DECENTRALIZED CONTROL OF COOPERATIVE ROBOTS WITHOUT VELOCITY–FORCE MEASUREMENTS

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Abstract: One of the main practical problems on cooperative robots is the complexity of integrating a large amount of expensive velocity/force sensors. In this paper, the control of cooperative robots using only joint measurements is considered to manipulate an object firmly. Experimental results are shown.

Keywords: Cooperative robot systems, observer design

1. INTRODUCTION

Cooperation control of multiple robots has been extensively studied since the 1980s. It is recognized that even the most advanced industrial robots at the present stage lack of versatility in fulfilling the various tasks imposed on them. A condition to be considered during execution of tasks is the dexterity property. Such a complicated behavior is only obtained by using two or more cooperating manipulators. In cooperation control the tasks characterized by physical contact between the end effector and a constraint surface are particularly interesting. A long list of such tasks can be given: scribing, writing, deburring, grinding, etc. Another example is when two cooperative manipulators are used in material handling, transporting objects beyond the load capacity of a single manipulator. Cooperation improves the capability of robots to carry out more complicated and dextrous tasks, which could not be accomplished by a single robot. To control two or more cooperative robots, there have been proposed mainly three kinds of approaches: master-slave model, centralized controller and decentralized architecture controller. In the decentralized architecture there is no need to handle high-dimensional matrices. Furthermore, the control laws for all the robots are the same, so its implementation is straightforward.

There has been a considerable amount of research regarding the development of nonlinear controllers for robot manipulators focused on reducing the number of sensors required to implement the control algorithm. However, the literature available on cooperative robots not requiring link velocity and end–effector force measurements is very limited. In fact, the lack of velocity measurements can be compensated by substituting measured data

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with numerical differentiation. However, recent experimental results have suggested that a (digitalized) observer in a control law has a better performance (Arteaga Pérez and Kelly, 2004). As to cooperative systems, McClamroch and Wang (1988) developed a nonlinear transformation to convert the constrained system into two reduced subsystems. From this approach, the elimination of expensive force sensors can be accomplished by utilizing asymptotic observers to estimate the contact force. In Huang and Tzeng (1991), two types of force observers are designed for constrained robot systems. In this model, the algebraic variables are regarded as state variables without governing differential equations. In de Queiroz *et al.* (1996), the problem of designing a position-force controller during constrained motion is considered. The proposed controller is based on exact knowledge of the system dynamics and does not require measurements of link velocity nor end–effector forces. Liu and Arimoto (1996) proposed a simple controller without force feedback by using the joint-space orthogonalization scheme. However, their approach still needs velocity measurement and no experimental results are presented.

In this paper, their method is used to design a decentralized position-force tracking controller for cooperative robot systems which does not require link velocities measurements nor end–effector contact forces. Experimental results are in good agreement with the developed theory. The approach is based on that presented in Gudiño Lau *et al.* (2004). However, two important improvements over the original algorithm are introduced: the observer is much simpler and, just as mentioned, no force measurements are required.

The paper is organized as follows. In Section 2, the system model and its properties are presented. Section 3 describes the proposed control and observer laws, while Section 4 shows experimental results. Finally, Section 5 gives some conclusions.

2. SYSTEM MODEL AND PROPERTIES

Consider a cooperative system with l–fingers, each of them with n_i degrees of freedom and m_i constraints arising from the contact with a held object. Then, the total number of degrees of freedom is given by $n = \sum_{i=1}^{l} n_i$ with a total number of $m = \sum_{i=1}^{l} m_i$ constraints, where $n_i > m_i$. The dynamics of the i–th finger is given by

$$\boldsymbol{H}_i(\boldsymbol{q}_i)\ddot{\boldsymbol{q}}_i + \boldsymbol{C}_i(\boldsymbol{q}_i,\dot{\boldsymbol{q}}_i)\dot{\boldsymbol{q}}_i + \boldsymbol{D}_i\dot{\boldsymbol{q}}_i + \boldsymbol{g}_i(\boldsymbol{q}_i) = \boldsymbol{\tau}_i + \boldsymbol{J}_{\varphi_i}^{\mathrm{T}}(\boldsymbol{q}_i)\boldsymbol{\lambda}_i, \tag{1}$$

where $\boldsymbol{q}_i \in \mathbb{R}^{n_i}$ is the vector of generalized joint coordinates, $\boldsymbol{H}_i(\boldsymbol{q}_i) \in \mathbb{R}^{n_i \times n_i}$ is the symmetric positive definite inertia matrix, $\boldsymbol{C}_i(\boldsymbol{q}_i,\dot{\boldsymbol{q}}_i)\dot{\boldsymbol{q}}_i \in \mathbb{R}^{n_i}$ is the vector of Coriolis and centrifugal torques, $\boldsymbol{g}_i(\boldsymbol{q}_i) \in \mathbb{R}^{n_i}$ is the vector of gravitational torques, $\boldsymbol{D}_i \in \mathbb{R}^{n_i \times n_i}$ is the positive semidefinite diagonal matrix accounting for joint viscous friction coefficients, $\boldsymbol{\tau}_i \in \mathbb{R}^{n_i}$ is the vector of torques acting at the joints, and $\boldsymbol{\lambda}_i \in \mathbb{R}^{m_i}$ is the vector of Lagrange multipliers (physically represents the force applied at the contact point). $\boldsymbol{J}_{\varphi_i}(\boldsymbol{q}_i) = \boldsymbol{\nabla}\boldsymbol{\varphi}_i(\boldsymbol{q}_i) \in \mathbb{R}^{m_i \times n_i}$ is assumed to be full rank in this paper. $\boldsymbol{\nabla}\boldsymbol{\varphi}_i(\boldsymbol{q}_i)$ denotes the gradient (or the Jacobian matrix) of the object surface vector $\boldsymbol{\varphi}_i \in \mathbb{R}^{m_i}$ which maps a vector onto the normal plane at the tangent plane that arises at the contact point described by

$$\boldsymbol{\varphi}_i(\boldsymbol{q}_i) = \boldsymbol{0}. \tag{2}$$

Note that equation (2) means that homogeneous constraints are being considered (Parra-Vega *et al.*, 2001). The complete system is subjected to m holonomic constraints given by $\boldsymbol{\varphi}(\boldsymbol{q}) = \boldsymbol{0}$, where $\boldsymbol{\varphi}(\boldsymbol{q}) = \boldsymbol{\varphi}(\boldsymbol{q}_1,\ldots,\boldsymbol{q}_l) \in \mathbb{R}^m$. Let us denote the largest (smallest) eigenvalue of a matrix by $\lambda_{\max}(\cdot)$ ($\lambda_{\min}(\cdot)$). Since revolute joints are considered, the following properties can be established:

Property 1. Each $\boldsymbol{H}_i(\boldsymbol{q}_i)$ satisfies $\lambda_{\mathrm{h}i}\|\boldsymbol{x}\|^2 \leq \boldsymbol{x}^{\mathrm{T}}\boldsymbol{H}_i(\boldsymbol{q}_i)\boldsymbol{x} \leq \lambda_{\mathrm{H}i}\|\boldsymbol{x}\|^2 \quad \forall\ \boldsymbol{q}_i,\boldsymbol{x} \in \mathbb{R}^{n_i}$, where $\lambda_{\mathrm{h}i} \triangleq \min_{\forall \boldsymbol{q}_i \in \mathbb{R}^{n_i}} \lambda_{\min}(\boldsymbol{H}_i)$, $\lambda_{\mathrm{H}i} \triangleq \max_{\forall \boldsymbol{q}_i \in \mathbb{R}^{n_i}} \lambda_{\max}(\boldsymbol{H}_i)$, and $0 < \lambda_{\mathrm{h}i} \leq \lambda_{\mathrm{H}i} < \infty$. △

Property 2. With a proper definition of $\boldsymbol{C}_i(\boldsymbol{q}_i,\dot{\boldsymbol{q}}_i)$, $\dot{\boldsymbol{H}}_i(\boldsymbol{q}_i) - 2\boldsymbol{C}_i(\boldsymbol{q}_i,\dot{\boldsymbol{q}}_i)$ is skew–symmetric. △

Property 3. It holds $\boldsymbol{C}_i(\boldsymbol{q}_i,\boldsymbol{x})\boldsymbol{y} = \boldsymbol{C}_i(\boldsymbol{q}_i,\boldsymbol{y})\boldsymbol{x}\ \forall\ \boldsymbol{x},\boldsymbol{y} \in \mathbb{R}^{n_i}$. △

Property 4. It is satisfied $\|\boldsymbol{C}_i(\boldsymbol{q}_i,\boldsymbol{x})\| \leq k_{\mathrm{c}i}\|\boldsymbol{x}\|$ with $0 < k_{\mathrm{c}i} < \infty$, $\forall\ \boldsymbol{x} \in \mathbb{R}^{n_i}$. △

Property 5. The vector $\dot{\boldsymbol{q}}_i$ can be written as

$$\dot{\boldsymbol{q}}_i = \boldsymbol{Q}_i(\boldsymbol{q}_i)\dot{\boldsymbol{q}}_i + \boldsymbol{J}_{\varphi_i}^{+}(\boldsymbol{q}_i)\dot{\boldsymbol{p}}_i, \tag{3}$$

where $\boldsymbol{Q}_i(\boldsymbol{q}_i) \triangleq \left(\boldsymbol{I}_{n_i \times n_i} - \boldsymbol{J}_{\varphi_i}^{+}\boldsymbol{J}_{\varphi_i}\right)$, and $\boldsymbol{J}_{\varphi_i}^{+} \triangleq \boldsymbol{J}_{\varphi_i}^{\mathrm{T}}\left(\boldsymbol{J}_{\varphi_i}\boldsymbol{J}_{\varphi_i}^{\mathrm{T}}\right)^{-1} \in \mathbb{R}^{n_i \times m_i}$ stands for the Penrose's pseudoinverse and $\boldsymbol{Q}_i \in \mathbb{R}^{n_i \times n_i}$ satisfies $\mathrm{rank}(\boldsymbol{Q}_i) = n_i - m_i$. These two matrices are orthogonal, *i.e.* $\boldsymbol{Q}_i\boldsymbol{J}_{\varphi_i}^{+} = \boldsymbol{O}$ (and $\boldsymbol{Q}_i\boldsymbol{J}_{\varphi_i}^{\mathrm{T}} = \boldsymbol{O}$). $\dot{\boldsymbol{p}}_i \triangleq \boldsymbol{J}_{\varphi_i}\dot{\boldsymbol{q}}_i \in \mathbb{R}^{m_i}$ is the so called constrained velocity. Furthermore, it holds

$$\dot{\boldsymbol{p}}_i = \boldsymbol{0} \quad \text{and} \quad \boldsymbol{p}_i = \boldsymbol{0}, \tag{4}$$

for $i = 1,\ldots,l$, since homogeneous constraints are being considered. $\boldsymbol{p}_i$ is called the constrained position. △

To be able to design the control–observer scheme, the following assumptions are made.

Assumption 1. The l robots of which the system is made up satisfy constraints (2) and (4) for all time. Furthermore, none of the robots is redundant nor it is in a singularity. $\triangle$

Assumption 2. The matrix $\boldsymbol{J}_{\varphi_i}$ is Lipschitz continuous, *i. e.* $\|\boldsymbol{J}_{\varphi_i}(\boldsymbol{q}_i)-\boldsymbol{J}_{\varphi_i}(\boldsymbol{q}_{\mathrm{d}i})\| \leq L_i\|\boldsymbol{q}_i-\boldsymbol{q}_{\mathrm{d}i}\|$, for a positive constant L_i and for all $\boldsymbol{q}_i$, $\boldsymbol{q}_{\mathrm{d}i} \in \mathbb{R}^{n_i}$. Besides, there exist positive finite constants c_{0i} and c_{1i} which satisfies $c_{0i} \triangleq \max_{\forall \mathbf{q}_i \in \mathbb{R}^{n_i}} \|\boldsymbol{J}^+_{\varphi_i}(\boldsymbol{q}_i)\|$, $c_{1i} \triangleq \max_{\forall \mathbf{q}_i \in \mathbb{R}^{n_i}} \|\boldsymbol{J}_{\varphi_i}(\boldsymbol{q}_i)\|$. $\triangle$

None of the manipulators can be redundant nor can be in a singularity so that (2) is satisfied only by a bounded vector $\boldsymbol{q}_i$. Of course, the closed kinematic loop that arises when the manipulators are holding an object is redundant.

3. CONTROL WITH VELOCITY ESTIMATION

In this section, the tracking control problem of a cooperative system of rigid robots is studied. Consider model (1) and define the tracking and observation errors as $\tilde{\boldsymbol{q}}_i \triangleq \boldsymbol{q}_i - \boldsymbol{q}_{\mathrm{d}i}$, $\boldsymbol{z}_i \triangleq \boldsymbol{q}_i - \hat{\boldsymbol{q}}_i$, where $\boldsymbol{q}_{\mathrm{d}i}$ is a desired smooth bounded trajectory satisfying constraint (2), and $(\hat{\cdot})$ represents the estimated value of $(\cdot)$. Other error definitions are $\Delta\boldsymbol{p}_i \triangleq \boldsymbol{p}_i - \boldsymbol{p}_{\mathrm{d}i}$, $\Delta\boldsymbol{\lambda}_i \triangleq \boldsymbol{\lambda}_i - \boldsymbol{\lambda}_{\mathrm{d}i}$, where $\boldsymbol{p}_{\mathrm{d}i}$ is the desired constrained position which satisfies (4). $\boldsymbol{\lambda}_{\mathrm{d}i}$ is the desired force to be applied by each finger on the constrained surface. Other definitions are

$$\begin{aligned}\dot{\boldsymbol{q}}_{\mathrm{r}i} &\triangleq \boldsymbol{Q}_i(\boldsymbol{q}_i)\left(\dot{\boldsymbol{q}}_{\mathrm{d}i} - \boldsymbol{\Lambda}_i\left(\hat{\boldsymbol{q}}_i - \boldsymbol{q}_{\mathrm{d}i}\right)\right) \\ &\quad + \boldsymbol{J}^+_{\varphi_i}(\boldsymbol{q}_i)\left(\dot{\boldsymbol{p}}_{\mathrm{d}i} - \beta_i\Delta\boldsymbol{p}_i\right) \end{aligned} \tag{5}$$

$$\begin{aligned}\boldsymbol{s}_i &\triangleq \dot{\boldsymbol{q}}_i - \dot{\boldsymbol{q}}_{\mathrm{r}i} = \boldsymbol{s}_{\mathrm{p}i} + \boldsymbol{s}_{\mathrm{f}i} \\ &= \boldsymbol{Q}_i(\boldsymbol{q}_i)\left(\dot{\hat{\boldsymbol{q}}}_i + \boldsymbol{\Lambda}_i\tilde{\boldsymbol{q}} - \boldsymbol{\Lambda}_i\boldsymbol{z}_i\right) \\ &\quad + \boldsymbol{J}^+_{\varphi_i}(\boldsymbol{q}_i)\left(\Delta\dot{\boldsymbol{p}}_i + \beta_i\Delta\boldsymbol{p}_i\right), \end{aligned} \tag{6}$$

where $\boldsymbol{\Lambda}_i = k_i\boldsymbol{I} \in \mathbb{R}^{n_i\times n_i}$ with $k_i > 0$, is a diagonal positive definite matrix, and β_i is a positive constant. It is important to notice that, contrary to $\dot{\boldsymbol{q}}_{\mathrm{r}i}$ given in Gudiño Lau *et al.* (2004), here the term $\Delta\boldsymbol{F}_i$ is not used. To get (6), the identity $\hat{\boldsymbol{q}}_i - \boldsymbol{q}_{\mathrm{d}i} = \tilde{\boldsymbol{q}}_i - \boldsymbol{z}_i$ has been used. Note also that $\boldsymbol{s}_{\mathrm{p}i}$ and $\boldsymbol{s}_{\mathrm{f}i}$ are orthogonal vectors. We propose the following substitution for $\ddot{\boldsymbol{q}}_{\mathrm{r}i}$

$$\begin{aligned}\ddot{\hat{\boldsymbol{q}}}_{\mathrm{r}i} &\triangleq \boldsymbol{Q}_i(\boldsymbol{q}_i)\left(\ddot{\boldsymbol{q}}_{\mathrm{d}i} - \boldsymbol{\Lambda}_i\left(\dot{\hat{\boldsymbol{q}}}_i - \dot{\boldsymbol{q}}_{\mathrm{d}i}\right)\right) \\ &\quad + \boldsymbol{J}^+_{\varphi_i}(\boldsymbol{q}_i)\left(\ddot{\boldsymbol{p}}_{\mathrm{d}i} - \beta_i\left(\dot{\boldsymbol{p}}_i - \dot{\boldsymbol{p}}_{\mathrm{d}i}\right)\right) \\ &\quad + \dot{\boldsymbol{Q}}_i(\dot{\boldsymbol{q}}_{\mathrm{o}i})\left(\dot{\boldsymbol{q}}_{\mathrm{d}i} - \boldsymbol{\Lambda}_i\left(\hat{\boldsymbol{q}}_i - \boldsymbol{q}_{\mathrm{d}i}\right)\right) \\ &\quad + \dot{\boldsymbol{J}}^+_{\varphi_i}(\dot{\boldsymbol{q}}_{\mathrm{o}i})\left(\dot{\boldsymbol{p}}_{\mathrm{d}i} - \beta_i\Delta\boldsymbol{p}_i\right), \end{aligned} \tag{7}$$

where $\dot{\boldsymbol{Q}}_i(\dot{\boldsymbol{q}}_{\mathrm{o}i})$ and $\dot{\boldsymbol{J}}^+_{\varphi_i}(\dot{\boldsymbol{q}}_{\mathrm{o}i})$ are defined in Gudiño Lau *et al.* (2004), and

$$\dot{\boldsymbol{q}}_{\mathrm{o}i} \triangleq \dot{\hat{\boldsymbol{q}}}_i - \boldsymbol{\Lambda}_i\boldsymbol{z}_i. \tag{8}$$

The dependence of $\dot{\boldsymbol{Q}}_i$ on $\boldsymbol{q}_i$ has been omitted. Define $\dot{\tilde{\boldsymbol{Q}}}_i(\boldsymbol{r}_i) \triangleq \dot{\boldsymbol{Q}}_i(\dot{\boldsymbol{q}}_i) - \dot{\boldsymbol{Q}}_i(\dot{\boldsymbol{q}}_{\mathrm{o}i})$, where

$$\boldsymbol{r}_i \triangleq \dot{\boldsymbol{q}}_i - \dot{\boldsymbol{q}}_{\mathrm{o}i} = \dot{\boldsymbol{z}}_i + \boldsymbol{\Lambda}_i\boldsymbol{z}_i. \tag{9}$$

After some manipulation, it is possible to get

$$\ddot{\hat{\boldsymbol{q}}}_{\mathrm{r}i} = \ddot{\boldsymbol{q}}_{\mathrm{r}i} + \boldsymbol{e}_i(\boldsymbol{r}_i), \tag{10}$$

where $\boldsymbol{e}_i(\boldsymbol{r}_i) = -\dot{\tilde{\boldsymbol{Q}}}_i(\boldsymbol{r}_i)\left(\dot{\boldsymbol{q}}_{\mathrm{d}i} - \boldsymbol{\Lambda}_i\tilde{\boldsymbol{q}}_i + \boldsymbol{\Lambda}_i\boldsymbol{z}_i\right) - \dot{\tilde{\boldsymbol{J}}}^+_{\varphi_i}(\boldsymbol{r}_i)\left(\dot{\boldsymbol{p}}_{\mathrm{d}i} - \beta_i\Delta\boldsymbol{p}_i\right)$. The proposed controller is then given for each single input by

$$\begin{aligned}\boldsymbol{\tau}_i &\triangleq \boldsymbol{H}_i(\boldsymbol{q}_i)\ddot{\hat{\boldsymbol{q}}}_{\mathrm{r}i} + \boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_{\mathrm{r}i})\dot{\boldsymbol{q}}_{\mathrm{r}i} + \boldsymbol{D}_i\dot{\boldsymbol{q}}_{\mathrm{r}_i} \\ &\quad + \boldsymbol{g}_i(\boldsymbol{q}_i) - \boldsymbol{K}_{\mathrm{R}_i}\left(\dot{\boldsymbol{q}}_{\mathrm{o}i} - \dot{\boldsymbol{q}}_{\mathrm{r}i}\right) - \boldsymbol{J}^{\mathrm{T}}_{\varphi_i}(\boldsymbol{q}_i)\boldsymbol{\lambda}_{\mathrm{d}i}, \end{aligned} \tag{11}$$

where $\boldsymbol{K}_{\mathrm{R}i} \in \mathbb{R}^{n_i\times n_i}$ is a diagonal positive definite matrix. Note that from (6) and (9) it is $\dot{\boldsymbol{q}}_{\mathrm{o}i} - \dot{\boldsymbol{q}}_{\mathrm{r}i} = \boldsymbol{s}_i - \boldsymbol{r}_i$. By substituting (11) into (1), the closed loop dynamics becomes after many manipulation

$$\begin{aligned}\boldsymbol{H}_i(\boldsymbol{q}_i)\dot{\boldsymbol{s}}_i &= -\boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\boldsymbol{s}_i - \boldsymbol{K}_{\mathrm{DR}_i}\boldsymbol{s}_i \\ &\quad + \boldsymbol{K}_{\mathrm{R}_i}\boldsymbol{r}_i + \boldsymbol{J}^{\mathrm{T}}_{\varphi_i}(\boldsymbol{q}_i)\Delta\boldsymbol{\lambda}_i \\ &\quad - \boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_{\mathrm{r}i})\boldsymbol{s}_i + \boldsymbol{H}_i(\boldsymbol{q}_i)\boldsymbol{e}_i(\boldsymbol{r}_i) \end{aligned} \tag{12}$$

where $\boldsymbol{K}_{\mathrm{DR}_i} \triangleq \boldsymbol{K}_{\mathrm{R}_i} + \boldsymbol{D}_i$. In order to get (12), Property 3 has been used. The observer is

$$\dot{\hat{\boldsymbol{q}}}_i = \dot{\hat{\boldsymbol{q}}}_{\mathrm{o}i} + \boldsymbol{\Lambda}_i\boldsymbol{z}_i + k_{\mathrm{d}i}\boldsymbol{z}_i \tag{13}$$

$$\ddot{\hat{\boldsymbol{q}}}_{\mathrm{o}i} = \ddot{\hat{\boldsymbol{q}}}_{\mathrm{r}i} + k_{\mathrm{d}i}\boldsymbol{\Lambda}_i\boldsymbol{z}_i, \tag{14}$$

where $k_{\mathrm{d}i}$ is a positive constant. Since from (13) you have $\ddot{\hat{\boldsymbol{q}}}_{\mathrm{o}i} = \ddot{\hat{\boldsymbol{q}}}_i - \boldsymbol{\Lambda}_i\dot{\boldsymbol{z}}_i - k_{\mathrm{d}i}\dot{\boldsymbol{z}}_i$, (14) becomes $\dot{\boldsymbol{s}}_i = \dot{\boldsymbol{r}}_i + k_{\mathrm{d}i}\boldsymbol{r}_i + \boldsymbol{e}_i(\boldsymbol{r}_i)$, in view of (10). By multiplying both sides by $\boldsymbol{H}_i(\boldsymbol{q}_i)$, using Property 3 again and more manipulation, it is

$$\begin{aligned}\boldsymbol{H}_i(\boldsymbol{q}_i)\dot{\boldsymbol{r}}_i &= -\boldsymbol{C}_i(\boldsymbol{q}_i, \dot{\boldsymbol{q}}_i)\boldsymbol{r}_i - \boldsymbol{H}_{\mathrm{rd}_i}\boldsymbol{r}_i \\ &\quad + \boldsymbol{C}_i(\boldsymbol{q}_i, \boldsymbol{s}_i + \dot{\boldsymbol{q}}_{\mathrm{r}i})\boldsymbol{r}_i - \boldsymbol{C}_i(\boldsymbol{q}_i, \boldsymbol{s}_i + 2\dot{\boldsymbol{q}}_{\mathrm{r}i})\boldsymbol{s}_i \\ &\quad - \boldsymbol{K}_{\mathrm{DR}_i}\boldsymbol{s}_i + \boldsymbol{J}^{\mathrm{T}}_{\varphi_i}(\boldsymbol{q}_i)\Delta\boldsymbol{\lambda}_i, \end{aligned} \tag{15}$$

where $\boldsymbol{H}_{\mathrm{rd}_i} \triangleq k_{\mathrm{d}i}\boldsymbol{H}_i(\boldsymbol{q}_i) - \boldsymbol{K}_{\mathrm{R}_i}$. Now, let us define

$$\boldsymbol{x}_i \triangleq [\,\boldsymbol{s}_i^{\mathrm{T}} \quad \boldsymbol{r}_i^{\mathrm{T}}\,]^{\mathrm{T}}, \tag{16}$$

as state for (12) and (15). The main idea of the control–observer design is to show that whenever

$\|\boldsymbol{x}_i\|$ tends to zero, the tracking errors $\tilde{\boldsymbol{q}}_i$, $\dot{\tilde{\boldsymbol{q}}}_i$, $\Delta\boldsymbol{p}_i$, $\Delta\dot{\boldsymbol{p}}_i$ and $\Delta\boldsymbol{\lambda}_i$ and the observation errors $\boldsymbol{z}_i$ and $\dot{\boldsymbol{z}}_i$ will do it as well. From (9), this is rather obvious for $\boldsymbol{z}_i$ and $\dot{\boldsymbol{z}}_i$. However, it is not clear for the other variables. The following lemma shows that this is the case under some conditions.

Lemma 1. If $\boldsymbol{x}_i$ is bounded by $x_{\max_i}$ and tends to zero, then the following facts hold:

a) $\Delta\boldsymbol{p}_i$, $\Delta\dot{\boldsymbol{p}}_i$ remain bounded and tend to zero.
b) $\tilde{\boldsymbol{q}}_i$ and $\dot{\tilde{\boldsymbol{q}}}_i$ remain bounded. Furthermore, if the bound $x_{\max_i}$ for $\|\boldsymbol{x}_i\|$ is chosen small enough so as to guarantee that $\|\tilde{\boldsymbol{q}}_i\| \leq \eta_i$ for all t, with η_i a positive and small enough constant, then both $\tilde{\boldsymbol{q}}_i$ and $\dot{\tilde{\boldsymbol{q}}}_i$ will tend to zero as well.
c) If, in addition, the velocity vector $\dot{\boldsymbol{q}}_i$ is bounded, then $\Delta\boldsymbol{\lambda}_i$ will remain bounded and tend to zero. $\triangle$

The proof of Lemma 1 can be found in Appendix A. It is interesting to note that, if $\|\boldsymbol{x}_i\|$ is bounded by $x_{\max_i}$, then it is always possible to find a bound for $\boldsymbol{e}_i(\boldsymbol{r}_i)$ in (10) which satisfies

$$\|\boldsymbol{e}_i(\boldsymbol{r}_i)\| \leq M_{\mathrm{e}i}(x_{\max_i})\|\boldsymbol{r}_i\| < \infty. \tag{17}$$

Consider now the following function

$$V_i(\boldsymbol{x}_i) = \frac{1}{2}\boldsymbol{x}_i^{\mathrm{T}}\boldsymbol{M}_i\boldsymbol{x}_i, \tag{18}$$

where $\boldsymbol{M}_i \overset{\triangle}{=} \text{block diag}\{\,\boldsymbol{H}_i(\boldsymbol{q}_i), \quad \boldsymbol{H}_i(\boldsymbol{q}_i\,\}$. Suppose that one may find a domain

$$\mathbb{D}_i = \{\boldsymbol{x}_i \in \mathbb{R}^{n_i} \mid \|\boldsymbol{x}_i\| \leq x_{\max_i}\}, \tag{19}$$

so that for all time $\dot{V}_i(\boldsymbol{x}_i) \leq 0$ with $\dot{V}_i(\boldsymbol{x}_i) = 0$ if and only if $\boldsymbol{x}_i = \boldsymbol{0}$. If $x_{\max_i}$ is small enough in the sense of Lemma 1, then from the former discussion one can conclude the convergence to zero of all error signals.

Theorem 1. Consider the cooperative system dynamics given by (1), (2) and (4), in closed loop, with the control law (11) and the observer (13)–(14), where $\boldsymbol{q}_{\mathrm{d}i}$ and $\boldsymbol{p}_{\mathrm{d}i}$ are the desired bounded joint and constrained positions, whose derivatives $\dot{\boldsymbol{q}}_{\mathrm{d}i}$, $\ddot{\boldsymbol{q}}_{\mathrm{d}i}$, $\dot{\boldsymbol{p}}_{\mathrm{d}i}$, and $\ddot{\boldsymbol{p}}_{\mathrm{d}i}$ are also bounded, and they all satisfy constraint (4). Consider also l given domains $\mathbb{D}_i \in \mathbb{R}^{n_i}$ defined by (19) for each subsystem, where the bounds $x_{\max_i}$, $i = 1, \ldots, l$, are chosen according to $x_{\max_i} \leq \frac{\eta_i\alpha_i}{(1+\sqrt{n_i})}$ with $\alpha_i \overset{\triangle}{=} k_i - |k_i - \beta_i| - \gamma_i$, k_i and β_i given in (5) and $\gamma_i \overset{\triangle}{=} c_{0i}L_i\left(v_{\mathrm{m}i} + \beta_i\eta_i\right)$, with c_{0i} and L_i given in Assumption 2 and $\|\dot{\boldsymbol{q}}_{\mathrm{d}i}\| \leq v_{\mathrm{m}i}\ \forall\ t$. Then, every dynamic and error signal remains bounded and asymptotic stability of tracking, observation and force errors arise, *i. e.* $\lim_{t\to\infty} \tilde{\boldsymbol{q}}_i = \boldsymbol{0}$, $\lim_{t\to\infty} \dot{\tilde{\boldsymbol{q}}}_i = \boldsymbol{0}$, $\lim_{t\to\infty} \boldsymbol{z}_i = \boldsymbol{0}$, $\lim_{t\to\infty} \dot{\boldsymbol{z}}_i = \boldsymbol{0}$, and $\lim_{t\to\infty} \Delta\boldsymbol{\lambda}_i = \boldsymbol{0}$, if the following conditions are satisfied

$$\lambda_{\min}(\boldsymbol{K}_{\mathrm{R}_i}) \geq \mu_{1i} + 1 + \delta_i \tag{20}$$

$$k_{\mathrm{d}i} \geq \frac{\lambda_{\max}(\boldsymbol{K}_{\mathrm{R}_i}) + \omega_i}{\lambda_{\mathrm{h}i}} \tag{21}$$

where $\omega_i = \mu_{2i} + \gamma_{2i} + \frac{1}{4}\left(\lambda_{\mathrm{D}i} + \mu_{3i} + \mu_{4i} + \gamma_{1i}\right)^2 + \delta_i$, with δ_i a positive constant and μ_{1i}, μ_{2i}, μ_{3i}, μ_{4i}, γ_{1i}, γ_{2i} and $\lambda_{\mathrm{D}i}$ defined in Appendix B. $\triangle$

For a proof of the Theorem 1 see Appendix B.

Remark 1. The result of Theorem 1 is only local. Also, it is rather difficult to find analytically a domain D_i, but it should be noticed that it *cannot* be made arbitrarily large. This is to guarantee the convergence to zero of the tracking errors $\tilde{\boldsymbol{q}}_i$ and $\dot{\tilde{\boldsymbol{q}}}_i$. However, this does not represent a serious drawback since for grasping purposes it is usual to give smooth trajectories with zero initial position errors. On the other hand, it is worthy pointing out that a controller–observer scheme is implemented for every robot separately, while only the knowledge of each constraint of the form (2) is required. Finally, it is easy to show that $S_{\mathrm{a}i} = \left\{\boldsymbol{x}_i \in \mathbb{R}^{n_i} \mid \|\boldsymbol{x}_i\| \leq \sqrt{\frac{\lambda_{1i}}{\lambda_{2i}}}\ x_{\max_i}\right\}$ is a region of attraction. $\triangle$

4. EXPERIMENTAL RESULTS

In this section, some experimental results are presented. To this end, a test bed with two industrial robots is used. The robots are at the Laboratory for Robotics of the National University of Mexico. They are the A465 and A255 of *CRS Robotics*. Only the first three joints of each robot are used for the experiments. Both robots own force sensors and crash protectors, so that one can verify whether the desired forces are being matched. The palm frame of the whole system is at the base of the robot A465, with its x–axis pointing towards the other manipulator. If the task consists in lifting the object and pushing with a desired force, then the constraints in Cartesian coordinates are simply given by $\varphi_i = x_i - b_i = 0$, for $i = 1, 2$ and b_i a positive constant. The desired trajectories are $x_{\mathrm{d}1} = 0.5530$[m], $x_{\mathrm{d}2} = 0.8522$[m], $y_{\mathrm{d}1,2} = 0.0095\sin(\omega(t - t_{\mathrm{i}}))$[m], and $z_{\mathrm{d}1,2} = (0.635 + 0.0095\cos(\omega(t - t_{\mathrm{i}})) - 0.0095)$[m]. The inverse kinematics of the manipulators has to be employed to compute $\boldsymbol{q}_{\mathrm{d}i}$, for $i = 1, 2$. These trajectories are valid from an initial time t_{i} to a final time t_{f}. For the experiments it has been set $t_{\mathrm{i}} = 20$s and $t_{\mathrm{f}} = 70$s. The robots will make a circle each second in the y–z plane. The desired pushing force $f_{\mathrm{d}x1,2}$ is given by

$$\begin{cases} 3.0(t-15) \text{ [N]} & 15 \le t < 25 \\ 30+10\sin(6\pi(t-25)/40) \text{ [N]} & 25 \le t \le 65 \\ 30-3.0(t-65) \text{ [N]} & 65 < t \le 75 \end{cases}$$

and $f_{\mathrm{d}y1,2} = f_{\mathrm{d}z1,2} = 0$[N]. The different control and observer parameters are $\boldsymbol{\Lambda}_1 = 21\boldsymbol{I}$, $\boldsymbol{\Lambda}_2 = 20\boldsymbol{I}$, $\boldsymbol{K}_{\mathrm{R}_1} = 80\boldsymbol{I}$, $\boldsymbol{K}_{\mathrm{R}_2} = \mathrm{diag}\{40 \quad 20 \quad 40\}$, $k_{\mathrm{d}1} = k_{\mathrm{d}2} = 12$. The experiment lasts 90s. The object is held at $t = 15$s. Before, the robots are in free movement and the control law (11) is used with the force part set to zero (*i. e.* $\boldsymbol{Q}_i = \boldsymbol{I}$ and $\boldsymbol{J}_{\varphi_i} = \boldsymbol{O}$). It is easy to prove that this scheme is stable for unconstrained motion. From $t = 15$s to $t = 20$s the object is lifted to the initial position to make the circles, while the desired pushing forces keep increasing. From $t = t_{\mathrm{i}} = 20$s to $t = t_{\mathrm{f}} = 70$s the robots are making the circles and the desired forces are sinus signals from $t = 25$s to $t = 65$s. From $t = 70$s to $t = 75$s the object is put down and the desired forces diminish to zero. Finally, from $t = 75$s to $t = 90$s the manipulators go back to their initial positions. For lack of room, only the outcomes for the desired forces are shown. They can be considered good, although the main force in the x direction shows some noise around the desired trajectory (see Figure 1). The results for tracking and observation errors were also good.

5. CONCLUSIONS

The position and force tracking control problem of cooperative robots with end effectors constrained on geometric surfaces and without velocity-force measurements is considered in this paper by using the joint-space orthogonalization scheme. The control law is a decentralized approach which takes into account motion constraints rather than the held object dynamics. By assuming that fingers dynamics are well known, the crucial point of this work is to show that our controller does not need any velocity-force feedback. A linear observer for each finger is proposed to estimate joint velocities which does not require any knowledge of the robots dynamics. Regarding the force control, our scheme only uses a feedforward of the desired force. Despite the fact that the stability analysis is complex, the controller and specially the observer are not. Experimental results have been carried out to test the proposed approach. Since both robots own force sensors, it was possible to check out that there was a good matching of real and desired forces. Also, the overall outcomes can be considered good as well.

Appendix A. PROOF OF LEMMA 1

In this appendix, item c) of Lemma 1 is proven. For a proof of items a) and b) see Gudiño Lau *et al.* (2004). Note that one only has to set $\boldsymbol{x}_i^{\mathrm{T}} = [\,\boldsymbol{s}_i^{\mathrm{T}} \quad \boldsymbol{r}_i^{\mathrm{T}}\,]^{\mathrm{T}}$ and $\boldsymbol{\xi}_i = \boldsymbol{O}$, where $\boldsymbol{\xi}_i$ is the gain for the force feedback $\Delta\boldsymbol{F}_i$, which is not used here.

As to c), when $\|\boldsymbol{x}_i\|$ is bounded and tends to zero, $\Delta\boldsymbol{\lambda}_i$ does not necessarily do it nor remains bounded. To prove that, one may use the fact $\boldsymbol{J}_{\varphi_i}(\boldsymbol{q}_i)\dot{\boldsymbol{s}}_i + \dot{\boldsymbol{J}}_{\varphi_i}(\boldsymbol{q}_i)\boldsymbol{s}_i = 0$. From (12) one gets

$$\begin{aligned}\Delta\boldsymbol{\lambda}_i = &-\left(\boldsymbol{J}_{\varphi_i}(\boldsymbol{q}_i)\boldsymbol{H}_i^{-1}(\boldsymbol{q}_i)\boldsymbol{J}_{\varphi_i}^T(\boldsymbol{q}_i)\right)^{-1}\cdot \\ &\cdot\left(\dot{\boldsymbol{J}}_{\varphi_i}(\boldsymbol{q}_i)\boldsymbol{s}_i + \boldsymbol{J}_{\varphi_i}(\boldsymbol{q}_i)\boldsymbol{H}_i^{-1}(\boldsymbol{q}_i)\boldsymbol{h}_i\right),\end{aligned} \quad \text{(A.1)}$$

with $\boldsymbol{h}_i = \boldsymbol{H}_i(\boldsymbol{q}_i)\boldsymbol{e}_i(\boldsymbol{r}_i) - \boldsymbol{C}_i(\boldsymbol{q}_i,\dot{\boldsymbol{q}}_i)\boldsymbol{s}_i + \boldsymbol{K}_{\mathrm{R}_i}\boldsymbol{r}_i - \boldsymbol{C}_i(\boldsymbol{q}_i,\dot{\boldsymbol{q}}_{\mathrm{r}i})\boldsymbol{s}_i - \boldsymbol{K}_{\mathrm{DR}_i}\boldsymbol{s}_i$. Because of the assumption of the boundedness of $\dot{\boldsymbol{q}}$ and $\boldsymbol{x}_i$, $\Delta\boldsymbol{\lambda}_i$ must be bounded as well from (A.1). Furthermore, if $\|\boldsymbol{x}_i\| \to 0$ then $\Delta\boldsymbol{\lambda}_i \to \boldsymbol{0}$. Finally, note that from (12) we can additionally conclude that $\dot{\boldsymbol{s}}_i$ is bounded and tends to zero.

Appendix B. PROOF OF THEOREM 1

We use the well–known theorem about asymptotical stability given in Khalil (2002)[pp. 114]. To take advantage of this theorem, we just have to find domains $\mathbb{D}_i$ for which each $V_i(\boldsymbol{x}_i)$ in (18) satisfies $\dot{V}_i(\boldsymbol{x}_i) < 0$ in $\mathbb{D}_i - \{\boldsymbol{0}\}$. Note that $V_i(\boldsymbol{x}_i)$ is positive definite in $\mathbb{R}^{n_i}$. In doing so, one can prove that $\boldsymbol{x}_i \to \boldsymbol{0}$ for all i. Then, Lemma 1 can be employed to analyze the behavior of the different error signals. Based on the discussion given in Appendix A, we define each domain $\mathbb{D}_i$ as in (19), where $x_{\max_i}$ is chosen $x_{\max_i} \le \frac{\eta_i\alpha_i}{1+\sqrt{n_i}}$. See Appendix I of Gudiño Lau *et al.* (2004) for details. In $\mathbb{D}_i$ one can define $\mu_{1i} \triangleq \max_{\|\boldsymbol{x}_i\|\le x_{\max_i}} \|\boldsymbol{C}_i(\boldsymbol{q}_i,\dot{\boldsymbol{q}}_{\mathrm{r}i})\|$, $\mu_{2i} \triangleq \max_{\|\boldsymbol{x}_i\|\le x_{\max_i}} \|\boldsymbol{C}_i(\boldsymbol{q}_i,\boldsymbol{s}_i + \dot{\boldsymbol{q}}_{\mathrm{r}i})\|$, $\mu_{3i} \triangleq \max_{\|\boldsymbol{x}_i\|\le x_{\max_i}} \|\boldsymbol{C}_i(\boldsymbol{q}_i,\boldsymbol{s}_i + 2\dot{\boldsymbol{q}}_{\mathrm{r}i})\|$, $\mu_{4i} \triangleq M_{\mathrm{e}i}(x_{\max_i})\lambda_{\mathrm{H}i}$, $\lambda_{\mathrm{D}i} \triangleq \lambda_{\max}(\boldsymbol{D}_i)$, $c_{2i} \triangleq \max_{\forall\boldsymbol{q}_i\in\mathbb{R}^{n_i}} \left\|\left[\boldsymbol{J}_{\varphi_i}\boldsymbol{H}_i^{-1}(\boldsymbol{q}_i)\boldsymbol{J}_{\varphi_i}^T\right]^{-1}\right\|$, $\|\dot{\boldsymbol{J}}_{\varphi_i}(\boldsymbol{q}_i)\| \le \alpha_{1i}\|\boldsymbol{s}_i\| + \alpha_{2i}\|\dot{\boldsymbol{q}}_{\mathrm{r}i}\|$, $\|\dot{\boldsymbol{q}}_{\mathrm{r}i}\| \le v_{\mathrm{m}i} + k_i\ \eta_i + \sqrt{n_i}\ x_{\max_i}$, $\sigma_{\mathrm{H}i} \triangleq \max_{\forall\boldsymbol{q}_i\in\mathbb{R}^{n_i}} \lambda_{\max}(\boldsymbol{H}_i^{-1})$, $\gamma_{1i} \triangleq c_{1i}c_{2i}(\alpha_{1i}x_{\max_i} + \alpha_{2i}\alpha_{3i}) + {c_{1i}}^2c_{2i}\sigma_{\mathrm{H}i}(\mu_{3i} + \lambda_{\max_i}(\boldsymbol{K}_{\mathrm{DR}_i}))$, $\gamma_{2i} \triangleq {c_{1i}}^2c_{2i}\sigma_{\mathrm{H}i}\lambda_{\max_i}(\boldsymbol{K}_{\mathrm{R}_i}) + {c_{1i}}^2c_{2i}M_{\mathrm{e}i}$, $\|\boldsymbol{r}_i^{\mathrm{T}}\boldsymbol{J}_{\varphi_i}^{\mathrm{T}}(\boldsymbol{q}_i)\Delta\boldsymbol{\lambda}_i\| \le \gamma_{1i}\|\boldsymbol{s}_i\|\|\boldsymbol{r}_i\| + \gamma_{2i}\|\boldsymbol{r}_i\|^2$, where α_{1i}, α_{2i} are positive constants, $M_{\mathrm{e}i}$ is given in (17), $\|\dot{\boldsymbol{q}}_{\mathrm{d}i}\| \le v_{\mathrm{m}i}$ $\forall$ t, and η_i small enough.

It is straightforward to show that the derivative of the Lyapunov function candidate in (18), satisfies

$$\begin{aligned}\dot{V}_i \le &-\lambda_{\min}(\boldsymbol{K}_{\mathrm{R}_i})\|\boldsymbol{s}_i\|^2 - k_{\mathrm{d}_i}\lambda_{\mathrm{h}_i}\|\boldsymbol{r}_i\|^2 \\ &+\lambda_{\max}(\boldsymbol{K}_{\mathrm{R}_i})\|\boldsymbol{r}_i\|^2 + \gamma_{2i}\|\boldsymbol{r}_i\|^2\end{aligned} \quad \text{(B.1)}$$

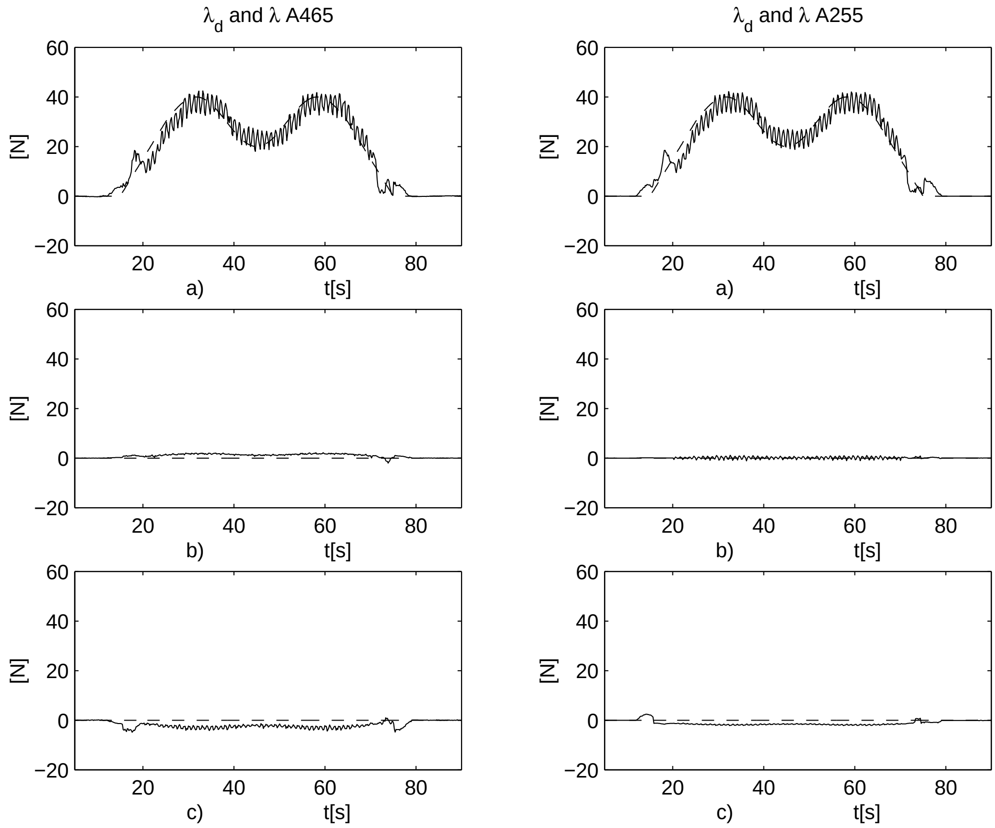

Fig. 1: Force measurements of robots A465 and A255. a) F_{x_1}. b) F_{y_1}. c) F_{z_1}. a) F_{x_2}. b) F_{y_2}. c) F_{z_2}. —— measured. - - - desired.

$$+\mu_{1i}\|\boldsymbol{s}_i\|^2+\mu_{2i}\|\boldsymbol{r}_i\|^2 \\ +(\lambda_{\mathrm{D}i}+\mu_{3i}+\mu_{4i}+\gamma_{1i})\|\boldsymbol{s}_i\|\|\boldsymbol{r}_i\|,$$

for $\boldsymbol{x}_i$ in $\mathbb{D}_i$. The next step is to choose the different gains to guarantee that $\dot{V}_i(\boldsymbol{x}_i) < 0$ in $\mathbb{D}_i - \{\mathbf{0}\}$. First of all, consider $\lambda_{\min}(\boldsymbol{K}_{\mathrm{R}_i})$ in (20) and $k_{\mathrm{d}i}$ in (21), such that (B.1) becomes $\dot{V}_i(\boldsymbol{x}_i) \leq -\delta_i\|\boldsymbol{x}_i\|^2$. Then, one concludes that $\boldsymbol{x}_i \to \mathbf{0}$. Now, from definition (9) one has directly $\lim_{t\to\infty} \boldsymbol{z}_i = \mathbf{0}$ and $\lim_{t\to\infty} \dot{\boldsymbol{z}}_i = \mathbf{0}$. Furthermore, in view of (19) one has $\|\boldsymbol{x}_i\| \leq x_{\max_i}$ and thus $\|\tilde{\boldsymbol{q}}_i\| \leq \eta_i$ (from the discussion in Appendix I of Gudiño Lau *et al.* (2004)). Thus, from Lemma 1, a) and b), we get $\lim_{t\to\infty} \Delta\dot{\boldsymbol{p}}_i = \mathbf{0}$, $\lim_{t\to\infty} \Delta\boldsymbol{p}_i = \mathbf{0}$, $\lim_{t\to\infty} \dot{\tilde{\boldsymbol{q}}}_i = \mathbf{0}$, and $\lim_{t\to\infty} \tilde{\boldsymbol{q}}_i = \mathbf{0}$. To applied c) of Lemma 1, we only need to show that $\dot{\boldsymbol{q}}_i$ is bounded. This is the case because $\dot{\tilde{\boldsymbol{q}}}_i$ and $\dot{\boldsymbol{q}}_{\mathrm{d}i}$ are bounded. Thus we get $\lim_{t\to\infty} \Delta\boldsymbol{\lambda}_i = \mathbf{0}$. Finally, the stability of the whole system can be proven using $V = \sum_{i=1}^{l} V_i(\boldsymbol{x}_i)$.

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