AN ADAPTIVE OBSERVER FOR SENSOR FAULT ESTIMATION IN LINEAR TIME VARYING SYSTEMS

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Abstract A method for sensor fault estimation in multiple-input multiple-output linear *time varying* systems is proposed in this paper. It is based on a new adaptive observer for joint estimation of states and sensor faults in a state-space formulation of the monitored system. The exponential convergence of the algorithm is proved under some persistent excitation condition.

Keywords: fault detection and isolation, linear time varying system, adaptive observer.

1. INTRODUCTION

In order to improve the safety and the reliability of more and more complex engineering systems, the problems of fault detection and isolation (FDI) have received considerable attention of researchers. The faults affecting a dynamic system can be typically classified as process faults, actuator faults and sensor faults. In a state-space formulation, sensor faults are usually modeled as some changes in the output equation.

There are two typical methods for dealing with sensor faults. One is to transform them into the state equation and to treat them as actuator faults (Massoumnia *et al.*, 1989). This method leads to a system model of higher order. The other method is to reject sensor faults by projecting the output measurements into some sub-space (Chen and Patton, 1999). It relies on sensor redundancy and does not consider fault estimation. In this paper, a new method is proposed to directly estimate sensor faults.

In this paper, linear time varying systems of the following form are considered

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + w(t) \tag{1a}$$

$$y(t) = C(t)x(t) + v(t) + f(t) \tag{1b}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^l$, $y(t) \in \mathbb{R}^m$ are respectively the state, input, output of the system; $A(t), B(t), C(t)$ are known *time varying* matrices of appropriate sizes; $w(t) \in \mathbb{R}^n$, $v(t) \in \mathbb{R}^m$ are bounded noises; the additional term $f(t) \in \mathbb{R}^m$ represents the possible sensor faults. The matrices $A(t), B(t), C(t)$ are all assumed piecewise continuous and bounded. Notice that *no whiteness* of the noises is assumed in this paper.

It is often assumed that only a subset of the sensors is possibly affected by faults. If the considered system remains observable when the possibly faulty subset of sensors is omitted (implying some sensor redundancy), then the estimation of the sensor faults $f(t)$ is trivial. Alternatively, if the system dynamics matrix $A(t)$ is asymptotically stable, the state $x(t)$ can be easily estimated by simulation, then the estimation of $f(t)$ is also trivial. Neither of these conditions is required in this paper. Instead, the observability of the system with its full set of sensors is assumed. It is also assumed that $f(t)$ can be expressed by some linear regression

$$f(t) = \theta_1\psi_1(t) + \cdots \theta_p\psi_p(t) \tag{2}$$

with given regressors $\psi_i(t) \in \mathbb{R}^m$ and unknown regression coefficients $\theta_i \in \mathbb{R}$. This model may come from some physical knowledge about the possible faults. For example, some disturbances with known frequencies may affect the output measurement. It can also be considered as a generic approximator of the fault signals. It is then assumed that the complexity of the regressors $\psi_1(t), \ldots, \psi_p(t)$ allows to reasonably approximate $f(t)$. In this case, the term $v(t)$ in (1b) includes also the approximation error of the regression estimator. Such a numerical example (approximating a chirp signal with a finite number of sinusoid functions) will be presented in Section 5.

For more compact notations, let

$$\Psi(t) = [\psi_1(t), \ldots, \psi_p(t)]$$
$$\theta = \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_p \end{bmatrix}$$

then system (1) is rewritten as

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + w(t) \tag{3a}$$
$$y(t) = C(t)x(t) + v(t) + \Psi(t)\theta \tag{3b}$$

The considered sensor fault estimation problem then amounts to the estimation of the parameter vector θ. The method proposed in this paper is based on a new adaptive observer that can jointly estimate the state vector $x(t)$ and the parameter vector θ.

A natural idea for joint estimation of states and parameters is to apply the *Kalman filter* to the *extended* system obtained by appending the parameter vector into the state vector. Compared with such an (extended) Kalman filter, one important advantage of the adaptive observer proposed in this paper is the possibility to be generalized to nonlinear systems by applying some techniques of high gain observer (Zhang and Besançon, 2005). Some other advantages, from both theoretic and practical points of view, will be discussed in Section 4.

Quite a few methods for the design of adaptive observers have been published in the literature (Kreisselmeier, 1977; Bastin and Gevers, 1988; Marino and Tomei, 1995; Besançon, 2000), and some of them have been used for FDI (Ding and Frank, 1993; Yang and Saif, 1995; Wang *et al.*, 1997; Zhang, 2000). These methods are restricted to time invariant systems. Recently, an adaptive observer for linear *time varying* systems has been proposed (Zhang, 2002). A related method has been developed for fault diagnosis (Xu and Zhang, 2004), dealing with *actuator* faults only, not *sensor* faults.

The result presented in this paper is to some extent similar to that of (Vemuri, 2001) which considers sensor *bias* fault only, whereas general sensor faults are considered in this paper. Another particularity of this paper is that linear *time varying* systems are considered, whereas most existing methods for sensor fault FDI consider *time invariant* systems. The result of this paper is also extended to nonlinear systems (Zhang and Besançon, 2005).

This paper is organized as follows. The proposed algorithm is described in Section 2 and its convergence is analyzed in Section 3. It is then compared with the Kalman filter in Section 4. A numerical example is presented in Section 5. Some concluding remarks are given in Section 6.

2. THE ADAPTIVE OBSERVER FOR SENSOR FAULT ESTIMATION

Let us first state some assumptions ensuring the convergence of the proposed algorithm.

Assumption 1. The matrix pair $(A(t), C(t))$ is such that a bounded (time-varying) matrix $K(t) \in \mathbb{R}^{n\times m}$ can be designed so that the system

$$\dot{\eta}(t) = [A(t) - K(t)C(t)]\eta(t) \tag{4}$$

is exponentially stable. □

This assumption implies that the fault free system has an exponential observer. It is known that, if the matrix pair $(A(t), C(t))$ is uniformly completely observable, then the Kalman gain $K(t)$ can fulfill Assumption 1 (Jazwinski, 1970).

Assumption 2. Let the matrix of signals $\Psi(t) \in \mathbb{R}^{m\times p}$ be filtered through the linear time varying filter

$$\dot{\Upsilon}(t) = [A(t) - K(t)C(t)]\Upsilon(t) - K(t)\Psi(t) \tag{5a}$$
$$\Omega(t) = C(t)\Upsilon(t) + \Psi(t) \tag{5b}$$

where $\Upsilon(t) \in \mathbb{R}^{n\times p}$ and $\Omega(t) \in \mathbb{R}^{m\times p}$ are respectively the state and the output of the filter. Assume that $\Psi(t)$ is persistently exciting, so that the filtered signals $\Omega(t)$ satisfies, for some positive constants α, T and for all $t \geq t_0$, the following inequality

$$\int_t^{t+T} \Omega^T(\tau)\Omega(\tau)d\tau \geq \alpha I_p \tag{6}$$

where I_p is the $p \times p$ identity matrix. □

Remark 1. Typically, the matrix $\Omega(\tau)$ has more columns than rows ($p > m$), then the matrix product $\Omega^T(\tau)\Omega(\tau)$ is rank deficient for each time instant τ. However, the integral in (6) can be made positive definite if the excitation $\Psi(t)$ generating $\Omega(t)$ is sufficiently rich. If $p \leq m$ (as assumed in many methods for *complete* fault isolation), then the integral in (6) is trivially positive definite (except degenerate case). □

Now the proposed adaptive observer, in the form of a set of ordinary differential equations (ODE), can be formulated:

$$\dot{\Upsilon}(t) = [A(t) - K(t)C(t)]\Upsilon(t) - K(t)\Psi(t) \quad (7a)$$

$$\begin{aligned}\dot{\hat{x}}(t) = {} & A(t)\hat{x}(t)+B(t)u(t) \\ & +K(t)\left[y(t) - C(t)\hat{x}(t) - \Psi(t)\hat{\theta}(t)\right] \\ & +\Upsilon(t)\Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T \\ & \cdot\left[y(t) - C(t)\hat{x}(t) - \Psi(t)\hat{\theta}(t)\right] \end{aligned} \quad (7b)$$

$$\begin{aligned}\dot{\hat{\theta}}(t) = {} & \Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T \\ & \cdot\left[y(t) - C(t)\hat{x}(t) - \Psi(t)\hat{\theta}(t)\right] \end{aligned} \quad (7c)$$

where $\Gamma \in \mathbb{R}^{p\times p}$ is a positive definite gain matrix. Notice that the last term of (7b) is equal to $\Upsilon(t)\dot{\hat{\theta}}(t)$.

It may not be obvious to understand the equations of this algorithm at a first view. Some heuristic explanation has been given in (Zhang, 2002) for a similar algorithm dealing with the case with the term $\Psi(t)\theta$ located in the state equation. The case of this paper with the term $\Psi(t)\theta$ located in the output equation may seem easier, since the unknown parameters are more close to the output measurements $y(t)$. However, the presence of $\Psi(t)\theta$ in the output equation makes the application of observer-like techniques more difficult, since any output feedback will involve the faults possibly affecting the output measurements.

3. CONVERGENCE ANALYSIS

The convergence property of algorithm (7) is first analyzed in the noise-free case, then in the noise-corrupted case.

3.1 The noise-free case

Theorem 1. In the noise-free case, that is, $w(t) = 0$ and $v(t) = 0$, under Assumptions 1 and 2, the algorithm (7) is a global exponential adaptive observer of system (3), *i.e.*, for any initial conditions $x(t_0), \hat{x}(t_0), \hat{\theta}(t_0), \Upsilon(t_0)$ and for any value of θ, when $t \to \infty$, the estimation errors $\hat{x}(t) - x(t)$ and $\hat{\theta}(t) - \theta$ tend to zero exponentially fast.

Proof of Theorem 1

Combining (7b) and (7c) yields

$$\begin{aligned}\dot{\hat{x}}(t) = {} & A(t)\hat{x}(t)+B(t)u(t) \\ & + K(t)\left[y(t) - C(t)\hat{x}(t) - \Psi(t)\hat{\theta}(t)\right] \\ & + \Upsilon(t)\dot{\hat{\theta}}(t) \end{aligned} \quad (8)$$

Define the error variables

$$\begin{aligned}\tilde{x}(t) &= \hat{x}(t) - x(t) \\ \tilde{\theta}(t) &= \hat{\theta}(t) - \theta\end{aligned}$$

Following (3a), (8), and the assumptions $w(t) = 0$, $\dot{\theta} = 0$, it is easy to get the error equation

$$\begin{aligned}\dot{\tilde{x}}(t) = {} & A(t)\tilde{x}(t) + K(t)\left[y(t) - C(t)\hat{x}(t) - \Psi(t)\hat{\theta}(t)\right] \\ & + \Upsilon(t)\dot{\tilde{\theta}}(t)\end{aligned}$$

Substitute (3b) into the last equation with $v(t) = 0$, then

$$\begin{aligned}\dot{\tilde{x}}(t) = {} & [A(t) - K(t)C(t)]\tilde{x}(t) - K(t)\Psi(t)\tilde{\theta}(t) \\ & + \Upsilon(t)\dot{\tilde{\theta}}(t)\end{aligned} \quad (9)$$

The key step of the proof is to define

$$\eta(t) = \tilde{x}(t) - \Upsilon(t)\tilde{\theta}(t) \quad (10)$$

then it is straightforward to obtain

$$\begin{aligned}\dot{\eta}(t) = {} & [A(t) - K(t)C(t)]\eta(t) \\ & + \Big([A(t) - K(t)C(t)]\Upsilon(t) - K(t)\Psi(t) \\ & - \dot{\Upsilon}(t)\Big)\tilde{\theta}(t)\end{aligned}$$

Because $\Upsilon(t)$ is generated by (7a), the last equation simply becomes

$$\dot{\eta}(t) = [A(t) - K(t)C(t)]\eta(t)$$

Then according to Assumption 1, $\eta(t) \to 0$ with exponential convergence.

Now from (7c), (3b) and $\dot{\theta} = 0$, the equation of $\tilde{\theta}(t)$ is derived:

$$\begin{aligned}\dot{\tilde{\theta}}(t) = {} & -\Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T \\ & \cdot\left[C(t)\tilde{x}(t) + \Psi(t)\tilde{\theta}(t)\right]\end{aligned} \quad (11)$$

Following (10), replace $\tilde{x}(t) = \eta(t) + \Upsilon(t)\tilde{\theta}(t)$ in the last equation, then

$$\begin{aligned}\dot{\tilde{\theta}}(t) = {} & -\Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T \\ & \cdot\left[C(t)\Upsilon(t) + \Psi(t)\right]\tilde{\theta}(t) \\ & - \Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T C(t)\eta(t)\end{aligned} \quad (12)$$

Now let us study the homogeneous part of the last equation, that is

$$\begin{aligned}\dot{\xi}(t) = {} & -\Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T \\ & \cdot\left[C(t)\Upsilon(t) + \Psi(t)\right]\xi(t)\end{aligned} \quad (13)$$

According to Assumption 2 and Lemma 1 stated in the Appendix, the homogeneous system (13) is exponentially stable.

The matrices $C(t), \Psi(t)$ are assumed bounded. The matrix $\Upsilon(t)$ generated through (5a) is also bounded following Assumption 1. Because the homogeneous part of the ODE (12) is exponentially stable and its non homogeneous term $-\Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T C(t)\eta(t)$ is exponentially vanishing, the error $\tilde{\theta}(t)$ governed by (12) is then exponentially vanishing.

Finally, $\tilde{x}(t) = \eta(t)+\Upsilon(t)\tilde{\theta}(t)$ is also exponentially vanishing. □

3.2 The noise-corrupted case

Theorem 2. Under Assumptions 1 and 2, when algorithm (7) is applied to system (3) with bounded noises $w(t)$ and $v(t)$, the estimation errors $\hat{x}(t) - x(t)$ and $\hat{\theta}(t) - \theta$ remain bounded. Moreover, if the noises $w(t), v(t)$ have zero means, then the estimation errors converge exponentially to zero in the mean.

Proof of Theorem 2 The proof of this theorem essentially relies on the result already established in the noise-free case. Like in the proof of Theorem1, the equations of the errors $\tilde{x}(t) = \hat{x}(t) - x(t)$ and $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$, similar to (9) and (11), are first derived, but now the noises are involved:

$$\dot{\tilde{x}}(t) = [A(t) - K(t)C(t)]\tilde{x}(t) - K(t)\Psi(t)\tilde{\theta}(t) + \Upsilon(t)\dot{\tilde{\theta}}(t) - w(t) + K(t)v(t) \tag{14a}$$

$$\dot{\tilde{\theta}}(t) = -\Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T \cdot \left[C(t)\tilde{x}(t) + \Psi(t)\tilde{\theta}(t)\right] + \Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T v(t) \tag{14b}$$

In order to put this error system in the standard ODE form, replace $\dot{\tilde{\theta}}(t)$ in (14a) with the right hand side of (14b):

$$\begin{aligned}\dot{\tilde{x}}(t) =& [A(t) - K(t)C(t)]\tilde{x}(t) - K(t)\Psi(t)\tilde{\theta}(t) \\ & -\Upsilon(t)\Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T \\ & \cdot \left[C(t)\tilde{x}(t) + \Psi(t)\tilde{\theta}(t)\right] \\ & +\Upsilon(t)\Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T v(t) \\ & -w(t) + K(t)v(t)\end{aligned} \tag{15a}$$

$$\dot{\tilde{\theta}}(t) = -\Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T \cdot \left[C(t)\tilde{x}(t) + \Psi(t)\tilde{\theta}(t)\right] + \Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T v(t) \tag{15b}$$

According to Theorem 1, when the noises $w(t) = 0$ and $v(t) = 0$, the errors $\tilde{x}(t), \tilde{\theta}(t)$ are exponentially vanishing. It means that the homogeneous part of the error system (15) (without the terms involving the noises) is exponentially stable.

For the same reasons as in the proof of Theorem 1, the matrices $K(t), C(t), \Upsilon(t), \Psi(t)$ are all bounded. Therefore, under the assumption that the noises $w(t), v(t)$ are bounded, the terms in (15) involving the noises are bounded. It then follows that the errors $\tilde{x}(t)$ and $\tilde{\theta}(t)$ driven by the noises through (15) remain bounded.

Let $\mathbf{E}w(t)$ denote the mean value of $w(t)$. Assume that $\mathbf{E}w(t) = 0$ and $\mathbf{E}v(t) = 0$. Take the mean at both sides of (14):

$$\begin{aligned}\frac{d\mathbf{E}\tilde{x}(t)}{dt} &= [A(t) - K(t)C(t)]\mathbf{E}\tilde{x}(t) \\ &\quad - K(t)\Psi(t)\mathbf{E}\tilde{\theta}(t) + \Upsilon(t)\frac{d\mathbf{E}\tilde{\theta}(t)}{dt} \\ \frac{d\mathbf{E}\tilde{\theta}(t)}{dt} &= -\Gamma\left[C(t)\Upsilon(t) + \Psi(t)\right]^T \\ &\quad \cdot \left[C(t)\mathbf{E}\tilde{x}(t) + \Psi(t)\mathbf{E}\tilde{\theta}(t)\right]\end{aligned}$$

These two equations are identical to (9) and (11), except that $\tilde{x}(t)$ and $\tilde{\theta}(t)$ are now replaced by their means $\mathbf{E}\tilde{x}(t)$ and $\mathbf{E}\tilde{\theta}(t)$. Following the same arguments as in the proof of Theorem 1, $\mathbf{E}\tilde{x}(t)$ and $\mathbf{E}\tilde{\theta}(t)$ tend to zero exponentially fast when $t \to \infty$. □

4. COMPARISON WITH THE KALMAN FILTER

Now let us compare the proposed adaptive observer with the Kalman filter applied to the *extended* system

$$\begin{bmatrix}\dot{x}(t)\\ \dot{\theta}(t)\end{bmatrix} = \begin{bmatrix}A(t) & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}x(t)\\ \theta(t)\end{bmatrix} + \begin{bmatrix}B(t)\\ 0\end{bmatrix}u(t) + \begin{bmatrix}w(t)\\ 0\end{bmatrix}$$

$$y(t) = \begin{bmatrix}C(t) & \Psi(t)\end{bmatrix}\begin{bmatrix}x(t)\\ \theta(t)\end{bmatrix} + v(t)$$

From theoretic point of view, it is important to know the condition guaranteeing the convergence of the Kalman filter. It is known that, for linear time varying systems, such a condition is given by the uniform complete observability (Jazwinski, 1970). In order to formulate the Gramian observability matrix of the *extended* system, its transition matrix must be first derived. Let $\Phi(t_0, t) \in \mathbb{R}^{n\times n}$ be the transition matrix of the non extended system, associated with the matrix $A(t)$. Then it is easy to check that the transition matrix of the extended system is

$$\begin{bmatrix}\Phi(t_0, t) & 0\\ 0 & I_p\end{bmatrix}$$

with I_p the $p \times p$ identity matrix. For time varying systems, in general it is difficult to analytically compute the transition matrix. It can be numerically computed by solving

$$\frac{d}{dt}\Phi(t_0, t) = A(t)\Phi(t_0, t), \qquad \Phi(t_0, t_0) = I_n$$

Note that this numerical solution is possible only if the matrix $A(t)$ has a good stability property to avoid numerical divergence.

Denote

$$G(\tau, s) = \begin{bmatrix}\Phi^T(\tau, s)C^T(\tau)C(\tau)\Phi^T(\tau, s) & \Phi^T(\tau, s)C^T(\tau)\Psi(\tau)\\ \Psi^T(\tau)C(\tau)\Phi(\tau, s) & \Psi^T(\tau)\Psi(\tau)\end{bmatrix}$$

If there exist positive constants α, β, T such that, for all $t \geq t_0$, the Gramian observability matrix of the extended system is bounded:

$$\alpha I_{n+p} \leq \int_t^{t+T} G(\tau, t+T)d\tau \leq \beta I_{n+p} \qquad (16)$$

then the Kalman filter applied to the extended system converges (Jazwinski, 1970).

The matrix $G(\tau, t+T)$ has the size $(n+p)\times(n+p)$. It is thus obvious that condition (16) is more difficult to be satisfied than the inequality (6) in Assumption 2 involving matrices of smaller sizes. Note that in order to ensure the existence of $K(t)$ used in the proposed adaptive observer, the uniform complete observability of the matrix pair $(A(t), C(t))$ should be checked, with the related $n \times n$ Gramian matrix.

From practical point of view, the application of the Kalman filter to the *extended* system requires the numerical solution of a $(n+p)$-th order Riccati equation, with a numerical complexity clearly higher than that of the proposed adaptive observer.

5. NUMERICAL EXAMPLE

In order to illustrate the proposed algorithm, let us consider a simulated flight control system. The linearized lateral dynamics of a remotely piloted aircraft (Chen and Patton, 1999, page 188) is modeled as

$$\dot{x}(t) = \begin{bmatrix} -0.277 & 0 & -32.9 & 9.81 & 0 \\ -0.1033 & -8.525 & 3.75 & 0 & 0 \\ 0.3649 & 0 & -0.639 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} -5.432 & 0 \\ 0 & -28.64 \\ -9.49 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} x(t)$$

with

$$x(t) = \begin{bmatrix} \text{side slip} \\ \text{roll rate} \\ \text{yaw rate} \\ \text{bank angle} \\ \text{yaw angle} \end{bmatrix} \qquad u(t) = \begin{bmatrix} \text{rudder} \\ \text{aileron} \end{bmatrix}$$

In the simulation, a saturated Gaussian noise (saturated for boundedness) with standard deviation equal to 0.2 is added to each output. A fault of the third sensor occurring at the 50-th second is simulated with the *chirp signal* $\sin(0.014t^2 - 0.6t)$. The fault estimator takes the form of a Fourier expansion $\theta_1 \cos 0.1t + \theta_2 \cos 0.2t + \theta_3 \cos 0.4t + \theta_4 \cos 0.8t + \theta_5 \sin 0.1t + \theta_6 \sin 0.2t + \theta_7 \sin 0.4t + \theta_8 \sin 0.8t$. The adaptive observer is applied with the parameters $\Gamma = 20 I_8$,

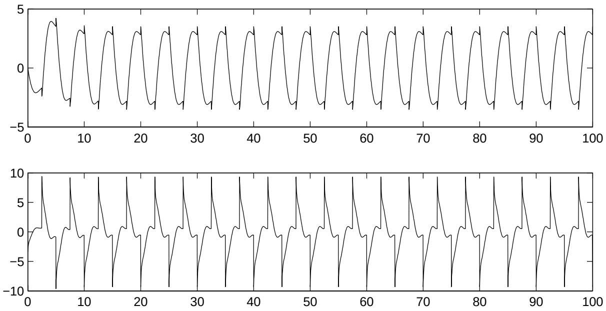

Figure 1. Input signals $u(t)$.

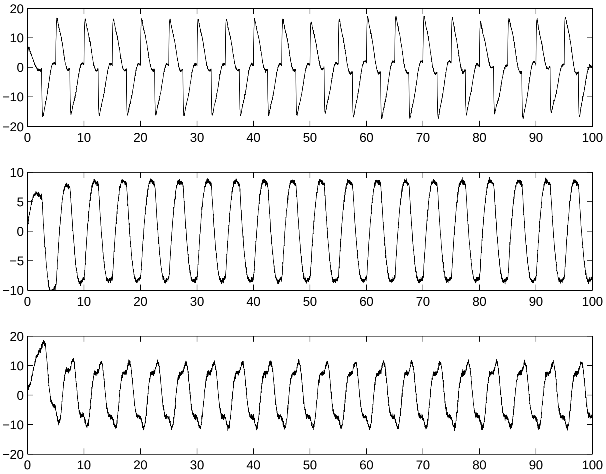

Figure 2. Output Signals $y(t)$

$$K = \begin{bmatrix} 0.0588 & 0.9135 & 0.0466 \\ 1.4054 & 0.3383 & 0.0031 \\ -0.7169 & -9.8948 & 0.5708 \\ 0.5638 & 4.5531 & 0.0603 \\ 0.0017 & 0.0201 & 2.9998 \end{bmatrix}$$

The initial values used in the simulation are $x(0) = [1, 1, 1, 1, 1]^T$, $\hat{x}(0) = 0.9x(0)$, $\hat{\theta}(0) = [0, 0, 0, 0, 0, 0, 0, 0]^T$, $\Upsilon(0) = 0_{3\times 4}$.

The input signals $u(t)$ generated by a simple proportional controller are shown in figure 1. The simulated output signals are illustrated in figure 2. The simulated fault (occurring at the 50-th second) is not easily noticeable by visual inspection of these signals.

The simulated fault, its estimate and their difference are plotted in figure 3. With as few as 4 frequencies in the Fourier expansion estimator, the fault signal is well estimated. Remark that the simulated fault is a chirp signal with continuously changing frequency. A filtering or smoothing algorithm can be used to reduce the noise of the estimation, but it would imply some delay for online processing.

6. CONCLUSION

A method has been proposed in this paper for sensor fault estimation based on a new adaptive observer. It is applicable to linear *time varying*

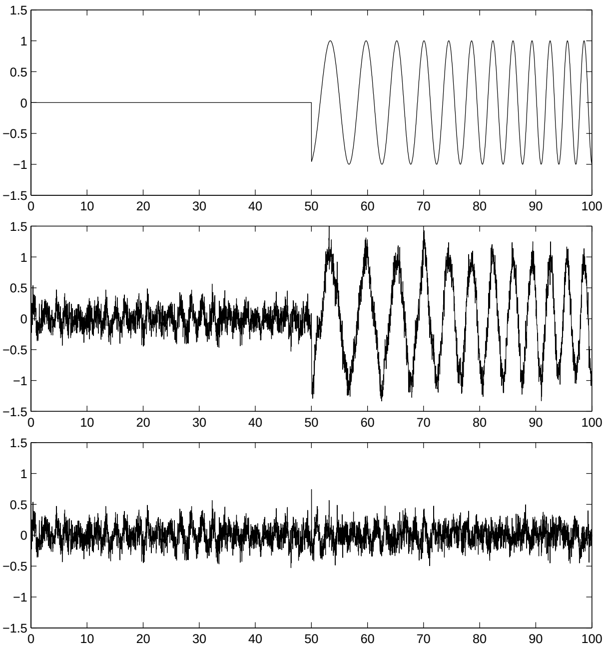

Figure 3. Simulated fault (top), its estimation (middle) and their difference (bottom)

systems subject to quite general sensor faults. An extension of this method to nonlinear systems is presented in (Zhang and Besançon, 2005).

The approach presented in this paper is to directly estimate sensor faults with the proposed adaptive observer. For the purpose of fault isolation, it is also possible to develop a residual generation approach, following the techniques used in (Xu and Zhang, 2004).

APPENDIX

Lemma 1. Let $\Omega(t) \in \mathbb{R}^{m\times p}$ be a bounded and piecewise continuous matrix and $\Gamma \in \mathbb{R}^{p\times p}$ be any symmetric positive definite matrix. If there exist positive constants T, α such that, for all $t \geq t_0$,

$$\int_t^{t+T} \Omega^T(\tau)\Omega(\tau)d\tau \geq \alpha I_p$$

then the system

$$\dot{\xi}(t) = -\Gamma\Omega^T(t)\Omega(t)\xi(t)$$

is exponentially stable.

A proof of this classical result can be found in (Narendra and Annaswamy, 1989, page 72).

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