GENERALIZED LYAPUNOV FUNCTION FOR STABILITY ANALYSIS OF UNCERTAIN SYSTEMS

Svetoslav Savov , Ivan Popchev

Institute of Information Technologies, Acad. G. Bonchev Str., Bl. 2
1113 Sofia, Bulgaria
e-mail: savovsg@yahoo.com

Abstract*:* An alternative approach for stability analysis of uncertain systems modelled in state-space is suggested. A new candidate for Lyapunov function and based on it stability condition which generalize most of the available results in the field are presented. The applicability of this approach is illustrated by an example, treated many times by different approaches.

Keywords: linear control systems, Lyapunov stability, uncertian dynamic systems, robust stability, matrix equations

1. INTRODUCTION[1]

Stability analysis for linear systems affected by structured parameter uncertainties is an active and important for practice field of research. Even though the problem is NP – hard in general, a number of more or less conservative tests are available. This paper is concerned with the class of uncertain systems described by a state space model

$$dx/dt = A(\alpha)x,\ A(\alpha) \in R^{n\times n}, \qquad (1)$$

where $\alpha = (\alpha_1 ... \alpha_p) \in R^p$ is a vector of uncertain parameters. The state matrix depends affinely on α, i.e.

$$A(\alpha) = A + \alpha_1 A_1 + ... + \alpha_p A_p \qquad (2)$$

and all A_i are fixed matrices.

Research in this area has been directed mainly to the following cases: (i) α is constant, but not exactly known, (ii) α is very fast time–varying and (iii) α has bounded rate of variation. The following assumptions are usually made:

a) each α_i ranges between two extremal values, i.e.
$\alpha_i \in [\alpha_i^-, \alpha_i^+],\ \alpha_i^- \le 0 \le \alpha_i^+$, so vector α is valued in a hyper-rectangle Ω_p with 2^p vertices,

b) r parameters $0 \le r \le p$, are time variant, their rate of variation being well defined and satisfying $\dot{\alpha}_i \in [r_i^-, r_i^+],\ r_i^- \le 0 \le r_i^+$, or similarly, $\dot{\alpha}$ is valued in another hyper-rectangle Ω_r with 2^r vertices.

A widely applied approach to solve the stability problem is based on Lyapunov theory and the usage of fixed structure quadratic in the state function $v(x,\alpha) = x^T P(\alpha)x$.

If $P(\alpha) = P(0) = P_0$, then

$v(x,\alpha) = v(x,0) \equiv v(x)$ is a parameter independent, or pi-function.

The functions

$$v_a(x,\alpha) = v(x) + x^T \sum_i^p \alpha_i P_i x,$$

$$v_q(x,\alpha) = v_a(x,\alpha) + x^T \sum_{i,j}^p \alpha_i \alpha_j P_{ij} x$$

are said to be affine and quadratically dependent on the uncertain parameters, or simply *a*-function and *q*-function, respectively. In general, fixed structure Lyapunov functions have the form

$$v(x,\alpha) = v(x) + x^T P(\alpha, v)x,$$

$$P(\alpha, v) = \sum_v \alpha^{[v]} P_v,\ \alpha^{[v]} = \alpha_1^{v_1} \alpha_2^{v_2} \alpha_3^{v_3} ... \alpha_p^{v_l}$$

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[1] This research is supported by the Institute of Information Technologies, BAS, under contract No 010069

where $\nu = (\nu_1 \nu_2 ... \nu_L) \in J, \nu_i \geq 0,$ are L – tuples of partial degrees in the finite set *J*.

A distinction is made between quadratic and robust stability in the literature. The notion of quadratic stability consists in seeking for a pi-function and it means stability for any (possibly infinite) time variation of α, which may be quite conservative in many applications, except for case (ii), mentioned above. Stability tests are based on convex optimization over linear matrix inequalities (LMI) (Boyd *et al*., 1994). Robust stability means stability for all possible (but frozen) values of α in case (i). The third case above, which is the most general one, has also been studied recently, but to a lesser degree. For time-invariant uncertain systems, robust stability domains assessed by parameter dependent functions are always less conservative. On the other hand, robust stability (even for *a*- or *q*-functions) cannot in general be analyzed using convex optimization techniques.

Since the work of (Barmish and De Marco, 1986), a great deal of interest has been devoted to robust stability analysis. Most of the works deal with model (1), (2) and are based on *a*-functions. The latter methods differ in the assumptions made regarding the uncertainty and in the overbounding techniques adopted. E.g., in (Gahinet *et al*., 1996), under suppositions a) and b) and using multiconvextity arguments, a sufficient condition for robust stability based on LMI is proposed. Several relaxation techniques to replace parametrized LMI by a finite set of LMI are developed in (Tuan and Apkarian, 1999). The resulting relaxed feasibility problems thus become convex and hence can be solved by interior point methods. A robust stability method for uncertain (possibly time-varying) system described by (1), (2), based on *q*-functions is proposed in (Trofino and Souza, 2001) and is referred to as biquadratic stability approach. It consists in LMI based sufficient condition for biquadratic stability, including quadratic and affine quadratic stability as particular cases. By considering the companion form of $A(\alpha)$, it can be shown that the respective Hermite matrix of its characteristic polynomial is a valid Lyapunov matrix $P(\alpha)$ ensuring stability. Based on this result, it is shown in (Henrion *et al*., to appear) that for robust stability analysis it is enough to seek for a parameter dependent function of degree at most np. If all matrices A_i are rank-one, the degree estimate becomes *2p*, independently from the system order. Robust stability can be assessed by global minimization of a multivariate scalar polynomial by means of the proposed hierarchy of LMI relaxations.

The main sources of conservatism for all similar approaches consist in:

- the a priori fixed structure of the Lyapunov function,
- the necessity to apply some convexifying techniques, required to put the problem in a numerically tractable form,
- the inevitable treatment of α and $\dot{\alpha}$ as independent uncertainties.

Another major group of approaches, e.g. (Gardiner, 1997), (Rern *et al*., 1994), (Tesi and Vicino, 1990), (Zhang *et al*., 2002), is based on the conversion of the original stability problem into nonsingularity analysis of a suitable uncertain matrix (Kronecker, Lyapunov or bialternate sum of $A(\alpha)$ with itself).The stability domain is calculated through a guardian map which involves the determinants of the respective matrices.

This paper is an attempt to suggest an alternative approach for stability analysis of uncertain systems modelled by (1) and (2). It proposes a candidate for Lyapunov function and based on it stability condition which to a great extent generalize most of the available approaches.

2. MAIN RESULT

Let X be some $t \times t$ matrix with spectrum $\sigma(X) \equiv \{\lambda_{\cdot 1},...,\lambda_t\}$. The following set notations are introduced: *H* is the set of Hurwitz matrices,
$H^- \equiv \{X : X^T + X < 0\}, S$ and S_S are the sets of symmetric and skew-symmetric matrices, and

$$F \equiv \{X : \lambda_i, \lambda_j \in \sigma(X) \Rightarrow \beta_S = \lambda_i + \lambda_j \neq 0$$
$$i, j = 1,...,t, i < j,$$
$$s = 1,...,b, b = 0.5t(t-1)\}. \quad (3)$$

Consider case (i) under assumption a), i.e. the uncertain vector parameter α is time-invariant and $\alpha \in \Omega_p$. From now on, it is assumed that "$X(\alpha) \in H$",
"rank $X(\alpha) = t$", "$X(\alpha) \in S$", etc., should be understand in sense that $X(\alpha)$ is Hurwitz, has rank t and is symmetric, etc., respectively, for all $\alpha \in \Omega_p$.

It is well known, that

$$A(\alpha) \in H \Leftrightarrow \forall \varpi, \varpi \in R \Rightarrow$$
$$\text{rank}[j\varpi I + A(\alpha)] = n, \quad (4)$$

since Ω_p is a compact set, $0 \in \Omega_p$ and therefore $A(0) = A \in H$. Due to the term "$\forall \varpi$" the above condition has only theoretical significance, but it also shows that the original stability problem could be suitably restated as a nonsingularity problem of increased order thus eliminating ϖ.

2.1. Linear matrix (vector) equations

Consider a linear (in the unknown matrix *X*) equation (LME):

$$YX + XY^T = Z; Y, Z \in R^{n\times n} \quad (5)$$

For *Z* general, symmetric or skew-symmetric matrix, LME (5) can be put in compact vector form, respectively, as

$$K(Y)\mathrm{vec}_k(X) = \mathrm{vec}_k(Z), \quad (6)$$

$$L(Y)\mathrm{vec}_l(X) = \mathrm{vec}_l(Z), \quad (7)$$

$$B(Y)\mathrm{vec}_b(X) = \mathrm{vec}_b(Z), \quad (8)$$

where $\mathrm{vec}_s(\bullet), s = k, l, b,$ denotes operator stacking the $k = n^2, l = 0.5n(n+1)$ and $b = 0.5n(n-1)$ entries columnwise of a general, symmetric or skew-symmetric matrix $(\bullet)$, respectively, in a suitable way. The coefficient matrices in (6) and (7) are known as the Kronecker sum of *Y* with itself and the Lyapunov matrix of *Y*, respectively. It is well known (Fuller, 1968) that

$$\sigma[K(Y)] = \{k_S = \lambda_i(Y) + \lambda_j(Y);$$

$$i, j = 1,..., n, s = 1,..., k\}$$

and $\sigma[L(Y)]$ is comprised of the *l* distinct eigenvalues of *K*(*Y*). Therefore, mat $[\mathrm{vec}_k(X)] \equiv X$ or mat $[\mathrm{vec}_l(X)] \equiv X$ is unique solution to (5) for any respective right-hand side matrix *Z* iff

$$\sigma(Y) \cap \sigma(-Y) \equiv \varnothing \Leftrightarrow \mathrm{rank}\, K(Y) = k$$

$$or \ \ \mathrm{rank}\, L(Y) = l. \quad (9)$$

The case when a skew-symmetric solution X is searched for represents special interest.

Theorem 2.1.1. (Savov and Popchev, 2003). LME (5) has unique solution $X \in S_S$ for any $Z \in S$ iff $Y \in \mathrm{F}$ (3).

Details of the proof are omitted, but it is based on the vector representation (8) of (5).It turns out that the coefficient matrix *B*(*Y*) is exactly the bialternate sum of *Y* with itself. In (Fuller, 1968) it is proved that

$$\sigma[B(Y)] \equiv \{\beta_s, s = 1,..., b\}. \quad (10)$$

Now let $Y = A(\alpha)$ in (3). Suppose that rank $K[A(\alpha)] = k$ or rank $L[A(\alpha)] = l.$ Then for any real $\varpi, j\varpi \notin \sigma[A(\alpha)]$ and in accordance with (4) this is the iff condition for $A(\alpha) \in H.$ The Lyapunov functions

$$v_S(x, \alpha) = x^T P_S(\alpha)x, P_S(\alpha) \equiv$$

$$\mathrm{mat}\,[\mathrm{vec}_S(X)], s = k, r, Z \in H^-,$$

ensuring robust stability for the uncertain system (1), (2) are obviously not structurally fixed.

2.2. Generalized Lyapunov function

Let $X \in S_S$ and *C* be some (possibly parameter dependent) $n \times n$ matrices. Consider the uncertain time invariant matrix $G(\alpha) = [g_{ij}(\alpha)] = (X + C)A(\alpha)$ and the associated with it parameter dependent matrix

$$\mathrm{F}[G(\alpha), P] = [f_{ij}(\alpha)] + P, \quad (11)$$

where for $i, j = 1,..., n, i \neq j,$

$$f_{ij}(\alpha) = \rho_{ij} g_{ij}(\alpha) - \rho_{ji} g_{ji}(\alpha),$$

$$f_{ji}(\alpha) = (1+\rho_{ij})g_{ij}(\alpha) - (1+\rho_{ji})g_{ji}(\alpha),$$

$\rho = [\rho_{ij}] \in R^{n(n-1)}$ is arbitrary real vector and $P \in S$ is arbitrary (possibly parameter dependent) matrix. Denote by $\rho = \rho^*$ the particular case when $\rho_{ij} = \rho_{ji}$.

Theorem 2.2.1. The uncertain system (1), (2) is robustly stable, i.e. $A(\alpha) \in H,$ if and only if there exist matrices $X \in S_S, C, P \in S$ and vector ρ, such that

$$L(\alpha) = A^T(\alpha)(C + C^T)A(\alpha) +$$

$$A^T(\alpha)\mathrm{F}[G(\alpha), P] + \mathrm{F}^T[G(\alpha), P]A(\alpha) < 0. \quad (12)$$

If condition (12) holds, the function $\mathrm{v}_g(x, \alpha) = x^T\{G(\alpha) + \mathrm{F}[G(\alpha), P]\}x$ is a Lyapunov function for system (1), (2).

Proof. Let $A(\alpha) \in H.$ Since $H \subset \mathrm{F}$, in accordance with Theorem 2.1.1., there exists unique matrix $X \in S_S$ satisfying LME (5) for $Y = A(\alpha)$ and any $Z \in S_S$, e.g. $Z = [CA(\alpha)]^T - CA(\alpha).$ Then (5) can be rewritten as

$$(X + C)A(\alpha) = [(x + C)A(\alpha)]^T = G(\alpha) \in S.$$

This implies

$$f_{ij}(\alpha) = (\rho_{ij} - \rho_{ji})g_{ij}(\alpha) = f_{ji}(\alpha) \Rightarrow$$

$$\mathrm{F}[G(\alpha), P] \in S, \forall P \in S.$$

Consider the matrix inequality (12), which can be always guaranteed by suitable choice of matrices *C*, *P* and vector ρ. E.g. $P = 0, \rho = \rho^*$ and $C \in H^-$, or $X + C = 0 \Rightarrow \mathrm{F}[0, P(\alpha)] \equiv P(\alpha) > 0$ and $A^T(\alpha)P(\alpha) + P(\alpha)A(\alpha) < 0,$ or for any C, ρ and $P(\alpha) > 0$, such that $L(\alpha) < 0$ ($P(\alpha)$ is independent from $G(\alpha)$), etc. This proves the necessity part.

Let (12) holds for some $A(\alpha)$ and let also $\mathrm{F}[G(\alpha), P]$ be taken as in (11). Condition (12) can be rewritten as

$$L(\alpha) = 2\{A^T(\alpha)(X+C)A(\alpha) +$$
$$A^T(\alpha)\mathrm{F}[G(\alpha), P]\}_S =$$
$$2A^T(\alpha)\{(X+C)A(\alpha) + \mathrm{F}[G(\alpha), P]\}_S =$$
$$2A^T(\alpha)\{G(\alpha) + \mathrm{F}[G(\alpha), P]\}_S =$$
$$2[A^T(\alpha)R(\alpha)]_S < 0 \qquad (13)$$

where $(\bullet)_S = 0.5[(\bullet) + (\bullet)^T]$ denotes the symmetric part of matrix $(\bullet)$. From the definition of $\mathrm{F}[G(\alpha), P]$ in (11) it follows that $R(\alpha) \in S$ for all $\alpha \in \Omega_p$, including the uncertainty free case ($\alpha = 0$). Therefore, $R(0) > 0$, due to $A(0) \equiv A \in H$. Since $L(\alpha) < 0$, it follows that rank $R(\alpha) = n$, which for this class of uncertain systems is equivalent to $R(\alpha) > 0$. As a consequence $A(\alpha) \in H$, which proves the sufficiency part.

Obviously $\mathrm{v}_g(x,\alpha) = x^T R(\alpha) x$ is a valid Lyapunov function for system (1), (2) under condition (12).

Theorem 2.2.1 includes most of the available robust stability analysis approaches as particular cases which becomes evident from the following.

1) For $X + C = 0$ and $P = P(\alpha)$, one has $G(\alpha) = 0$, $\mathrm{F}[0, P(\alpha)] \equiv P(\alpha)$ and a standard parameter dependent function with fixed structure $x^T P(\alpha) x$ is searched to ensure condition (12).

2) Suppose that rank $R(\alpha) = n$ and unique matrix $X \in S_S$ is searched for, such that LME (5) be satisfied for $Y = A^T(\alpha)$ and arbitrary $Z = Z_1^T - Z_1$. This in accordance with Theorem 2.1.1, is equivalent to nonsingularity of matrix $B[A(\alpha)]$. Then, (5) can be rewritten as $[X + Z_1 A^{-1}(\alpha)]A(\alpha) = G_1(\alpha) \in S$ and for $Z_1 = CA(\alpha), C \in H^-$, $\rho = \rho^*$ and $P = 0$, one has

$$G_1(\alpha) = (X+C)A(\alpha) = G(\alpha),$$
$$\mathrm{F}(0,0) \equiv 0 \Rightarrow L(\alpha) < 0.$$

The proposed parameter dependent function $\mathrm{v}_g(x,\alpha)$ has a structure, which enables some extensions in robust stability analysis. As it was already said, the case $X + C = 0$ is only a particular one. Suppose that *X* and *C* are some parameter dependent matrices. By means of a suitable choice for them, vector ρ and matrix $P = P(\alpha)$ it becomes possible to get more chances to convexify condition (12) (in the sense of (Gahinet *et al.*, 1996) or (Tuan and Apkarian, 1999)), in comparison with the standard robust stability approach based on functions $\mathrm{v}_a(x,\alpha)$ or $\mathrm{v}_q(x,\alpha)$. Note also, that the vector form (8) of LME (5) can be suitably used to show that for $Y = A(\alpha)$ a solution vector (not even unique) $\mathrm{vec}_b(X)$ exists for some (not all) right-hand side vectors, by applying the solution of uncertain systems of linear equations approach.

As far as time-varying (with bounded rate of variation) uncertain systems are considered, the proposed structurally fixed version of the function $\mathrm{v}_g(x,\alpha)$ is suitable for the application of the procedures presented in (Gahjinet et al., 1996), or Trifino and Souza, 2001). The advantages outlined above for time-invariant uncertain systems are retained in this case as well.

3. UNCERTAIN SYSTEMS OF LINEAR EQUATIONS

Consider an interval systems $[X]y = [z]$, where [*X*] and [*z*] are $m \times m$ and $m \times 1$ interval matrix and vector, respectively. The set of all possible solutions is $D \equiv \{y : \forall X \in [X], \forall z \in [z], Xy = z\}$. This model supposes that the entries of [*X*] and [*z*] represent independent uncertainties. In many engineering and control problems one faces the case when these entries depend on a single interval vector $\alpha \in \Omega$, where Ω is a compact set, e.g. a polyhedron Ω_p. Define the augmented $(p+1) \times 1$ vector $\overline{\alpha} = (1\ \alpha)^T$. Such an uncertain system is modelled as

$$\sum_{j=1}^{m} x_{ij} y_j = z_i,\ x_{ij} = c_{ij}\overline{\alpha},\ z_i = d_i\overline{\alpha},$$
$$i = 1,\ldots,m \qquad (14)$$

where c_{ij} and d_i are given real fixed row vectors. Then (14) is called an uncertain linear system with dependent coefficients, or simply a parametrised linear system (PLS). Note that when $A(\alpha)$ has an affine structure, i.e.

$A(\alpha) = A_0 + \alpha_1 A_1 + \ldots + \alpha_p A_p$ the vector equation (1) exactly matches the PLS (14).

The solution set is

$$D(\alpha) \equiv \{y : \forall \alpha \in \Omega,\ X(\alpha)y = z(\alpha)\}.$$

Obviously PLS can be viewed as interval systems as well, but in general this leads to contraction of Ω. One of the problems dealt with in interval analysis is to get an outer estimation $D^*(\alpha)$ of the interval hull of $D(\alpha)$, i.e. $D(\alpha) \subseteq D^*(\alpha)$, guaranteeing that any solution of the PLS lies within some enclosure set, e.g. Ω_{ph}. A fundamental necessary and sufficient condition for $y \in D(\alpha)$ is $\alpha \in \Omega \Rightarrow X(\alpha)y - z(\alpha) \subseteq 0$.

A significant number of approaches are developed to solve the problem for both interval and PLS. Among them one should mention the application of the interval Gauss-Seidel method in (Popova, 2000), (Shary, 2001) and the procedure of consecutive uncertain parameters elimination in (Alefeld *et al*., 2003). An important fact concerning PLS(1) is that it is required to find some set $D^*(\alpha)$ thus guaranteeing that $D(\alpha)$ is not empty.

4. EXAMPLE

Consider the uncertain time – invariant system (1), where

$$A(\alpha) = \begin{bmatrix} -2+\alpha_1 & 0 & -1+\alpha_1 \\ 0 & -3+\alpha_2 & 0 \\ -1+\alpha_1 & -2+\alpha_2 & -4+\alpha_1 \end{bmatrix}.$$

This example is oftentimes treated and it can be seen that the exact stability domain Ω_2^*, is described as $-\infty < \alpha_1 < 1.75$ and $-\infty < \alpha_2 < 3$.

case 1 (Lyapunov function with fixed structure). Let $\mu \in R, \mu < 0$. Consider the matrix $G(\alpha) = CA(\alpha), C = [c_{ij}], c_{ij} = 0$ for $(i,j) \neq (2,2)$ and $c_{22} = \mu$. For any $\mu, G(\alpha) \in S$. Let $\rho = \rho^*$ and P be the solution to the fixed Lyapunov equation $A^T P + PA = -(A^T A)^{1/2}$. Condition (12) becomes

$$2A^T(\alpha)CA(\alpha) + A^T(\alpha)P + PA(\alpha) =$$
$$2M(\alpha) + N(\alpha) < 0$$

where $N(\alpha) = A^T(\alpha)P + PA(\alpha) = [n_{ij}]$. Since $M(\alpha)$ has the same structure as C with $m_{22} = \mu \upsilon_3^2$, provided that $\upsilon_3 \neq 0 \Leftrightarrow \alpha_2 < 3$ and for $\mu = \upsilon_3^{-2}(\mu_1 - n_{22})\mu_1 < 0$, one gets a convex with respect to α problem. It turns out that for $\mu_1 = -10^9, -1000 \leq \alpha_1 \leq 1.7499$ and $-500 \leq \alpha_2 \leq 2.999, L(\alpha) < 0$ in (12). The upper bound for α is practically the exact one, while the lower one is absolutely satisfactory and much better in comparison with the available results for this case. Note that neither relaxations, nor solutions to LMI are required in this case.

case 2 (Lyapunov function with not fixed structure). Consider matrix

$$G(\alpha) = (X + \rho I)A(\alpha),\ X = -X^T,$$
$$\rho \in R, \rho < 0.$$

It is desired to justify the existence of matrix X such that $G(\alpha) \in S$ and rank $A(\alpha) = 3, \alpha \in \Omega_2^*$. If this is so, for $\rho = \rho^*, P = 0 \Rightarrow F[G(\alpha), P] = 0$ and inequality (12) will be satisfied, thus generating the necessary and sufficient stability condition.

The symmetry condition for $G(\alpha)$ can be rewritten in the form (14) as

$$(-5 + \alpha_1 + \alpha_2)x_{12} + (-2 + \alpha_2)x_{13} -$$
$$(-1 + \alpha_2)x_{23} = 0$$
$$(-6 + 2\alpha_2)x_{13} = 0$$
$$-(-1 + \alpha_1)x_{12} + (-7 + \alpha_1 + \alpha_2)x_{23} =$$
$$\rho(-2 + \alpha_2)$$

Let $x_{13} = 0$. Application of the Gauss-Seidel approach for the reduced PLS results in

$$x_{12} = (-5 + \alpha_1 + \alpha_2)^{-1}(-1 + \alpha_1)x_{23},$$
$$x_{23} = (-7 + \alpha_1 + \alpha_2)^{-1}[(-1 + \alpha_1)x_{12} +$$
$$\rho(-2 + \alpha_2)].$$

Let $x_{12} \in [-3,3]$ and $x_{23} \in [-1,1]$. The right-hand sides of the reduced PLS are convex in α and therefore they achieve their extremal values at the respective vertices. It can be easily seen that for $\alpha \in \Omega_2^*$ and $\rho \to 0^-$ the PLS has a solution and therefore $G(\alpha) \in S$.

For $\alpha \in \Omega_2^*$ one has $-2 + \alpha_1 = \upsilon_1 \neq 0$ and $-3 + \alpha_2 = \upsilon_2 \neq 0$. Consider the triangular matrix

$$T(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ (1-\alpha_1)\upsilon_1^{-1} & (2-\alpha_2)\upsilon_2^{-1} & 1 \end{bmatrix}$$

and the matrix product

$$T(\alpha)A(\alpha) = \begin{bmatrix} -2+\alpha_1 & 0 & -1+\alpha_1 \\ 0 & -3+\alpha_2 & 0 \\ 0 & 0 & (\upsilon_1\upsilon_4 - \upsilon_3^2)\upsilon_1^{-1} \end{bmatrix},$$

where $\nu_3 = 1 - \alpha_1$ and $\nu_4 = -4 + \alpha_1$.

For $\alpha \in \Omega_2^*$,

rank $T(\alpha)A(\alpha) = 3 \Leftrightarrow \text{rank } A(\alpha) = 3$.

Application of the proposed here approach for stability analysis of uncertain systems results in determining the exact stability domain for α in this case.

5. CONCLUSION

An attempt to generalize most of the available approaches for stability analysis of uncertain systems is made. The main contribution is due to the derived here candidate Lyapunov function which includes the available similar functions as particular cases. The search for a solution to a PLS can be viewed as an alternative to the guardian map approach in stability analysis. The applicability of the proposed method is illustrated by an example.

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