NONLINEAR SYSTEM SENSOR FAULT ESTIMATION

Qinghua Zhang [*] **Gildas Besançon** [**]

[*] *IRISA-INRIA, Campus de Beaulieu, 35042 Rennes Cedex, France*
Email: zhang@irisa.fr
[**] *LAG-ENSIEG, BP 46, 38402 Sant-Martin d'Hères, France*
Email: Gildas.Besancon@inpg.fr

Abstract Based on the techniques of high gain observer and adaptive estimation, an algorithm is proposed in this paper for sensor fault estimation in nonlinear systems. It is essentially assumed that a high gain observer exists for the fault-free system. A high gain adaptive observer is then designed for sensor fault estimation. The convergence of the algorithm is established under a persistent excitation condition.

Keywords: fault detection and isolation, nonlinear system, high gain observer, adaptive estimation.

1. INTRODUCTION

Fault detection and isolation (FDI) have been extensively studied for linear dynamic systems, see, e.g., (Frank, 1990; Basseville and Nikiforov, 1993; Gertler, 1998; Chen and Patton, 1999). Due to the importance of nonlinearities in many applications, nonlinear system FDI have recently become an active research topic (Hammouri *et al.*, 1998; De-Persis and Isidori, 2001; Staroswiecki and Comtet-Varga, 2001; Xu and Zhang, 2004). Most known results on nonlinear system FDI deal with faults affecting the state equation in the state space model of a nonlinear system, typically actuator faults. The present paper considers sensor fault affecting the output equation.

More specifically, the considered class of nonlinear systems subject to sensor fault is in the form of

$$\dot{\bar{x}}(t) = \bar{f}(\bar{x}(t)) + \bar{g}(\bar{x}(t))u(t) \tag{1a}$$

$$y(t) = \bar{h}(\bar{x}(t)) + q(t) \tag{1b}$$

where $\bar{x}(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^l$ the input, $y(t) \in \mathbb{R}$ the output, $q(t) \in \mathbb{R}$ represents the sensor fault eventually affecting the system. It is assumed that the nonlinear functions $\bar{f} : \mathbb{R}^n \to \mathbb{R}^n$, $\bar{g} : \mathbb{R}^n \to \mathbb{R}^{n\times l}$ and $\bar{h} : \mathbb{R}^n \to \mathbb{R}$ are such that a high gain observer exists for the *fault-free* system (a short introduction to high gain observer will be given in Section 2). The input $u(t)$ is assumed to ensure well-defined state trajectory for $t \in [t_0, +\infty)$. The term $q(t)$ is equal to zero when no fault is present. *The purpose of this paper* is to propose a method for the estimation of the sensor fault $q(t)$. For this estimation to be possible, it is assumed that $q(t)$ can be modeled by a linear regression

$$q(t) = \theta_1\psi_1(t) + \cdots + \theta_p\psi_p(t) \tag{2}$$

with given regressor functions $\psi_1(t), \ldots, \psi_p(t)$. This model may come from some physical knowledge about the possible fault. For example, some disturbances with known frequencies may affect the output measurement. It can also be considered as a generic approximator of the fault signal. It is then assumed that the complexity of the regressors $\psi_1(t), \ldots, \psi_p(t)$ allows to reasonably approximate $q(t)$. Such a numerical example (approximating a chirp signal with a finite number of sinusoid functions) will be presented in Section 4.

Only single output system is considered in this paper. For a multi-output system, if a high gain observer can be designed with each individual output for which the sensor fault is to be estimated, then the method of this paper can be applied separately with each of these outputs. When this condition is not satisfied, the result of this paper can still be generalized to some particular multi-output systems, in a way similar to (Besançon *et al.*, 2004), or based on some other recent results on nonlinear observers (Hou *et al.*, 2000; Gauthier and Kupka, 2001).

If the fault-free output $\bar{h}(x(t))$ can be correctly predicted from the input signal $u(t)$ only, then the estimation of $q(t)$ is trivial. However, due to modeling uncertainty and possibly due to lack of model stability, quite often such a simple prediction is not sufficiently accurate. State observers or Kalman filter-like algorithms are known to be more reliable.

With the above formulation, the problem of sensor fault estimation amounts to the estimation of the parameters $\theta_1, \ldots, \theta_p$. A similar problem, with faults affecting the *state equation*, has been studied in (Xu and Zhang, 2004). Some related adaptive observer algorithms also consider unknown parameters in state equation only, as in (Bastin and Gevers, 1988; Marino and Tomei, 1995; Cho and Rajamani, 1997; Besançon, 2000).

Remark that it is possible to move the parameters $\theta_1, \ldots, \theta_p$ from the output equation into the state equation in the following way. Introduce a new state variable $x^*(t)$ (corresponding to the integral of $y(t)$) in addition to the state vector $\bar{x}(t)$, with the extra state equation

$$\dot{x}^*(t) = \bar{h}(\bar{x}(t)) + \theta_1\psi_1(t) + \cdots + \theta_p\psi_p(t)$$

Replace the output variable $y(t)$ by its integral $y^*(t)$, so that the new output equation becomes simply $y^*(t) = x^*(t)$. Then the parameters $\theta_1, \ldots, \theta_p$ are moved into the state equation. However, with this method, the state vector is unnecessarily increased, and it may be difficult to put the new system model into the form required by (Xu and Zhang, 2004). In the present paper, the *main result* is a nonlinear adaptive observer which allows to *directly* estimate the fault parameters in the output equation, and incidentally to estimate the state vector. The *global* exponential convergence of the algorithm is established under appropriate assumptions.

Compared to the result of (Xu and Zhang, 2004), in addition to the already mentioned difference about the location of the considered faults, this paper presents an important algorithm simplification which is now also known to be applicable to the algorithm of (Xu and Zhang, 2004).

This paper is organized as follows. Section 2 recalls the basic high gain observer. The proposed algorithm is presented in Section 3. A numerical example is given in Section 4. Some conclusions are drawn in Section 5.

2. THE BASIC HIGH GAIN OBSERVER

In order to introduce some technical elements used in this paper, let us recall the basic high gain observer, essentially following (Deza, 1991; Gauthier *et al.*, 1992). See also (Gauthier and Kupka, 2001) for some more advanced developments.

It is shown in (Gauthier *et al.*, 1992) that, in the fault-free case ($q(t) \equiv 0$), if the nonlinear system (1) is observable for all inputs, the coordinate change [1]

$$x(t) = [h(\bar{x}(t)), L_f h(\bar{x}(t)), \ldots, L_f^{n-1}h(\bar{x}(t))]^T \quad (3)$$

transforms the system into the form

$$\begin{aligned} \dot{x}(t) &= A_o x(t) + f(x(t)) + g(x(t))u(t) && (4a) \\ y(t) &= c_o x(t) && (4b) \end{aligned}$$

where

$$A_o = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \qquad c_o = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \quad (5)$$

$f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times l}$ are two nonlinear functions in the triangular form:

$$f(x)=\begin{bmatrix} f_1(x_1) \\ f_2(x_1, x_2) \\ \vdots \\ f_n(x_1 \ldots x_n) \end{bmatrix}, \; g(x)=\begin{bmatrix} g_1(x_1) \\ g_2(x_1, x_2) \\ \vdots \\ g_n(x_1 \ldots x_n) \end{bmatrix} \quad (6)$$

For a positive real number ρ, define the diagonal matrix

$$\Delta = \begin{bmatrix} \rho & 0 & \cdots & 0 \\ 0 & \rho^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \rho^n \end{bmatrix} \quad (7)$$

Let S be the solution of the matrix equation

$$A_o^T S + S A_o + S = c_o^T c_o \quad (8)$$

It is known that S is a positive definite matrix (see Appendix B of (Xu and Zhang, 2004)).

Define also

$$\kappa_o = \frac{1}{2} S^{-1} c_o^T \quad (9)$$

Theorem 1. Consider system (4) with $A_o, c_o, f(x)$ and $g(x)$ as defined in (5) and (6). If the functions $f(x)$ and $g(x)$ are globally Lipschitz, and if the input $u(t)$ is bounded, then, for sufficiently large ρ, the ordinary differential equation (ODE)

$$\begin{aligned} \dot{\hat{x}}(t) = A_o\hat{x}(t) + f(\hat{x}(t)) + g(\hat{x}(t))u(t) \\ + \Delta\,\kappa_o[y(t) - c_o\hat{x}(t)] \end{aligned} \quad (10)$$

with Δ and κ_o as defined in (7) and (9) is a global exponential observer for system (4), *i.e.*, for all

[1] $L_f h(x)$ denotes the Lie derivative of $h(x)$ along $f(x)$.

initial conditions $x(t_0)$ and $\hat{x}(t_0)$, the estimation error $\hat{x}(t) - x(t)$ tends to zero exponentially fast when $t \to \infty$. $\square$

Intuitively, after the state transformation $z = \Delta^{-1}x$ and $\hat{z} = \Delta^{-1}\hat{x}$, for large ρ, the nonlinear terms in the error system are dominated by the linear terms. It is thus not surprising that the high gain observer is similar to the linear Luenberger observer. See (Deza, 1991; Gauthier *et al.*, 1992) for formal proofs.

3. SENSOR FAULT ADAPTIVE ESTIMATION

Let us rewrite the linear regression (2) as

$$q(t) = \psi(t)\theta$$

with the *row* vector

$$\psi(t) = [\psi_1(t), \ldots, \psi_p(t)]$$

and the *column* vector

$$\theta = [\theta_1, \ldots, \theta_p]^T$$

Now with the sensor fault present in system (1), the coordinate change (3) transforms it into the form

$$\dot{x}(t) = A_o x(t) + f(x(t)) + g(x(t))u(t) \quad (11a)$$
$$y(t) = c_o x(t) + \psi(t)\theta \quad (11b)$$

where A_o, c_o are as defined in (5), and f, g as in (6). The sensor fault estimation problem then amounts to the estimation of the parameter vector θ.

The same problem for linear time varying systems is considered in (Zhang, 2005). A similar problem, with $\psi(t)\theta$ affecting the state equation instead of the output equation, has been considered in (Xu and Zhang, 2004). Inspired by these results, the following *adaptive observer* is proposed for recursive joint estimation of $x(t)$ and θ.

$$\dot{\Upsilon}(t) = \rho(A_o - \kappa_o c_o)\Upsilon(t) - \rho\kappa_o\psi(t) \quad (12a)$$
$$\begin{aligned}\dot{\hat{x}}(t) &= A_o\hat{x}(t) + f(\hat{x}(t)) + g(\hat{x}(t))u(t) \\ &\quad + \Delta\left\{\kappa_o + \rho^{-1}\Upsilon(t)\Gamma\left[c_o\Upsilon(t) + \psi(t)\right]^T\right\} \\ &\quad \cdot [y(t) - c_o\hat{x}(t) - \psi(t)\hat{\theta}(t)] \end{aligned} \quad (12b)$$
$$\begin{aligned}\dot{\hat{\theta}}(t) &= \Gamma\left[c_o\Upsilon(t) + \psi(t)\right]^T \\ &\quad \cdot [y(t) - c_o\hat{x}(t) - \psi(t)\hat{\theta}(t)] \end{aligned} \quad (12c)$$

where $\Upsilon(t) \in \mathbb{R}^{n\times p}$ is a matrix of signals generated by linearly filtering $\psi(t)$, $\hat{x}(t) \in \mathbb{R}^n$ is the state estimate, $\hat{\theta}(t) \in \mathbb{R}^p$ is the parameter estimate, ρ is a positive real number, Δ is as defined in (7), κ_o as in (9), and $\Gamma \in \mathbb{R}^{p\times p}$ is a positive definite gain matrix.

The state estimation equation (12b) is clearly related to the high gain observer (10). Besides the basic observer gain κ_o, extra terms have been introduced so that this equation can be written as

$$\begin{aligned}\dot{\hat{x}}(t) &= A_o\hat{x}(t) + f(\hat{x}(t)) + g(\hat{x}(t))u(t) \\ &\quad + \Delta\kappa_o[y(t) - c_o\hat{x}(t) - \psi(t)\hat{\theta}(t)] \\ &\quad + \rho^{-1}\Delta\Upsilon(t)\dot{\hat{\theta}}(t) \end{aligned} \quad (13)$$

where the term $\rho^{-1}\Delta\Upsilon(t)\dot{\hat{\theta}}(t)$ is needed due to the fact that the prediction error $y(t) - c_o\hat{x}(t) - \psi(t)\hat{\theta}(t)$ is computed with the parameter estimate $\hat{\theta}(t)$ instead of the true parameter θ. The term $\rho^{-1}\Delta\Upsilon(t)\dot{\hat{\theta}}(t)$ will play an important role in the convergence analysis of the proposed adaptive observer (12).

Like all parameter estimation problem, the estimation of θ requires some persistent excitation condition.

Assumption 1. Assume that $\psi(t)$ is persistently exciting, so that, for any $\rho > 0$ sufficiently large, the *row* vector of signals $\phi(t) \in \mathbb{R}^p$ obtained by linearly filtering $\psi(t)$ with the filter defined by the state space equations

$$\begin{aligned}\dot{\Upsilon}(t) &= \rho(A_o - \kappa_o c_o)\Upsilon(t) - \rho\kappa_o\psi(t) \\ \phi(t) &= c_o\Upsilon(t) + \psi(t) \end{aligned} \quad (14)$$

satisfies, for some positive constants α, T and for all $t \geq t_0$, the following inequality

$$\int_t^{t+T} \phi^T(\tau)\phi(\tau)d\tau \geq \alpha I_p \quad (15)$$

$\square$

Notice that $\phi(\tau)$ is a row vector and that $\phi^T(\tau)\phi(\tau)$ is a $p \times p$ matrix. For each instant τ, the rank of the matrix $\phi^T(\tau)\phi(\tau)$ is equal to one, however, its integral can be positive definite if $\psi(t)$ is sufficiently rich.

Theorem 2. Let $\Gamma \in \mathbb{R}^{p\times p}$ be any symmetric positive definite matrix, the notations Δ and κ_o as in (7) and (9). If $f(x)$, $g(x)$ are globally Lipschitz, $u(t)$ and $\psi(t)$ are bounded, then, under Assumption 1, for sufficiently large $\rho > 0$, the ordinary differential equations (12) constitute a *global* exponential adaptive observer for system (11) *i.e.*, for any initial conditions $x(t_0), \hat{x}(t_0), \hat{\theta}(t_0)$ and for all $\theta \in \mathbb{R}^n$, the errors $\hat{x}(t) - x(t)$ and $\hat{\theta}(t) - \theta$ tend exponentially to zero when $t \to \infty$.
$\square$

Proof. Remind that equation (13) has been derived from (12b) and (12c). Define

$$\tilde{x}(t) = \hat{x}(t) - x(t)$$
$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta$$

Following (11), (13) and noticing that $\dot{\theta} = 0$,

$$
\begin{aligned}
\dot{\tilde{x}}(t) &= A_o\tilde{x}(t) + f(\hat{x}(t)) - f(x(t)) \\
&\quad + g(\hat{x}(t))u(t) - g(x(t))u(t) \\
&\quad - \Delta\kappa_o[c_o\tilde{x}(t) + \psi(t)\tilde{\theta}(t)] \\
&\quad + \rho^{-1}\Delta\Upsilon(t)\dot{\tilde{\theta}}(t)
\end{aligned}
$$

Now let us define the scale transformation

$$
\begin{aligned}
x(t) &= \rho^{-1}\Delta z(t) \\
\hat{x}(t) &= \rho^{-1}\Delta\hat{z}(t) \\
\tilde{x}(t) &= \rho^{-1}\Delta\tilde{z}(t)
\end{aligned}
$$

Due to the special forms of Δ, A_o, c_o, it is easy to check the equalities

$$
\begin{aligned}
\Delta^{-1}A_o\Delta &= \rho A_o \\
c_o\Delta &= \rho\, c_o
\end{aligned}
$$

Then the scale transformation leads to

$$
\begin{aligned}
\dot{\tilde{z}}(t) &= \rho(A_o - \kappa_o c_o)\tilde{z}(t) + \xi(t) - \rho\kappa_o\psi(t)\tilde{\theta}(t) \\
&\quad + \Upsilon(t)\dot{\tilde{\theta}}(t)
\end{aligned}
$$

with

$$
\begin{aligned}
\xi(t) &= \rho\Delta^{-1}[f(\rho^{-1}\Delta\hat{z}(t)) - f(\rho^{-1}\Delta z(t))] \\
&\quad + \rho\Delta^{-1}[g(\rho^{-1}\Delta\hat{z}(t)) \\
&\quad - g(\rho^{-1}\Delta z(t))]u(t)
\end{aligned}
\tag{16}
$$

Now define the linear combination of the estimation errors

$$
\eta(t) = \tilde{z}(t) - \Upsilon(t)\tilde{\theta}(t) \tag{17}
$$

then

$$
\begin{aligned}
\dot{\eta}(t) &= \rho(A_o - \kappa_o c_o)[\eta(t) + \Upsilon(t)\tilde{\theta}(t)] + \xi(t) \\
&\quad - \rho\kappa_o\psi(t)\tilde{\theta}(t) + \Upsilon(t)\dot{\tilde{\theta}}(t) \\
&\quad - [\dot{\Upsilon}(t)\tilde{\theta}(t) + \Upsilon(t)\dot{\tilde{\theta}}(t)] \\
&= \rho(A_o - \kappa_o c_o)\eta(t) + \xi(t) \\
&\quad + [\rho(A_o - \kappa_o c_o)\Upsilon(t) - \rho\kappa_o\psi(t) \\
&\quad - \dot{\Upsilon}(t)]\tilde{\theta}(t)
\end{aligned}
\tag{18}
$$

Remind that $\Upsilon(t)$ is generated by (12a), therefore equation (18) simply becomes

$$
\dot{\eta}(t) = \rho(A_o - \kappa_o c_o)\eta(t) + \xi(t) \tag{19}
$$

Though the matrix $A_o - \kappa_o c_o$ is asymptotically stable (see Appendix B of (Xu and Zhang, 2004)), it is still not clear if $\eta(t)$ tends to zero, because of the nonlinear term $\xi(t)$ depending on $\tilde{\theta}(t)$. So the behavior of $\eta(t)$ has to be analyzed together with $\tilde{\theta}(t)$.

Now following (12c), (11b) and the equality $\dot{\theta} = 0$, the equation of $\tilde{\theta}(t)$ is derived:

$$
\dot{\tilde{\theta}}(t) = -\Gamma\left[c_o\Upsilon(t) + \psi(t)\right]^T\left[c_o\tilde{z}(t) + \psi(t)\tilde{\theta}(t)\right]
$$

Notice that $\tilde{z}(t) = \eta(t) + \Upsilon(t)\tilde{\theta}(t)$ following the definition of $\eta(t)$ in (17), then

$$
\begin{aligned}
\dot{\tilde{\theta}}(t) &= -\Gamma\left[c_o\Upsilon(t) + \psi(t)\right]^T c_o\eta(t) \\
&\quad - \Gamma\left[c_o\Upsilon(t) + \psi(t)\right]^T\left[c_o\Upsilon(t) + \psi(t)\right]\tilde{\theta}(t) \\
&= -\Gamma\phi^T(t)c_o\eta(t) - \Gamma\phi^T(t)\phi(t)\tilde{\theta}(t)
\end{aligned}
$$

where $\phi(t)$ is as defined in (14)

Now the problem is to study the stability of the joint error system

$$
\dot{\eta}(t) = \rho(A_o - \kappa_o c_o)\eta(t) + \xi(t) \tag{20a}
$$

$$
\dot{\tilde{\theta}}(t) = -\Gamma\phi^T(t)c_o\eta(t) - \Gamma\phi^T(t)\phi(t)\tilde{\theta}(t) \tag{20b}
$$

A similar problem has been encountered in (Xu and Zhang, 2004). Because of the difficulty of the stability analysis, the solution of (Xu and Zhang, 2004) was to modify their algorithm by considering a collection of systems corresponding to different delayed versions of the original system. It allowed to overcome the main difficulty of the error stability analysis, but also considerably increased the numerical complexity of the algorithm. In this paper, for the analysis of the error system (20), a different method is used in the following. This method, similar to the one used in (Zhang *et al.*, 2003), allows to preserve the simplicity of the algorithm (12).

Let us first consider the homogeneous part of the differential equation (20b), namely, the linear time varying system

$$
\dot{e}(t) = -\Gamma\phi^T(t)\phi(t)e(t) \tag{21}
$$

Based on the persistent excitation condition (15) and a classical result on linear time varying system stability, system (21) is exponentially stable (Narendra and Annaswamy, 1989, page 72). This stability implies that, for any symmetric positive definite matrix $Q(t) \in \mathbb{R}^{p\times p}$, in particular, for $Q(t) = I_p$, there exists a symmetric positive definite matrix $P(t) \in \mathbb{R}^{p\times p}$ satisfying the equation

$$
\begin{aligned}
\dot{P}(t) &= [\Gamma\phi^T(t)\phi(t)]^T P(t) \\
&\quad + P(t)[\Gamma\phi^T(t)\phi(t)] - I_p
\end{aligned}
\tag{22}
$$

The matrix $P(t)$ is known to have positive lower and upper bounds.

Now it is ready to study the stability of the error system (20). Consider the Lyapunov function candidate:

$$
V(t) = \eta^T(t)S\eta(t) + \tilde{\theta}(t)^T P(t)\tilde{\theta}(t)
$$

with S being the (positive definite) solution of (8) and $P(t)$ the solution of (22). Then

$$
\begin{aligned}
\dot{V}(t) = &-\rho\eta^T(t)S\eta(t) + 2\eta^T(t)S\xi(t) + \dot{\tilde{\theta}}^T(t)P(t)\tilde{\theta}(t) \\
&+ \tilde{\theta}^T(t)\dot{P}(t)\tilde{\theta}(t) + \tilde{\theta}^T(t)P(t)\dot{\tilde{\theta}}(t) \\
= &-\rho\eta^T(t)S\eta(t) + 2\eta^T(t)S\xi(t) \\
&- \eta^T(t)[P(t)\Gamma\phi^T(t)c_o]^T\tilde{\theta}(t) \\
&- \tilde{\theta}^T(t)[P(t)\Gamma\phi^T(t)c_o]\eta(t) - \tilde{\theta}^T(t)\tilde{\theta}(t)
\end{aligned}
$$

where, for the first equality, the equations (8) and (9) have been used with (20a).

Because f and g are globally Lipschitz and are triangular, the nonlinear term $\xi(t)$ as defined in (16) satisfies the inequality

$$
\begin{aligned}
\|\xi(t)\| &\le \mu(\rho^{-1})\|\tilde{z}(t)\| \\
&= \mu(\rho^{-1})\|\eta(t) + \Upsilon(t)\tilde{\theta}(t)\|
\end{aligned}
$$

with $\mu(\rho^{-1}) > 0$, a polynomial in ρ^{-1} depending on the Lipschitz constants of f and g and on the upper bound of $u(t)$. Therefore, there exist two polynomials $\mu_1(\rho^{-1})$ and $\mu_2(\rho^{-1})$ such that

$$
\begin{aligned}
2\eta^T(t)S\xi(t) \le \mu_1(\rho^{-1})\|\eta(t)\|^2 \\
+ \mu_2(\rho^{-1})\|\eta(t)\| \cdot \|\tilde{\theta}(t)\|
\end{aligned}
$$

Because $P(t)$ and $\Upsilon(t)$ have upper bounds, there exists a constant $c > 0$ such that

$$
2\|P(t)\Gamma\phi^T(t)c_o\| \le c
$$

The inequality $ab \le \frac{1}{2}\left(a^2 + b^2\right)$ with $a = [\mu_2(\rho^{-1}) + c]\|\eta\|$ and $b = \|\tilde{\theta}\|$ leads to

$$
\begin{aligned}
\dot{V}(t) \le &-\eta^T(t)[\rho S - \mu_1(\rho^{-1})I]\eta(t) - \|\tilde{\theta}\|^2 \\
&+ [\mu_2(\rho^{-1}) + c]\|\tilde{\theta}\| \cdot \|\eta\| \\
\le &-\eta^T(t)[\rho S - \mu_1(\rho^{-1})I]\eta(t) - \|\tilde{\theta}\|^2 \\
&+ \frac{(\mu_2(\rho^{-1}) + c)^2}{2}\|\eta(t)\|^2 + \frac{1}{2}\|\tilde{\theta}\|^2 \\
= &-\eta^T(t)\left[\rho S - \mu_1(\rho^{-1})I - \frac{(\mu_2(\rho^{-1}) + c)^2}{2}I\right] \\
&\cdot \eta(t) - \frac{1}{2}\|\tilde{\theta}\|^2
\end{aligned}
$$

Let us choose a value of ρ sufficiently large, such that

$$
\rho S - \mu_1(\rho^{-1})I - \frac{(\mu_2(\rho^{-1}) + c)^2}{2}I > \frac{1}{2}I
$$

then

$$
\begin{aligned}
\dot{V}(t) &\le -\frac{1}{2}\|\eta\|^2 - \frac{1}{2}\|\tilde{\theta}\|^2 \\
&\le -\frac{1}{2}\left[\eta^T\frac{S}{\lambda_{\max}(S)}\eta + \tilde{\theta}^T\frac{P(t)}{\lambda_{\max}(P(t))}\tilde{\theta}\right] \\
&\le -\frac{1}{2}\min\left(\frac{1}{\lambda_{\max}(S)}, \frac{1}{\lambda_{\max}(P(t))}\right)V
\end{aligned}
$$

with $\lambda_{\max}(S)$ being the largest eigenvalue of S.

Then it is concluded that $\eta(t)$ and $\tilde{\theta}(t)$ tend exponentially to zero, so do $\tilde{z}(t) = \eta(t) + \Upsilon(t)\tilde{\theta}(t)$ and $\tilde{x}(t) = \rho^{-1}\Delta\tilde{z}(t)$.
□

4. NUMERICAL EXAMPLE

Let us illustrate the proposed algorithm with a simple bio-reactor simulated with the Contois model (Contois, 1959) which has been used in (Gauthier *et al.*, 1992) as an example of high gain observer application. Denoting by x_1' and x_2' the concentrations of micro-organisms and substrate, the evolution of the two concentrations is described by the equations

$$
\begin{aligned}
\dot{x}_1' &= \frac{a_1 x_1' x_2'}{a_2 x_1' + x_2'} - u x_1' \\
\dot{x}_2' &= -\frac{a_3 a_1 x_1' x_2'}{a_2 x_1' + x_2'} - u x_2' + a_4 u
\end{aligned}
$$

where u is the dilution rate and a_1, a_2, a_3, a_4 are model parameters.

After the state transformation

$$
\begin{aligned}
x_1 &= x_1' \\
x_2 &= \frac{a_1 x_1' x_2'}{a_2 x_1' + x_2'}
\end{aligned}
$$

the system equation becomes

$$
\begin{aligned}
\dot{x}_1 &= x_2 - u x_1 \\
\dot{x}_2 &= \frac{a_2 x_2 (x_2^2 - a_1 u x_1^2) + (a_1 x_1 - x_2)^2 (a_4 u - a_3 x_2)}{a_1 a_2 x_1^2}
\end{aligned}
$$

As in (Gauthier *et al.*, 1992), the parameter values used in the simulation are: $a_1 = a_2 = a_3 = 1$, $a_4 = 0.1$, the initial states $x_1(0) = 0.05$, $x_2(0) = 0.025$, and the input signal used in the simulation

$$
u(t) = \begin{cases} 0.08 & \text{for } 0 \le t < 10 \\ 0.02 & \text{for } 10 \le t < 20 \\ 0.08 & \text{for } t \ge 20 \end{cases}
$$

Assume that the sensor measuring x_1 is affected by a fault

$$
y(t) = x_1(t) + q(t)
$$

where the fault $q(t)$ is simulated as a *chirp* signal

$$
q(t) = \begin{cases} 0 & \text{for } t < 20 \\ 0.01\sin(0.06t^2 - 0.4t) & \text{for } t \ge 20 \end{cases}
$$

In the adaptive observer, the fault $q(t)$ is *approximated* with the Fourier expansion

$$
\begin{aligned}
&\theta_1 \cos 4t + \theta_2 \cos 8t + \theta_3 \cos 16t + \theta_4 \cos 32t \\
&+ \theta_5 \sin 4t + \theta_6 \sin 8t + \theta_7 \sin 16t + \theta_8 \sin 32t
\end{aligned}
$$

The algorithm parameters $\rho = 1$ and $\Gamma = 400 I_8$ with I_8 being the 8×8 identity matrix. In figure 1 are plotted the simulated fault (top), its estimation given by the adaptive observer (middle) and their difference (bottom).

5. CONCLUSION

An adaptive observer is proposed in this paper for sensor fault estimation. The convergence analysis

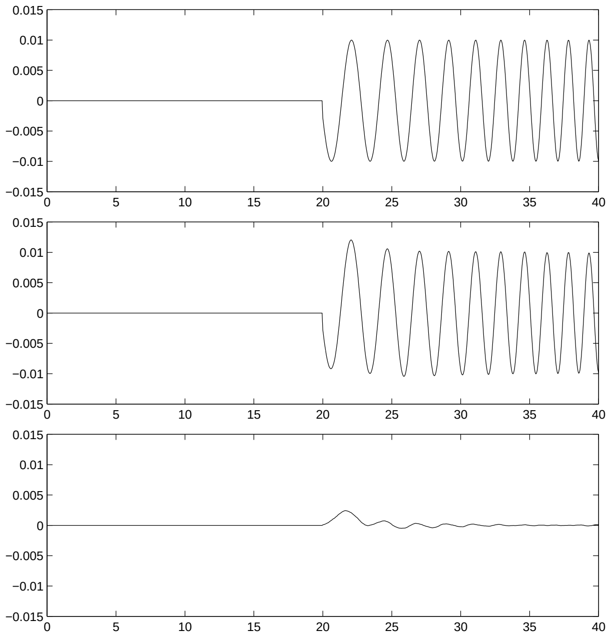

Figure 1. Sensor fault signal (top), its estimation (middle) and their difference (bottom)

of the proposed algorithm has been made under the assumption of perfect modeling, In practice, modeling and measurement uncertainties are inevitable, implying estimation errors. Though simulations show the robustness of the proposed algorithm to such errors, as in the numerical example of this paper, the theoretic study of such impact on nonlinear observer based approaches remains a challenging problem.

REFERENCES

Basseville, M. and I. Nikiforov (1993). *Detection of Abrupt Changes – Theory and Applications*. Information and System Sciences Serie. Prentice Hall. Englewood Cliffs, N.J.

Bastin, Georges and Michel Gevers (1988). Stable adaptive observers for nonlinear time varying systems. *IEEE Trans. on Automatic Control* **33**(7), 650–658.

Besançon, Gildas (2000). Remarks on nonlinear adaptive observer design. *Systems and Control Letters* **41**(4), 271–280.

Besançon, Gildas, Qinghua Zhang and Hassan Hammouri (2004). High-gain observer based state and parameter estimation in nonlinear systems. In: *IFAC Symposium on Nonlinear Control Systems (NOLCOS)*. Stuttgart.

Chen, Jie and Ron J. Patton (1999). *Robust model-based fault diagnosis for dynamic systems*. Kluwer Academic Publishers. Boston, Dordrecht, London.

Cho, Young Man and Rajesh Rajamani (1997). A systematic approach to adaptive observer synthesis for nonlinear systems. *IEEE Trans. on Automatic Control* **42**(4), 534–537.

Contois, D. (1959). Kinetics of bacteria growth relationship between population density and specific growth rate of continuous cultures. *Journal of Genetic Macrobiology* **21**, 40–50.

DePersis, C. and A. Isidori (2001). A geometric approach to nonlinear fault detection and isolation. *IEEE Trans. on Automatic Control* **46**(6), 853–865.

Deza, François (1991). Contribution à la synthèse d'observateurs exponentiels – application à un procédé industriel. PhD thesis. Institut National des Sciences Appliquées de Rouen. Rouen, France. In French.

Frank, P. M. (1990). Fault diagnosis in dynamic systems using analytical and knowledge based redundancy - a survey and new results. *Automatica* **26**, 459–474.

Gauthier, J-P., H. Hammouri and S. Othman (1992). A simple observer for nonlinear systems, application to bioreactors. *IEEE Trans. on Automatic Control* **37**(6), 875–880.

Gauthier, Jean-Paul and Ivan Kupka (2001). *Deterministic observation theory and applications*. Cambridge University Press.

Gertler, Janos J. (1998). *Fault detection and diagnosis in engineering systems*. Marcel Dekker, Inc.. New York, Basel, Hong Kong.

Hammouri, H., M. Kinnaert and E. H. El Yaagoubi (1998). Observer-based approach to fault detection and isolation for nonlinear systems. *IEEE Trans. on Automatic Control* **44**(10), 1879–1884.

Hou, M., K. Busawon and M Saif (2000). Observer design based on triangular form generated by injective map. *IEEE Trans. on Automatic Control* **45**(7), 1350–1355.

Marino, Riccardo and Patrizio Tomei (1995). *Nonlinear control design*. Information and system sciences. Prentice Hall. London, New York.

Narendra, K. S. and A. M. Annaswamy (1989). *Stable adaptive systems*. Prentice Hall. Boston.

Staroswiecki, M. and G. Comtet-Varga (2001). Analytic redundancy for fault detection and isolation in algebraic dynamic system. *Automatica* **37**, 687–699.

Xu, Aiping and Qinghua Zhang (2004). Nonlinear system fault diagnosis based on adaptive estimation. *Automatica* **40**(7), 1181–1193.

Zhang, Qinghua (2005). An adaptive observer for sensor fault estimation in linear time varying systems. In: *Submitted to IFAC World Congress*.

Zhang, Qinghua, Aiping Xu and Gildas Besançon (2003). An efficient nonlinear adaptive observer with global convergence. In: *13th IFAC/IFORS Symposium on System Identification (SYSID)*. Rotterdam. pp. 1737–1742.