Generalized Predictive Control with Flexible Inequality Constraints

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Abstract: Generalized predictive control with flexible constraints is investigated. To apply fuzzy technique, the flexible parts in the inequality constraints are fuzzified as fuzzy constraints. The optimization problem of the generalized predictive control with flexible inequality constraints is converted into two sub-problems to get the lower and upper bound of the objective values, and then the objective of generalized predictive control can be fuzzified too. When the constraints and objective of the control system are all in fuzzy uncertainty environment, fuzzy decision making method can be used to solve the problem. Simulation result proves the validity of this algorithm.

Keywords: Generalized predictive control, Fuzzification, Constraint satisfaction, Decision making

1. INTRODUCTION

Generalized predictive control (GPC) (Clarke, et al, 1987) which owes to its easy implementation and its ability to take into account key control objectives has become a popular means of controlling scalar systems. In the absence of input/output constraints, the main computational demand in GPC concerns the minimization of a simple quadratic cost function. The optimal solution obtained in this way, however, may be far from optimal in the presence of input and output constraints.

There are many methods solving the constrained predictive control problem. The formulations of the constrained predictive control problem using linear programming (LP) have been proposed by Allwright and Papavasiliou (1992), and Dave et al (1997). The formulations using quadratic programming (QP) have been proposed by Ricker (1985), and Kuznetsov and Clarke (1994). Others, for example, Sarimveis and Bafas (2003) employ genetic algorithms to solve the optimization problem. Perhaps a more critical issue is with regard to potential infeasibilities in the LP/QP, that is, a combination of input/output constraints can easily result in LP/QP in which there is no feasible solution, so the control algorithm cannot compute any control move to be implemented at that sampling instance. Such behavior cannot be tolerated, so it has become common to use a soft constraint formulation in (Zheng, et al, 1995) and (Li, et al, 2000) to handle the constraints, in which penalty terms on the constraints are included in the objective function to avoid infeasibility problems in MPC.

Obviously, there are excessive methods to deal with constraints, but all these investigation assume that the goal and constraints are definite and unchanged with their bounds. However, there exist some cases that the constraint bound is not so rigid, but flexible. For example, the operator may moderately change the limitation on the use of resource to get better quality product. However, there is two-conflicted process: one is that we hope to save the resource as possible otherwise we hope products have the better quality as possible. Then, classical dynamic optimization techniques become unsuccessful to deal with problems that contain flexible bounds. To overcome this problem, Chen C.L., et al (2002) propose a solution strategy for optimising the dynamic system with flexible inequality constraint but is only consider the static optimisation of dynamic systems.

Moreover, the flexible portion of the inequality constraints seems to be a stochastic or probabilistic problem. However, we consider the operator does not change the constraints bound randomly, but changes on preference that largely depends on the operator's subjective consideration. One good way is to use fuzzy membership function to express the acceptability of the flexible portion of the inequality constraints. Fuzzy goal and fuzzy constraints have become a popular method to deal with systems with fuzzy uncertainty. This cost function is usually a sum of an error measure of one or more output variables. Alternatively, to get more flexibility for expressing the control goals, in which such flexibility in operation largely depends on the operator's subjective consideration, techniques from fuzzy multi-criteria decision-making can be used (Sousa, et al, 2001), the optimization problem is formulated as a multi-criteria decision making problem with fuzzy goals and constraints. In these methods, goal and constraints are defined as fuzzy functions separately, however, for generalize predictive control with flexible inequality constraints, the goal is correlative with the constraint bound.

In this paper, we present a new constrained generalized predictive control algorithm with flexible inequality constraints. The optimisation problem is converted into fuzzy decision making through the fuzzification of the constraint and objective of the system.

2. INTRODUCTION FORMULATION of COSTRAINED PREDICTIVE CONTROL

2.1 Model-based predictive control

In essence it utilizes systems predictive information to optimize the performance index within a finite horizon. In order to overcome the uncertainty, we take the receding horizon strategy in predictive control. The predictive output $\hat{y}(k+i)$, $i = N_1, \cdots, N_2$ is derived from the information at current time t and the future control signal $u(k+i)$, $i = 1, \cdots, N_u$, where $[N_1, N_2]$ is the predictive horizon. The objective to be optimized is:

$$J = \sum_{i=N_1}^{N_2} (\hat{e}(k+i))^2 + \sum_{i=1}^{N_u} \lambda_i (\Delta u(k+i-1))^2 \qquad (1)$$

where $\hat{e}(k+i)$ is the predictive error, $\Delta u(k+i-1)$ is the control increment, λ_i is the weight coefficient of the control signal.

Although the complete sequence of optimal control signals within the predictive horizon is computed, only the control signal $u(k)$ is applied to the process. At the next sampling instant, the process output $y(k+1)$ becomes known and the prediction horizon is shifted by one sampling period so that the optimization and the prediction can be repeated with the updated values. This is called the receding horizon principle.

Since the objective function reflects the control goals, it is advantageous to have additional freedom for specifying more complicated control goals. This additional freedom can be achieved by choosing a different representation of the objective function, e.g. as a combination fuzzy goals and constraints.

The CARIMA model can describe the system:

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + \frac{C(q^{-1})\xi(t)}{\Delta} \qquad (2)$$

where

$$A(q^{-1}) = 1 + a_1 q^{-1} + \cdots + a_p q^{-p}$$
$$B(q^{-1}) = b_0 + b_1 q^{-1} + \cdots + b_m q^{-m}$$
$$C(q^{-1}) = 1 + c_1 q^{-1} + \cdots + c_l q^{-l}$$

The predictive equation is:

$$\hat{y} = G\tilde{u} + f$$

where

$$\left[\hat{y}^T(t+1\,|\,t), \;\; \cdots, \;\; \hat{y}^T(t+N_2\,|\,t)\right]^T, \;\; (1\times N_2 p)$$
$$\tilde{u} = \left[\Delta u^T(t), \;\; \cdots, \;\; \Delta u^T(t+N_u)\right]^T, \;\; (1\times N_u m)$$
$$f = \left[f_1^T(t), \;\; \cdots, \;\; f_{N_2}^T(t)\right]^T, \;\; (1\times N_2 p)$$

The control law without constraints can be obtained:

$$u(t) = u(t-1) + g^T(\boldsymbol{w} - \boldsymbol{f}) \qquad (3)$$

where g^T are the former m lines of the matrix $(\boldsymbol{G}^T\boldsymbol{G} + \lambda I)^{-1}\boldsymbol{G}^T$, significance of parameters seeing also (Clarke, et al, 1987).

2.2 Fuzzification of flexible constraints

In the traditional constraint programming, the constraint conditions cannot be exceeded and changed, but in the practical control process, some of constraints are adjustable, called 'soft constraints'. Thus, every constraint variable can be adjusted within a limit boundary and has a function to reflect the fuzziness of constraint variable boundary defined by decision-maker. We can use the fuzzy variable to describe this case. For fuzzy variable $\tilde{b}$, we define the membership function $\mu(\tilde{b})$ $0 \le \mu \le 1$, which express the degree of membership. $\mu = 1$ Indicates the corresponding fuzzy variable belongs to this set, conversely for $\mu = 0$. In fact, we can understand μ as the degree of satisfactory degree. Fig. 1 is a kind of fuzzy boundary, where the membership function is linear function (of course, we can assume other function) to simplify the computation. Then the degree of membership is expressed as follows:

$$\mu_C(b) = \begin{cases} 0, \; b < b_{min} - p_1 \\ 1 - \dfrac{b_{min} - b}{p_1}, \; b_{min} - p_1 \le b < b_{min} \\ 1, \; b_{min} \le b \le b_{max} \\ 1 - \dfrac{b - b_{max}}{p_2}, \; b_{max} < b \le b_{max} + p_2 \\ 0, \; b_{max} < b \end{cases} \qquad (4)$$

where p_1, p_2 is called fuzzy width or tolerant width, $b_{\min}$, $b_{\max}$ is the expected boundary of fuzzy variable $\tilde{b}$. Obviously, when the fuzzy width is *zero*, it corresponds the 'hard constraint'.

Adjusting the '*soft constraint*' is based on the man-machine interaction, which is actuarially the interaction of the experience decision and the knowledge base and rules base with computer. It is natural to set soft constraint bounds according to the specified industry process, and make the decision on various inputs and output states real-time and at the same time adjust the boundary, so as to realize the receding-horizon optimization. The decision-maker takes part in the control only in a special case. In other words, at this time, decision-maker makes proposal and order to the whole system at a higher level (such as change the production plan, implement a new completely standard, etc), while the simple logical, the knowledge base and rule base needed for this kind of expert system are not very large because they are designed for a special industry environment, and the cost of building and operating is also very feasible.

2.3 Fuzzification of objective function

In this section, we discuss in detail how to deal with the fuzzy boundary optimization. Firstly, consider the control variable u and output variable y in constraint equation. They are all decided by the control increment Δu during the receding horizon optimization in GPC algorithm. The boundary condition (4) can be expressed as:

$$\mathbf{A}\Delta\mathbf{u}(t) \le \mathbf{b}(t) \tag{5}$$

where

$$\mathbf{A} = \begin{bmatrix} C_1^T & -C_1^T & C_2^T & -C_2^T & C_3^T & -C_3^T \end{bmatrix}^T \tag{6}$$

$$\mathbf{b}(t) = \begin{bmatrix} b_1(t) & b_2(t) & b_3(t) & b_4(t) & b_5(t) & b_6(t) \end{bmatrix}^T \tag{7}$$

$$\Delta\mathbf{u} = \left[\Delta u^T(t+1), \cdots, \Delta u^T(t+N_u)\right]^T \tag{8}$$

The matrixes are defined as follows:

$$C_1 = I,$$

$$C_2 = \begin{bmatrix} I & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ I & \cdots & \cdots & I \end{bmatrix},$$

$$C_3 = G$$

$$b_1(t) = \left[u_{\max}^T, \quad \cdots, \quad u_{\max}^T\right]^T$$

$$b_2(t) = \left[-u_{\min}^T, \quad \cdots, \quad -u_{\min}^T\right]^T$$

$$b_3(t) = \left[(\Delta u_{\max} - u(t-1))^T, \quad \cdots, \quad (\Delta u_{\max} - u(t-1))^T\right]^T$$

$$b_4(t) = \left[(-\Delta u_{\min} + u(t-1))^T, \quad \cdots, \quad (-\Delta u_{\min} + u(t-1))^T\right]^T$$

$$b_5(t) = \left[(y_{\max} - f_{N_1}(t))^T, \quad \cdots, \quad (y_{\min} - f_{N_2}(t))^T\right]^T$$

$$b_6(t) = \left[(-y_{\min} + f_{N_1}(t))^T, \quad \cdots \quad (-y_{\min} + f_{N_2}(t))^T\right]^T$$

Notice that for the constraint of control variable and its change rate, we should consider the future N_u cycles, whereas for the output constraint, the cycles are from N_1 to N_2. However, from the perspective of computing, the computation complexity of solving the optimization with constraints has much to do with the number of constraint conditions. Therefore, sometimes we only consider the constraints in the near cycles to decrease the computation.

What's more, as $\mathbf{b}(t)$ in equation (7) is transformed from the boundary expression (4), and this

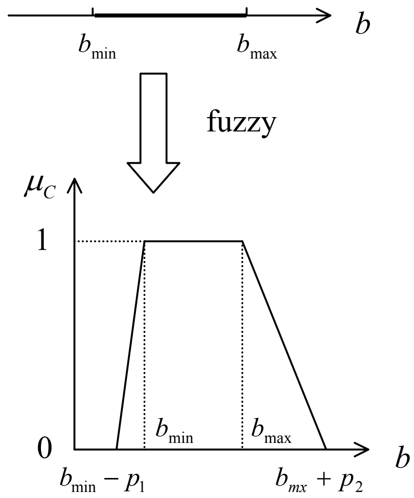

Fig.1 Fuzzy boundary

transformation is only a series of displacement and inverse, thus under the fuzzy boundary condition, the derived $\tilde{\mathbf{b}}(t)$ still has the same form as the nonfuzzy constraints, and the fuzzy width of all fuzzy variables are not changed. It can be expressed as:

$$\mathbf{A}\Delta\mathbf{u}\ \ (t) \le \mathbf{b}(t) + \mathbf{p} \triangleq \tilde{\mathbf{b}}(t) \tag{10}$$

And the vector $\mathbf{p}$ represents the fuzzy width of various fuzzy variable of $\tilde{\mathbf{b}}(t)$:

$$\Big[\overbrace{p_{u_{\max}}^T, \cdots, p_{u_{\max}}^T}^{N_u},\ \overbrace{p_{u_{\min}}^T, \cdots, p_{u_{\min}}^T}^{N_u},\ \overbrace{p_{\Delta u_{\max}}^T, \cdots, p_{\Delta u_{\max}}^T}^{N_u},$$

$$\overbrace{p_{\Delta u_{\min}}^T, \cdots, p_{\Delta u_{\min}}^T}^{N_u},\ \overbrace{p_{y_{\max}}^T, \cdots, p_{y_{\max}}^T}^{N_2 - N_1 + 1},\ \overbrace{p_{y_{\min}}^T, \cdots, p_{y_{\min}}^T}^{N_2 - N_1 + 1}\Big]^T$$

where p_x stands for the fuzzy width of fuzzy variable x.

So, the constrained predictive control problem can be formulated as follows:

$$\begin{cases} min \;\; J \\ s.t. \;\; \mathbf{A}\Delta\mathbf{u}(t) \le \tilde{\mathbf{b}}(t) \end{cases} \tag{11}$$

Here, the so-called fuzzy flexible constrained optimization means that when the reasonable limiting value b_k and the acceptable maximal tolerance p_k can be preliminarily defined, all those values that are smaller than $b_k + p_k$. The optimization result will vary with the flexible parameter p_k, therefore, two new sub-problems are solved at first: one is restricted by constraints with optimistic boundary $b_k + p_k$, while the other one is confined by the constraints 'with pessimistic boundary b_k, that is:

$$S0: \begin{cases} min \;\; J \triangleq J^0 \\ s.t. \;\; \mathbf{A}\Delta\mathbf{u}(t) \le \mathbf{b}(t) + \mathbf{p} \end{cases} \tag{12}$$

and

$$S1: \begin{cases} min \;\; J \triangleq J^1 \\ s.t. \;\; \mathbf{A}\Delta\mathbf{u}(t) \le \mathbf{b}(t) \end{cases} \tag{13}$$

Because the constraint domain of optimization problem: $S0$ contains that of problem: $S1$, the minimum of optimization problem: $S0$ must be less than or equal to $S1$, i.e. $J^0 \le J^1$. Obviously, the optimization problem $S0$ and $S1$ must be feasible, otherwise we could not implement the present algorithm. In case of the infeasibility, the constraints can be violated temporarily with the algorithm in (Zheng, et al, 1995), and simultaneously, the corresponding constraints bound also need to be modified temporarily.

Problem $S0$ and $S1$ can be solved using standard constrained optimization approach online (Garcia, C.E, et al). J^0 and J^1 represent the global optimal for the two sub-problems, respectively. Moreover, owing to various acceptability for constraints changed from b_k to $b_k + p_k$, it will have different satisfaction for the cost function J as it changes from J^0 to J^1. It means that values closer to J^1 have the lower satisfaction and values closer to J^0 have higher satisfaction. For objective values in between J^0 and J^1, lower J value results in increased degree of satisfaction.

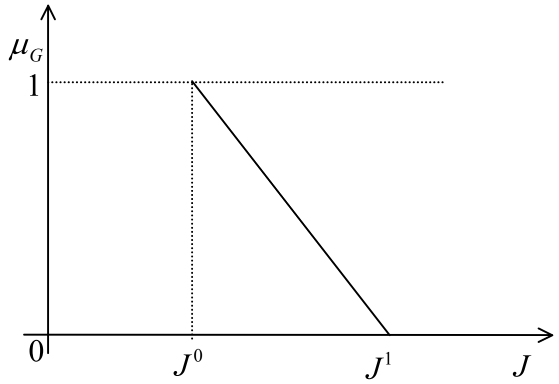

Fig.2 Fuzzy membership for control goals

As the constrained optimization computation has great burden, and the *receding horizon* used in MPC need online optimization, so we pre-defined the fuzzy goals as follows for the online applications as Eq.(14) and Fig. 2.

$$\mu_G = \begin{cases} 1 & if \;\; J < J^0 \\ g(J, J^0, J^1) & J^0 \le J \le J^1 \\ 0 & if \;\; J > J^1 \end{cases} \tag{14}$$

The following two functions are frequent to use as membership function, which are listed as

$$g(J, J^0, J^1) = \begin{cases} \dfrac{J - J^1}{J^0 - J^1} \\ \dfrac{1 - \exp[-\alpha(J - J^1)/(J^0 - J^1)]}{1\text{-}\exp[-\alpha]} \end{cases} \tag{15}$$

Here, α is an adjusting factor and $\alpha \ne 0$. For simplicity, we will employ the linear type function as the membership function of fuzzy goal.

So far, for online control application based on the *receding horizon* in MPC, the optimization problem with fuzzy constraints (4) and fuzzy goals (14) is formulated. To the fuzzy optimization problem, Liu in (Liu, *et al*, 1999) and (Liu, 2002), and Lu and Fang in (2001) have developed some efficient algorithm, but these approach are fit to use offline.

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3. FUZZY GOAL PRGRAMMING AND SATISFYING DECISION MAKING

In a standard goal programming formulation, goals and constraints are defined precisely. In fact, to ask a decision-maker (DM) what attainments are desired for each objective function is a difficult job. Applying fuzzy set theory into goal programming at this junction has the advantage of allowing for the vague aspirations of DMs, which can then be quantified by some natural language rules. In this paper, we will present a fuzzy satisfying solution to the goal programming.

In Bellman and Zadeh's (1970) setting fuzziness of the environment is modeled by fuzzy goals, fuzzy constraints, and a fuzzy decision.

A fuzzy goal is defined as a fuzzy set in $X : \mu_G(x)$; and a fuzzy constraints is similarly defined as a fuzzy set in $X : \mu_C(x)$. The general problem formulation is: attain G *and* satisfy C, which leads to a fuzzy decision:

$$\mu_D(x) = \mu_G(x) \wedge \mu_C(x) \tag{16}$$

for each $x \in X$, where $a \wedge b = \min(a, b)$, "$\wedge$" may be replaced by another appropriate operation as, a *t*-norm, such an operation may reflect various relevant attitudes and aspects of the decision.

The optimal decision is defined as an $x^* \in X$, such that

$$\mu_D(x^*) = \max_{x \in X} \mu_D(x) \tag{17}$$

This basic framework can be extended to handle multiple fuzzy goals and fuzzy constraints. Namely, if we have $p > 1$ fuzzy goals $G_1, \cdots, G_p$, defined in Y, $q > 1$ fuzzy constraints $C_1, \cdots, C_q$, defined in X, and a function $f : X \to Y,\ y = f(x)$, we have

$$\begin{aligned}\mu_D(x) = \mu_{G^1}[f(x)] \wedge \mu_{G^2}[f(x)] \wedge \cdots \wedge \mu_{G^p}[f(x)] \\ \wedge \mu_{C^1}(x) \wedge \cdots \wedge \mu_{C^q}(x)\end{aligned} \tag{18}$$

and the maximizing decision is (17), i.e. $\mu_D(x^*) = \max_{x \in X} \mu_D(x)$ .In this paper, the branch and bound search technique(Sousa, J.M., Babuska R. and Verbruggen H.B., 1997) is used to solve the non-convex optimisation problem. For what concerns the practical the optimum of the fuzzy decision making problem, the interested reader is referred to Sousa and Kaymak, (2001) and Lai and Hwang (1994).

By the way, in present paper we name the constraint condition like $\mathbf{A}\Delta\mathbf{u}(t) \le \mathbf{b}(t)$ as inner constraint, the constraint condition like $\mathbf{A}\Delta\mathbf{u}(t) \le \mathbf{b}(t) + \mathbf{p}$ as outer constraint and the constraint ranging from $\mathbf{A}\Delta\mathbf{u}(t) \le \mathbf{b}(t)$ to $\mathbf{A}\Delta\mathbf{u}(t) \le \mathbf{b}(t) + \mathbf{p}$ as fuzzy flexible constraint.

4. SOLUTION STEPS

The on-line constrained predictive control algorithm based on fuzzy constraints and fuzzy goal are summarized as follows:

1. Preliminary preparations (offline):

Step 1. Select suitable flexible boundary for the constraints for $u(t), \Delta u(t),$ and $y(t)$;

Step 2. Define the fuzzy membership function μ_{C_i} for $u(t), \Delta u(t),$ and $y(t)$ as Eq. (4) according to the operation conditions, the trapezoid-type functions will be used in this paper.

Step 3. Select the MPC parameters N_1, N_2, and N_u.

2. Dynamic Programming (online):

Step 1. Within the time horizon $[t, t+N]$, solve Eqs. (12) and (13) by any existing constrained predictive control method to determine J^0 and J^1;

Step 2. Define the fuzzy membership function μ_{G_i} for control objective as Eq. (14), the line-type functions will be used in this paper; obtain the control action $\Delta u(t)$ by Eq.(18);

Step 3. $t = t + 1$, with the receding horizon $[t+1, t+N+1]$, repeat the Step 1 to get the $\Delta u(t+1)$ using the new measurable system's inputs and outputs.

5. SIMULATION

Consider a dynamic system whose transfer function is

$$\frac{y(s)}{u(s)} = \frac{1}{s^2 + s} \tag{19}.$$

The constraint conditions are listed as two groups. The first constraint group are $-0.5 \le u(t) \le 0.5$, $-0.3 \le \Delta u(t) \le 0.3$, and $0 \le y(t) \le 1.2$, corresponding flexible parts are (0.2,0.2), (0.15,0.15), (0.2,0.2). The second constraint group are $-0.2 \le u(t) \le 0.3$, $-0.25 \le \Delta u(t) \le 0.25$ and $0 \le y(t) \le 1$, corresponding flexible parts are (0.2,0.2), (0.15,0.15), (0.2,0.2). In the following, we define the acceptability membership function of constraints according to the Equation (4), and set $N_1 = 1$, $N_2 = 5$, $N_u = 3$, $Q = 1$, and $R = 1$. According to the algorithm presented in this paper, at every sampling time instant, two sub-optimal problems are solved firstly, i.e. under inner constraint and outer constraint, and then the trade-off solution under fuzzy flexible constraint is solved by fuzzy decision-making.

In order to show the validity of present algorithm, the simulation is implemented under inner constraint, outer constraint and fuzzy flexible constraint conditions. For the first constraint group, the control objective is to let the output $y(t)$ track the step input and for the second constraint group, the control objective is to let the output $y(t)$ track the sin reference input. The trajectory of the controlled variable $y(t)$ and the control profile of manipulate variable $u(t)$ are shown in the fig.3 and fig.4, respectively.

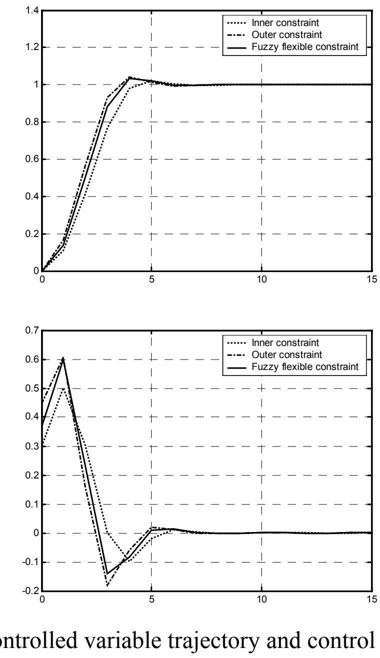

Fig.3 Controlled variable trajectory and control profile for tracking step input

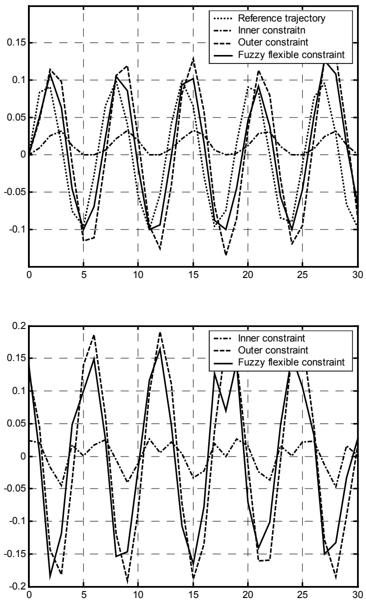

Fig.4 Controlled variable trajectory and control profile for tracking sin reference input

From the results of simulation, we can see it is deficient for the control ability of the solution derived by generalized predictive control under the inner constraint because the tight constraints confine the control action, however it is superfluous and overdoing for the control ability of the solution under the outer constraint. In fact, we need a compromised solution under flexible constraint, i.e. at every sampling time instant generalize predictive control algorithm need tradeoff between the constraint acceptability and optimality of performance index. From fig.3 and fig.4, we can see the present algorithm have good effect and satisfy the operator's request.

6. CONCLUSION

Based on the fuzzy constraints and fuzzy goals, we present a generalized predictive control algorithm with flexible constraints. To apply fuzzy technique, the flexible parts in the inequality constraints are fuzzified as fuzzy constraints. The optimization problem of the generalized predictive control with flexible inequality constraints is converted into two sub-problems to get the lower and upper bound of the objective values, and then the objective of generalized predictive control can be fuzzified too. When the constraints and objective of the control system are all in fuzzy uncertainty environment, fuzzy decision making method can be used to solve the problem to obtain the control action. In this way, at every sampling instant, the control action is the most acceptable and most satisfying one to the operator.

ACKNOWLEDGMENT

This work was supported by the National Natural Science Foundation of China (No. 60474051) and the Key Technology and Development Program of Shanghai Science and Technology Department (No. 04DZ11008).

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