ROBUST FAULT DETECTION AND ISOLATION FILTERS DESIGN WITH SENSITIVITY CONSTRAINT FOR LPV SYSTEMS

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Abstract The fault sensitivity of residual generator filters to multiple simultaneous faults is optimized for linear parameter varying (LPV) systems. The filters are synthesized by robust filtering design techniques under a geometric eigenvector assignment constraint. The goal is to design a residual generator providing both high disturbance attenuation and enhanced fault transmission level.

Keywords: Fault Detection and Isolation, Linear Matrix Inequalities, LPV systems, $\mathcal{H}_\infty$ optimization.

1. INTRODUCTION

The Fault Detection and Isolation Problem (FDIP) in dynamical systems consists of generating a diagnostic signal, which has to be different from zero during a specific fault occurrence and insensitive to other inputs, such as disturbances and other fault signals. The paper considers the case of multiple simultaneous faults. Our approach consists of designing a number of observers equal to the number of faults to be detected. Each observer is designed so as to detect a single fault occurrence and reject all other fault signals and disturbances. An earlier solution to this problem was given in (Balas *et al.*, 2002) and (Bokor *et al.*, 2002), where the notion of (C, A)-invariant subspace is extended to LPV systems. On the contrary, here the design problem leads to an optimal $\mathcal{H}_\infty$ filtering problem which can be solved by a family of linear matrix inequality (LMI) optimization problems. The aim of the paper is to design a residual generator filter which maximizes the transmission from a fault to the residual while minimizing the transmission from nuisances (disturbances and signals from other faults) to the residual. A signal/noise ratio is introduced as a performance criterion in the observer design. The main contribution of this paper is the introduction of a novel optimization technique which solves the FDI problem for LPV systems and ensures the highest disturbance attenuation level compatible with a predefined lower-bound to the zero frequency gain from fault to residual. The paper is organized as follows: Section 2 introduces the affine linear parameter varying (ALPV) dynamical systems, while Section 3 illustrates the FDIP with the sensitivity constraint. In Section 4 we discuss the projection matrix used in the FDIP. In Section 5 a numerical example is reported for showing the effectiveness of the proposed method. Some conclusions end the paper.

2. THE DYNAMICAL MODEL

Consider the following ALPV dynamical system subject to r simultaneous different fault signals $f_i(t)$

$$\begin{cases} \dot{x}(t)=A(\rho)x(t)+B(\rho)u(t)+B_w(\rho)w(t)+\sum_{i=1}^{r} F_i f_i(t) \\ y(t)=Cx(t)+D_n n(t) \end{cases} \tag{1}$$

where $\rho = \rho(t)$ is assumed to be a bounded continuously differentiable function of time

$$\rho : [0, \infty) \to \mathbb{R}^d \quad for\, all\, t \geq 0 \tag{2}$$

It is assumed that each entry of the vector $\rho = (\rho_1, \ldots, \rho_d)$ is bounded and ranges between known extremal values $\underline{\rho}_j$ and $\overline{\rho}_j$

$$\rho_i \in [\underline{\rho}_j, \overline{\rho}_j] \tag{3}$$

The parameter vector ρ takes value in an hyperrectangle, with the set Ω of its $m = 2^d$ vertices defined by

$$\Omega \doteq \left\{(v_1, \ldots, v_m) : \quad v_j \in \{\underline{\rho}_j, \overline{\rho}_j\}\right\} \tag{4}$$

Hence, all possible trajectories of $\rho(t)$ belong to the set $co\Omega$, i.e. the convex hull of Ω. In (1), the matrices $A(\rho), B(\rho), B_w(\rho)$ are defined as follows

$$\begin{aligned} A(\rho) &\doteq A_0 + \sum_{j=1}^{d} \rho_j A_j \\ B(\rho) &\doteq B_0 + \sum_{j=1}^{d} \rho_j B_j \\ B_w(\rho) &\doteq B_0^w + \sum_{j=1}^{d} \rho_j B_j^w \end{aligned} \tag{5}$$

where $A_0, \ldots, A_d \in \mathbb{R}^{n \times n}$, $B_0, \ldots, B_d \in \mathbb{R}^{n \times m}$, $B_0^w, \ldots, B_d^w \in \mathbb{R}^{n \times \bar{w}}$ and $\rho_j : [0, \infty) \to \mathbb{R}$ for $j = 1, \ldots, d$ are bounded continuously differentiable functions of the time. Furthermore, it is assumed that the parameter $\rho(t)$ is measurable online. In (1), $C \in \mathbb{R}^{p \times n}$, $D_n \in \mathbb{R}^{p \times \bar{n}}$ with $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ indicating the state and, respectively, the input vector; $w(t) \in \mathbb{R}^{\bar{w}}$ and $n(t) \in \mathbb{R}^{\bar{n}}$ the input disturbance and, respectively, the measurement noise vectors signals, while $f_i(t) \in \mathbb{R}$ and $F_i \in \mathbb{R}^n$ for $i = 1, \ldots, r$ represent the r faults signals and, respectively, the fault injection matrices. It is assumed that the fault direction is fixed, then, w.l.o.g. each matrices F_i has been assumed independent from the parameter ρ. In order to make the residual generator design problem well-posed, we also assume that:

(A1) the pair $(A(\rho), C)$ is uniformly observable for all possible trajectories of $\rho(t) \in co\Omega$;
(A2) for $i = 1, \ldots, r$, all F_i are monic, viz. if $f_i(t) \neq 0$ then $F_i f_i(t) \neq 0$;
(A3) for $i = 1, \ldots, r$, the conditions $CF_i \neq 0$ hold true;
(A4) the $\text{rank}(C[F_1, \ldots, F_r]) = r$.

Assumption (A1) is required for solvability reasons. That is, if the system (1) is uniformly observable, the state of the system may be determined from observations of the inputs and outputs at any time. A sufficient condition for uniform observability of (1) is the existence of a bounded continuous matrix $L_i(\rho)$, which is affine over ρ, such that

$$A_{cl}(\rho) \doteq A(\rho) - L_i(\rho)C \tag{6}$$

is quadratically stable in the Lyapunov sense for all $\rho(t) \in co\Omega$. Hence, there exist an observer gain $L_i(\rho)$ and a symmetric positive definite matrix P such that

$$A'_{cl}(\rho)P + PA_{cl(\rho)} \prec 0 \qquad \forall \rho \in co\Omega \tag{7}$$

See (Apkarian and Becker, 1995), (Becker and Packard, 1994), (Silverman and Meadows, 1967), (Willems and Mitter, 1971) and (Szigeti, 1992). Assumptions (A2)-(A3) are related to the input observability property. Notice that the signal $f_i(t)$ is said to be input observable if and only if

$$f_i(t) \neq 0 \quad t \geq 0 \Longrightarrow r(t) \neq 0 \quad t > 0 \tag{8}$$

i.e. it is possible to detect $f(t)$ from the observations of the residual $r(t)$. If (A2)-(A3) hold true then $f_i(t)$ is input observable, see (Hou and Patton, 1998) and (Sain and Massey, 1969). Assumption (A4) is related to output observability property, i.e. each fault f_i can be isolated from the others. If (A4) holds true then (1) is output separable. See (Massoumnia *et al.*, 1989) and (Chung and Speyer, 1998). In the following we consider the design of a residual generator filter capable to detect and isolate the i-th fault occurrence. Hence, for the sake of notational simplicity, let us to rewrite (1) as follows

$$\begin{cases} \dot{x}(t) \;=\; A(\rho)x(t) + B(\rho)u(t) + B_w(\rho)w(t) + F_i f_i(t) + \hat{F}_i \hat{f}_i(t) \\ y(t) \;=\; Cx(t) + D_n n(t) \end{cases} \tag{9}$$

where

$$\hat{F}_i \doteq [F_1, \ldots, F_{i-1}, F_{i+1}, \ldots, F_r] \in \mathbb{R}^{n \times r}$$

and

$$\hat{f}_i(t) \doteq [f_1(t), \ldots, f_{i-1}, f_{i+1}(t), \ldots, f_r(t)]' \in \mathbb{R}^r$$

with $\hat{f}_i(t)$ indicating a nuisance signal.

3. THE FDI PROBLEM

In order to isolate the i-th fault occurrence, let us to define the observer for the ALPV system (1) as follows

$$\begin{cases} \dot{\hat{x}}(t) = A(\rho)\hat{x}(t) + B(\rho)u(t) + L_i(\rho)(y(t) - C\hat{x}(t)) \\ \hat{y}(t) = C\hat{x}(t) \end{cases} \tag{10}$$

where $L_i(\rho) \in \mathbb{R}^{n \times p}$ is the observer gain matrix to be determined. In (10), $\hat{x} \in \mathbb{R}^n$ and $\hat{y} \in \mathbb{R}^p$ indicate the observer state and, respectively, the output estimation. Define the state error by

$$e(t) \doteq x(t) - \hat{x}(t) \tag{11}$$

Hence, the error dynamic is given by

$$\begin{aligned} \dot{e}(t) \;=\; & A_{cl}(\rho)e(t) + B_w(\rho)w(t) + \\ & -L_i(\rho)D_n n(t) + F_i f_i(t) + \hat{F}_i \hat{f}_i(t) \end{aligned} \tag{12}$$

and the output error $\tilde{y}$ becomes

$$\tilde{y}(t) = y(t) - \hat{y}(t) = Ce(t) + D_n n(t) \tag{13}$$

Moreover, we define the residual as the projection of $\tilde{y}$ via a projection matrix $H_i \in \mathbb{R}^{p \times p}$ to be defined, viz.

$$r(t) \doteq H_i \tilde{y}(t) \tag{14}$$

In conclusion, our aim is to design matrices $L_i(\rho)$ and H_i in a such way that

$$\begin{cases} r \approx 0 \;\; f_i(t) \equiv 0 \\ r \neq 0 \;\; f_i(t) \neq 0 \end{cases} \tag{15}$$

whatever $\hat{f}_i(t)$, $w(t)$ and $n(t)$ are in L_2. To this end, a preliminary nuisance attenuation problem is introduced.

$$\min_{L(\rho)} \gamma^2$$
$$s.t. \begin{cases} \|r\|_2^2 \le \gamma^2(\|\hat{f}_i\|_2^2 + \|n\|_2^2 + \|w\|_2^2) \\ A_{cl}(\rho)\ quadratically\ stable \end{cases} \tag{16}$$

The problem is that of finding a matrix $L_i(\rho)$ that stabilizes (12) and minimizes the $\mathcal{H}_\infty$ gain from the nuisances and disturbances ($\hat{f}_i$, w and n) to the residual (r), for all $\rho(t) \in co\Omega$. In order to solve the optimization problem (16), consider the following Lyapunov function

$$V(x) = x'Px \tag{17}$$

where P is a positive definite matrix. Then, the constraint

$$\|r\|_2^2 \le \gamma^2(\|\hat{f}_i\|_2^2 + \|n\|_2^2 + \|w\|_2^2) \tag{18}$$

is fulfilled if the following inequality is satisfied

$$\frac{dV(x)}{dt} + r'r - \gamma^2(\hat{f}_i'\hat{f}_i + n'n + w'w) < 0 \tag{19}$$

for all $\rho \in co\Omega$ and for all signals $\hat{f}_i$, w, n in L_2. A sufficient condition for (19) to hold true is the existence of a positive definite matrix $P = P' \succ 0$ such that the following three LMI constraints are feasible for all allowable values of $\rho \in co\Omega$.

$$\begin{cases} \begin{bmatrix} A_{cl}(\rho)'P + PA_{cl}(\rho) & P\hat{F}_i & (H_iC)' \\ * & -\gamma I & 0 \\ * & 0 & -\gamma I \end{bmatrix} \prec 0 \\ \begin{bmatrix} A_{cl}(\rho)'P + PA_{cl}(\rho) & PB_w(\rho) & (H_iC)' \\ * & -\gamma I & 0 \\ * & 0 & -\gamma I \end{bmatrix} \prec 0 \\ \begin{bmatrix} A_{cl}(\rho)'P + PA_{cl}(\rho) & -PL_i(\rho)D_n & (H_iC)' \\ * & -\gamma I & (H_iD_n)' \\ * & * & -\gamma I \end{bmatrix} \prec 0 \end{cases} \tag{20}$$

Thanks to the affine linear parameter dependence of (20), it is enough to verify these constraints only over a finite set of values of ρ, i.e. for all $\rho \in \Omega$.

Remark 1. Recall that (20), in the case of constant and known ρ , reduces to the the Bounded Real Lemma, (Boyd *et al.*, 1994). □

Then, we can rewrite (16) as follows

$$\min_{L_i(\rho)} \gamma^2$$
$$s.t. \begin{cases} (20) \\ P = P' \succ 0 \\ \forall \rho \in \Omega \end{cases} \tag{21}$$

Remark 2. If $\dot{\rho}$ is bounded and known, the use of a constant matrix P in the Lyapunov function could be conservative. A way to reduce such a conservativeness is to resort to parameterized Lyapunov functions, (Gahinet *et al.*, 1996). □

The solution of (21) gives rise to a matrix $L_i(\rho)$ that minimizes the transmission from disturbances and nuisances to the residual. However, we are also interested in maximizing the transmission from fault to residual. To this end, consider the response of (12) from $f_i(t)$ when $\hat{f}_i \equiv 0$, $n \equiv 0$ and $w \equiv 0$. If Assumption (A1) holds true there exists $L_i(\rho)$ such that the following ALPV system

$$\begin{cases} \dot{e} = A_{cl}(\rho)e + F_if_i \\ \tilde{y} = Ce \end{cases} \tag{22}$$

is quadratically stable. Rewrite $\tilde{y}(t)$ as

$$\tilde{y}(t) = C\Phi_\rho(t,0)e(0) + C\int_0^t \Phi_\rho(t,\tau)F_if_i(\tau)d\tau \tag{23}$$

where $\Phi_\rho(t,0)$ denotes the state transition matrix, solution of

$$\begin{cases} \dot{\Phi}_\rho(t,0) = A_{cl}(\rho)\Phi_\rho(t,0) \\ \Phi_\rho(0,0) = I \end{cases} \tag{24}$$

In order to enhance the fault sensitivity we impose the following eigenvalue-eigenvector constraint

$$A_{cl}(\rho)F_i = \lambda F_i \quad \lambda \in \mathbb{R} \tag{25}$$

to the closed-loop dynamic for all ρ. A suitable choice for $L_i(\rho)$ that satisfies (25) is as follows

$$L_i(\rho) = (A(\rho) - \lambda I)F_i(CF_i)^+ + \bar{L}_i(\rho)(I - (CF_i)(CF_i)^+) \tag{26}$$

with $\bar{L}_i(\rho) \in \mathbb{R}^{n\times p}$ of the form

$$\bar{L}_i(\rho) \doteq \bar{L}_i^0 + \sum_{j=1}^d \bar{L}_i^j \rho_j \tag{27}$$

to be determined in such a way that $A_{cl}(\rho)$ is quadratically stable and the $\mathcal{H}_\infty$-gain γ defined in (18) is minimized. Obviously, λ has to be chosen in $(-\infty, 0)$, for stability reasons. In (26), $(CF_i)^+$ denotes the left pseudo-inverse of CF_i, viz.

$$(CF_i)^+ = ((CF_i)'CF_i)^{-1}(CF_i)' \tag{28}$$

If the Assumption (A3) holds true then $(CF_i)^+$ is well-defined. Thanks to the Peano-Baker formula we can express $\Phi_\rho(t,0)$ with respect to $A_{cl}(\rho)$ as follows

$$\Phi_\rho(t,0) = I + \int_0^t A_{cl}(\rho(\tau_1))d\tau_1 + \\ + \int_0^t \int_0^{\tau_1} A_{cl}(\rho(\tau_1))A_{cl}(\rho(\tau_2))d\tau_2 d\tau_1 + \cdots + \\ \int_0^t \cdots \int_0^{\tau_{n-1}} A_{cl}(\tau_1)\cdots A_{cl}(\tau_n)d\tau_n \cdots d\tau_1 + \cdots \tag{29}$$

Then, if we consider the integrand part of (23) and take into account the constraint (25) and the formula (29), we have that

$$\Phi_\rho(t,\tau)F_i = F_i + \lambda(t-\tau)F_i + \lambda^2\frac{(t-\tau)^2}{2}F_i + \cdots + \lambda^n\frac{(t-\tau)^n}{n!}F_i + \cdots \tag{30}$$

which can also be rewritten as

$$\Phi_\rho(t,\tau)F_i = e^{\lambda(t-\tau)}F_i \tag{31}$$

With this relation we can rewrite (23) as

$$\tilde{y}(t) = C\Phi_\rho(t,0)e(0) + C\int_{t_f}^t e^{\lambda(t-\tau)}F_if_i(\tau)d\tau \tag{32}$$

The zero frequency gain from fault f_i to residual r is given by

$$G_i(0) \doteq -\frac{1}{\lambda}H_iCF_i \tag{33}$$

Define now the fault sensitivity as the minimum singular value of the zero frequency gain

$$\underline{\sigma}(G_i(0)) = \frac{1}{|\lambda|}\|H_iCF_i\| \tag{34}$$

Higher values of (34) are related to better fault sensitivity properties. In order to obtain the maximum nuisance attenuation level and guaranteeing a minimum level of fault sensitivity we introduce the following ratio

$$\mu = \frac{\gamma^2}{\underline{\sigma}^2(G_i(0))} = \frac{\gamma^2\lambda^2}{\|H_iCF_i\|^2} \tag{35}$$

Hence, smaller values of μ are related to better fault sensitivity properties and highest disturbance attenuation. Our intent is to find a matrix $L_i(\rho)$ such that (12) is asymptotically stable and (35) is minimized. To this end, we introduce the following problem

$$\min_{\gamma,\lambda,\bar{L}_i} \ \mu$$
$$s.t. \begin{cases} \|r\|_2^2 \le \gamma^2(\|\hat{f}_i\|_2^2 + \|n\|_2^2 + \|w\|_2^2) \\ A_{cl}(\rho) \ quadratically\ stable \\ L_i(\rho) = (A(\rho)-\lambda I)F_i(CF_i)^+ + \bar{L}_i(I-(CF_i)(CF_i)^+) \end{cases} \tag{36}$$

Problem (36) is a non-convex Bilinear Matrix Inequalities optimization problem. Notice that if we freeze λ to a predefined constant value, problem (36) becomes a convex LMI optimization problem. Hence, a sub-optimal solution of (36) can be obtained by solving a family of LMI problems by gridding along λ. The sub-optimal solution is related to the value of λ which gives the smaller μ. Then, using (26), we can rewrite (21) for a predefined constant value of λ as

$$\min_{\gamma,P,K_0,\dots,K_d} \ t$$
$$s.t. \begin{cases} \begin{bmatrix} \mathcal{A}(v_k) & P\hat{F}_i & (H_iC)' \\ * & -\gamma & 0 \\ * & 0 & -\gamma I \end{bmatrix} \prec 0 \\ \begin{bmatrix} \mathcal{A}(v_k) & PB_w(v_k) & (H_iC)' \\ * & -\gamma & 0 \\ * & 0 & -\gamma I \end{bmatrix} \prec 0 \\ \begin{bmatrix} \mathcal{A}(v_k) & \mathcal{K}(v_k) & (H_iC)' \\ * & -\gamma & (H_iD_n)' \\ * & * & -\gamma I \end{bmatrix} \prec 0 \\ \begin{bmatrix} t\|H_iCF_i\|^2 & \lambda\gamma \\ \lambda\gamma & 1 \end{bmatrix} \succeq 0 \\ P = P' \succ 0 \\ \gamma > 0 \\ \text{for all } k = 1,\dots,m \end{cases} \tag{37}$$

where

$$\begin{aligned} \mathcal{A}(v_k) \doteq\ & \bar{A}'(v_k)P + P\bar{A}(v_k) + K_i^0 + (K_i^0)' + \\ & - \left(\sum_{j=1}^{d} v_k^j K_i^j \bar{C}_i\right)' - \sum_{j=1}^{d} v_k^j K_i^j \bar{C}_i \end{aligned} \tag{38}$$

$$\mathcal{K}(v_k) \doteq -K_i^0 D_n - \sum_{j=1}^{d} v_k^j K_i^j D_n \tag{39}$$

and

$$\bar{A}(v_i) \doteq A(v_i) - (A(v_i) - \lambda I)F_i(CF_i)^+C \tag{40}$$

$$\bar{C}_i \doteq I - (CF_i)(CF_i)^+ \tag{41}$$

$$K_i^j \doteq P\bar{L}_i^j \tag{42}$$

Moreover, for the solvability of (37), along with (A1)-(A4), we also assume that

(A5) the pair $(\bar{A}(\rho), \bar{C}_i)$ is uniformly observable for all $\rho \in \Omega$.

Finally, it can be shown that the matrix $\bar{L}_i$ is obtained in the form

$$\bar{L}_i^j = P^{-1}K_i^j \tag{43}$$

In conclusion, we solve the problem (37) over a grid of values for λ and choose, as a solution, the one at which the ratio (35) is minimal.

Remark 3. Notice that λ is also an eigenvalue for the error dynamics (12). The error dynamic will be slower if λ is chosen close to zero. □

Remark 4. Notice that the use the same matrix P in (37) for all the three matrix constraints could be restrictive. A way to reduce this kind of conservativeness is to exploit the results exposed in (Scherer, 2000). □

Main Result 1. Consider the ALPV system (1) and let assumptions (A1)-(A5) be satisfied. Then, (37) has solution for any fixed $\lambda \in (-\infty, 0)$. □

4. THE PROJECTION MATRIX

In order to define the projection matrix H_i consider the reachability matrix of the system (22). The reachability matrix $\mathbf{R}$ is defined as

$$\begin{aligned} \mathbf{R} &= [p_0, \dots, p_n - 1] \\ p_{k+1} &= -A_{cl}(\rho)p_k + \dot{p}_k \\ p_0 &= F_i \end{aligned} \tag{44}$$

Then, thanks to the eigenvector assignment constraint (25), $\mathbf{R}$ is equal to

$$\mathbf{R} = [F_i \quad -\lambda F_i \quad \cdots \quad (-1)^{n-1}\lambda^{n-1}F_i] \tag{45}$$

which obviously has rank one. Moreover it can be shown that $Im(\mathbf{R})$ is also the smallest parameter-varying (C, A)-invariant subspace that contains the image of F_i, (Balas *et al.*, 2003). Then, the projection matrix can be defined as follows

$$H_i \doteq I - C\hat{F}_i(C\hat{F}_i)^+ \tag{46}$$

In order to clarify the Assumption (A4) consider the mapping in the output subspace of $Im(\mathbf{R})$ which is given by $Im(C\mathbf{R})$.

$$C\mathbf{R} = [CF_i \quad -\lambda CF_i \quad \cdots \quad (-1)^{n-1}\lambda^{n-1}CF_i]$$

Hence, if we have r faults to detect we need that $Im(C\mathbf{R})$ for each fault is pairwise disjoint in the output subspace. Consider two distinct faults i and j. Then, if we have that CF_i and CF_j are co-linear, the two faults cannot be isolated. In conclusion, we are able to isolate only faults that satisfy (A4).

5. EXAMPLE

Consider a two-tanks system where the faulty condition is represented by the possibility of a leakage in each tank. The dynamical equations of the two-tanks system are

$$\begin{cases} S_1\dot{h}_1 = -a_1\sqrt{2gh_1} + u - a_f\sqrt{2gh_2} \\ S_2\dot{h}_2 = a_1\sqrt{2gh_1} - a_2\sqrt{2gh_2} - a_f\sqrt{2gh_2} \end{cases} \tag{47}$$

where S_1, S_2 indicate the tanks cross sections, a_1, a_2 the cross sections of the outlet pipes, u is the liquid input flow and a_f indicates the cross section of the leakages. It is possible to rewrite (47) in a quasi-LPV form.

$$\begin{cases} \dot{h} = A(h)h + Bu + F_1 f_1 + F_2 f_2 \\ y = Ch \end{cases} \tag{48}$$

where $h = [h_1 \; h_2]'$ is the state variable.

$$A(h) = \begin{bmatrix} -\frac{a_1}{S_1}\frac{\sqrt{2g}}{\sqrt{h_1}} & 0 \\ \frac{a_1}{S_2}\frac{\sqrt{2g}}{\sqrt{h_1}} & -\frac{a_2}{S_2}\frac{\sqrt{2g}}{\sqrt{h_2}} \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{S_1} \\ 0 \end{bmatrix} \quad C = I$$
$$F_1 = [1 \quad 0]' \qquad\qquad F_2 = [0 \quad 1]'$$

In (48), $f_1(t)$, $f_2(t)$ indicate the fault signals defined by

$$f_i(t) = \frac{a_f}{S_i}\sqrt{2gh_i(t)} \quad \textit{for } i = 1, 2 \tag{49}$$

Assume that the tank water levels are constrained in a prescribed range, say

$$h_1 \in [\bar{h}_1 \quad \underline{h}_1] \qquad h_2 \in [\bar{h}_2 \quad \underline{h}_2] \tag{50}$$

Define the variable parameter $\rho \doteq [\rho_1 \quad \rho_2]'$ as follows

$$\rho_1 = \frac{\frac{1}{\sqrt{h_1}}}{\frac{1}{\sqrt{\bar{h}_1}} - \frac{1}{\sqrt{\underline{h}_1}}} \qquad \rho_2 = \frac{\frac{1}{\sqrt{h_2}}}{\frac{1}{\sqrt{\bar{h}_2}} - \frac{1}{\sqrt{\underline{h}_2}}} \tag{51}$$

with $\bar{h}_i$ and $\underline{h}_i$ indicating the maximum and, respectively, the minimum water levels. Then, we can rewrite $A(h)$ in an affine form with respect to the parameter ρ as

$$A(\rho) = \rho_1 A_1 + \rho_2 A_2 \tag{52}$$

with

$$A_1 = \frac{1}{2}(\Phi_1 + \Phi_2 - \Phi_3 - \Phi_4) \tag{53}$$

$$A_2 = \frac{1}{2}(\Phi_1 + \Phi_3 - \Phi_2 - \Phi_4) \tag{54}$$

and

$$\begin{array}{l} \Phi_1 = A(\bar{h}_1, \bar{h}_2) \;\; \Phi_2 = A(\bar{h}_1, \underline{h}_2) \\ \Phi_3 = A(\underline{h}_1, \bar{h}_2) \;\; \Phi_4 = A(\underline{h}_1, \underline{h}_2) \end{array} \tag{55}$$

indicating the vertices of the box that contains all matrices, viz.

$$A(\rho) \in co\{\Phi_1, \Phi_2, \Phi_3, \Phi_4\} \tag{56}$$

Hence, the nonlinear model (47) has been rewritten in a quasi-LPV form because the parameter ρ is a function of the state h and is not exogenous. The quasi-LPV model (48) is well-defined if $\rho(t)$ is a bounded continuous function, i.e. for all $h_i \neq 0$, $i = 1, 2$. See (Packard and Kantner, 1992). In order to set the water level h_1 to a predefined level $\bar{h}_1$, the input flow u is obtained through the following control law

$$u(t) = k_p(\bar{h}_1 - h_1(t)) + k_i \int_0^t \bar{h}_1 - h_1(\tau) d\tau \tag{57}$$

Recall that the main objective is the fault detection. Hence, our aim is to design two distinct observers, each one capable to isolate its associate tank leakage. In order to show the effectiveness of the proposed design method, we present some

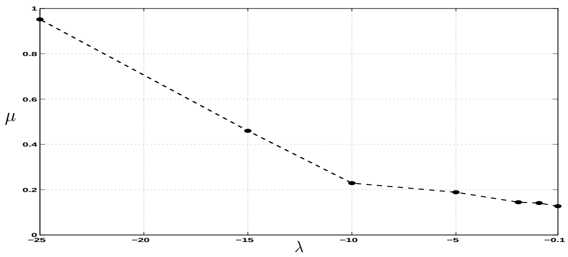


Figure 1. The cost functional (35) for the 1^{st} observer

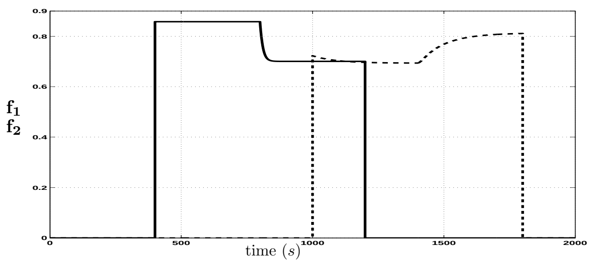


Figure 2. The fault signals: f_1 solid line, f_2 dash line.

simulations achieved on the nonlinear plant (48). Fig. 1 shows the cost functional (35) for different values of λ. Notice that for smaller values of lambda we have smaller value of (35), viz. better sensitivity to faults. The faulty conditions are defined by tank leakages, one in each tank. Fig. 2 explains the behavior of the fault signals f_1 and f_2. If $f_i \neq 0$ we have an additional liquid outflow in the relative tank, due to the leakage. The ALPV residual generators are compared with linear time invariant (LTI) residual generators. Fig. 5 illustrates the behavior of the residuals obtained from the ALPV and the linear filters. Note that although the two faults are overlapped the ALPV filters rejects efficiently the nuisances and each fault is well isolated from the others. On the contrary the residual r_1 given by the LTI filter does not detect in an efficient way the leakage in the first tank. Moreover, the residual r_2 given by the LTI filter shows a false leakage before the real one. The initial spikes in Fig. 5 are due to the fact that $e(0) \neq 0$. Similar results have been achieved with different fault conditions. In this example, the ALPV residual generator filters are obtained in few minutes by the numerical solver.

6. CONCLUSION

This paper describes an approach to the FDI problem for ALPV system, which takes into account the design of a residual generator with the maximum fault sensitivity. The main contribution is the introduction of a sensitivity constraint in the fault detection problem formulation and also the use of a cost functional to select the best residual generator filter. Nuisance attenuation is obtained through an $\mathcal{H}_\infty$ filtering design technique. Further work is needed to reduce some conservativeness, as explained in Remarks 2 and 4.

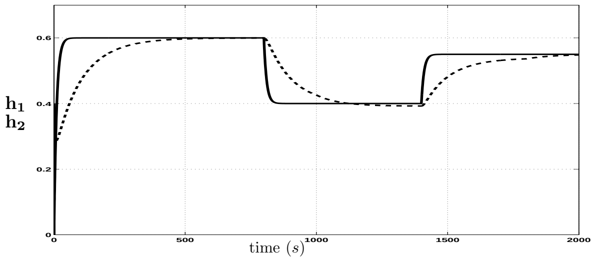


Figure 3. The tank water levels: h_1 solid line, h_2 dash line

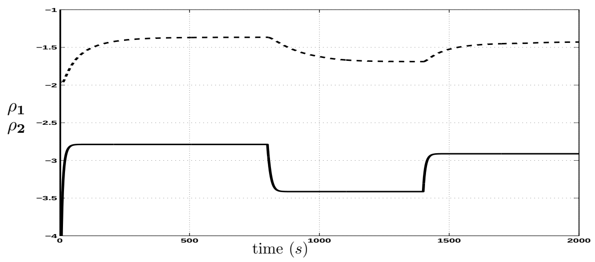
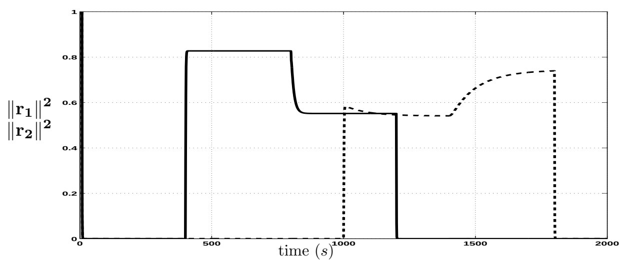
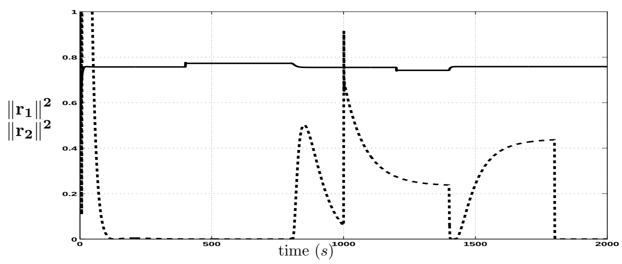


Figure 4. The variable parameter $\rho(t)$: $\rho_1(t)$ solid line, $\rho_2(t)$ dash line.



(a) ALPV residual generator filters



(b) LTI residual generator filters

Figure 5. The residual signals: r_1 solid line, r_2 dash line.

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