar arguments in Song *et al*. (2001), the assertion is true.

4. STOCHASTIC APPROXIMATION BASED ON PERTURBATION ANALYSIS

This section describes a stochastic approximation algorithm to find the optimal planned start lead-times by minimising the cost function $J(\mathbf{s})$. The general form of stochastic approximation (Rubinstein, 1992) is:

$$\mathbf{s}_{n+1}=\mathbf{s}_n - \gamma_n \cdot \nabla J_n \qquad (5)$$

where $\mathbf{s}_n$ is the parameter vector at the beginning of iteration n, ∇J_n is an estimator of $\nabla J(\mathbf{s}_n)$ (where $\nabla J(\mathbf{s}_n) := (\partial J(\mathbf{s}_n)/\partial s_N, \partial J(\mathbf{s}_n)/\partial s_{N-1}, \ldots, \partial J(\mathbf{s}_n)/\partial s_1)^{\mathrm{T}}$), γ_n is a positive sequence of step sizes such that (a) it decreases to zero; (b) the sum of all the sequence $\{\gamma_n\}$ is infinite and (c) the sum of its squares is bounded. Typically, the harmonic sequence $1/n$ satisfies all above assumptions for γ_n. When ∇J_n is an unbiased estimator of $\nabla J(\mathbf{s}_n)$, (5) is called a Robbins-Monro (RM) algorithm and when a finite difference estimator is used, it is called a Kiefer-Wolfowitz (KW) algorithm. The RM algorithm has faster convergence rates than the KW algorithm (Fu and Hu, 1997). Since theorem 1 provides an unbiased gradient estimator, the equation (5) yields an RM algorithm.

The Perturbation Analysis Stochastic Approximation (PASA) algorithm consists of six steps:

Step 1. Set initialise $\mathbf{s}_0$, the maximum number of iterations $\underline{n}$ and initial iteration loop n=0.

Step 2. Set the step size γ_n.

Step 3. For a given solution $\mathbf{s}_n$, run K sample processes to calculate the gradient estimator by Perturbation Analysis: $\nabla J_n = ([\sum_k \partial L(\mathbf{s}, \omega_k)/\partial s_N]/K \ldots, [\sum_k \partial L(\mathbf{s}, \omega_k)/\partial s_1]/K)^{\mathrm{T}}$.

Step 4. Calculate a new solution $\mathbf{s}_{n+1}=\mathbf{s}_n - \gamma_n \cdot \nabla J_n$.

Step 5. Adjust the solution $\mathbf{s}_{n+1}$ to meet the part precedence constraints and make its elements non-negative.

Step 6. If $n < \underline{n}$, set $n=n+1$ and go to Step 2; otherwise return $\mathbf{s}_n$ as the optimal planned start lead-time vector.

In this paper, the step size is chosen by $\gamma_n=1/v_n$, where v_n is the number of times the gradient changed signs. This choice has been found to perform satisfactorily in stochastic approximation (Hasan and Spearman, 1999). In addition, by the Large Number Law and Theorem 2, it is clear that $\lim_{K\to\infty} [\sum_k \partial L(\mathbf{s}, \omega_k)/\partial s_i]/K = \partial J(\mathbf{s})/\partial s_i$ almost surely. Therefore, if K is appropriately large, the gradient estimator ∇J_n is accurate enough.

For the multistage assembly systems, it is very difficult to obtain either analytical optimal solutions or tight lower bounds. To demonstrate the effectiveness of the PASA procedure, a random search method, Simulated Annealing, is applied to make comparison.

5. SIMULATED ANNEALING METHOD

Simulated Annealing (SA) is a technique that has been regarded as suitable for optimisation problems of large scale (Kirkpatrick *et al*., 1983, Aarts and Korst, 1989). There are two kinds of loops involved within SA procedure. In the outer loop, the temperature (T) cools until the ground state T=0 is reached. In the inner loop, for fixed temperature T the equilibrium state is found (i.e. no further improvement is achieved). This is done as follows. At each step a neighbour to the current solution is generated at random. If the objective function associated with the neighbour is better (i.e. $J_{\text{new}}\le J_{\text{old}}$) it is accepted as the starting point of the next step. Otherwise, it is accepted with a probability (which equals $\exp[(J_{\text{old}}-J_{\text{new}})/T]$).

In the situation under consideration, the solution is characterised by a real vector $\mathbf{s}$. Therefore, the neighbourhood of a solution can be obtained by making changes to the elements of $\mathbf{s}$. The SA algorithm can be tailored to tackle the numerical optimisation problem in stochastic situations by appropriately modifying the cost function evaluation step, e.g. the cost function can be evaluated by averaging over K sample processes.

The notation used in the SA algorithm is defined as follows. Let $\mathbf{s}_0$ denote the initial solution vector; T_0 denote the initial temperature; γ_0 denote the initial width for the step sizes for variation of the solution; α denote the temperature cooling and step size reduction factor. Termination rules are defined as follows. Let N_I (N_O) denote the maximum number of consecutive inner (outer) loop trials in which no improvement is achieved for the cost function. Let $\underline{n}$ be the maximum number for outer loop trials. The inner loop search will be terminated if no improvement is achieved during consecutive N_I inner loop trials. The whole procedure will be terminated if

no improvement is achieved during consecutive N_O outer loop trials or the total outer loop trials exceeds $\underline{n}$. The SA algorithm to optimise planned start lead-times consists of seven steps.

Step 1. Set initial $\mathbf{s}_0$, T_0, α, γ_0, N_I, N_O, $\underline{n}$, $\mathbf{s}^*=\mathbf{s}'=\mathbf{s}_0$, $n^*=0$ and $n=0$, where $\mathbf{s}^*$ is the optimal solution up to now; $\mathbf{s}'$ is a temporal vector used in the inner loop; n^* denotes the outer loop in which the optimal solution up to now is achieved.

Step 2. Set $l=1$ and $NI=0$, where NI denotes the number of consecutive inner loop trials in which no improvement is achieved for the cost function.

Step 3.

(i) Set $\mathbf{s}^{(n,l)} = \mathbf{s}' + \gamma_n \cdot \mathbf{z}$, where each element of $\mathbf{z}$ follows a uniform distribution, i.e. $z_i \sim U(-1/2,1/2)$.

(ii) Adjust the solution to meet the precedence constraints and to be non-negative.

(iii) For fixed parameter $\mathbf{s}^{(n,l)}$, run K sample processes to calculate the average cost function $J(\mathbf{s}^{(n,l)}) := \sum_{j=1}^{K} L(\mathbf{s}^{(n,l)}, \omega_j)/K$.

(iv) If $J(\mathbf{s}^{(n,l)}) < J(\mathbf{s}^*)$, set $NI=0$, $n^*=n$ and $\mathbf{s}^*= \mathbf{s}^{(n,l)}$; otherwise, set $NI=NI+1$.

(v) If $J(\mathbf{s}^{(n,l)}) < J(\mathbf{s}')$, set $\mathbf{s}' = \mathbf{s}^{(n,l)}$; otherwise, draw a uniform random number $\chi \sim U(0,1)$ and if $\chi \le \exp((J(\mathbf{s}') - J(\mathbf{s}^{(n,l)}))/T_n)$, set $\mathbf{s}' = \mathbf{s}^{(n,l)}$.

Step 4. If $NI>N_I$, go to Step 6.

Step 5. Set $l=l+1$, go to Step 3.

Step 6. If $n - n^* > N_O$ or $n>\underline{n}$, return $\mathbf{s}^*$ as an optimal planned start lead-time vector.

Step 7. Set $\mathbf{s}'=\mathbf{s}^*$, $T_{n+1}=\alpha \cdot T_n$ and $\gamma_{n+1}=\alpha \cdot \gamma_n$; Set $n=n+1$ and go to Step 2.

In general, random search methods, e.g. Simulated Annealing, Genetic Algorithms, require a large number of trials and cost evaluations. In stochastic situations, each cost evaluation needs multiple sample processes to do averaging. Therefore, random search methods could be very timing consuming to deal with stochastic optimisation problems.

6. NUMERICAL EXAMPLES

In this Section, ten case studies, whose data are obtained from a make to order company that manufactures capital products (Hicks, 1998), are used to test the effectiveness of the methods.

Deterministic backward scheduling sets the planned start lead-times based on mean data, i.e. $s_i = s_{\rho(i)} - \mu_i$ starting from the final product to the components, where $\mu_i = E\, x_i$. Intuitively, since no safety lead-times are included for any activity, high tardiness penalty costs could be resulted due to the stochastic processing times. This plan is used as an initial solution for both PASA and SA methods.

In SA algorithm, the initial control parameters, especially the inner loop termination parameter N_I, the initial temperature T_0 and the temperature-cooling rate α, may affect the search speed and solution accuracy. Preliminary experiment showed that both N_I and α should not be too small. Different combinations of T_0 and α are tested and a good pair is selected and used in all case studies.

In each case study, a single product is produced. The total operation (including both machining and assembly) number in each case varies from 24 to 268, as shown in table 1. All processing times are assumed to be normally distributed. The mean processing times are obtained from the company and the standard deviation is assumed to be the corresponding mean multiplied by 0.2. This assumption is reasonable since in reality larger processing times often have larger variability. The holding cost of a part is proportional to the sum of mean processing times on this part and on all the previous parts within its branch. This reflects the fact that holding costs increase with added value and time. The product tardiness penalty coefficient is twice the product earliness penalty coefficient. The product due date is 500 days.

Table 1 Ten cases with different machining and assembly operations

Case	Machining operations	Assembly operations
1	18	6
2	34	5
3	46	6
4	56	6
5	74	16
6	102	7
7	100	13
8	146	20
9	176	24
10	229	39

Table 2 Cost and CPU time (in second) for deterministic planning, PASA and SA methods

Case	Deterministic		PASA		SA	
	Cost	CPU(s)	Cost	CPU(s)	Cost	CPU(s)
1	37	<1	25	4	25	128
2	478	<1	271	6	269	227
3	63	<1	26	9	27	349
4	564	<1	326	10	327	356
5	1894	<1	1070	15	1072	555
6	252	<1	93	18	100	957
7	393	<1	148	18	164	918
8	3678	<1	1806	27	1796	1464
9	4336	<1	2201	33	2190	2614
10	9228	<1	4036	46	4085	2962

Let K=100. The costs and CPU times (in seconds) for deterministic backward scheduling, PASA (with $\underline{n}$=500) and SA (with T_0=1.0 and α=0.95) methods are given in table 2. Other initial parameters in SA algorithm are: $\gamma_0=\min\{\mu_i,\ i=1,\ 2,\ \ldots,\ N\}$, N_I=100, N_O=50 and $\underline{n}$=200. The PASA and SA algorithms are written by *c* language and executed in SunOS 5.7 environment with 400MHz processor.

From table 2, it is found that the PASA and SA substantially reduce the expected total costs compared with the deterministic backward scheduling method (e.g. costs are reduced by more than 50% in five case studies); PASA and SA achieve very similar costs, but SA requires much more CPU time than the PASA. Particularly, as the problem complexity increases, the difference of CPU times between PASA and SA becomes larger. In addition, it can be derived by intuition that the difference of CPU times between PASA and SA will increase dreadfully as the number of sample processes K increases.

7. CONCLUSION

A Perturbation Analysis algorithm is first developed to estimate the gradient of the objective function with respect to the planned start lead times in stochastic assembly manufacturing systems. This estimator is shown to be unbiased. Then a Perturbation Analysis Stochastic Approximation (PASA) procedure is presented to optimise the planned lead-times. A Simulated Annealing (SA) method is also applied to solve the same problem. Ten case studies from an industrial make-to-order companies show that: the solutions from the PASA and the SA perform significantly better than those produced by the deterministic backward scheduling method; the PASA and the SA achieve very close performance in all case studies; the PASA saves substantial computation time compared with the SA. This reveals that the PASA is recommended to deal with planned lead-time design problems in stochastic complex manufacturing systems.

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