ELO MODEL REDUCTION AND CASE STUDY OF EVENLY DISTRIBUTED RC INTERCONNECT

Ben Wang[1], Sheng-Guo Wang[2], and Baoguo Yuan[3]

[1]IBM, PO Box 12195, 3039 Cornwallis Rd, RTP, NC 27709, USA
[2]College of Engineering, University of North Carolina at Charlotte, Charlotte, NC 28223-0001, USA, swang@uncc.edu
[3]Div. of Electrical & Electronic Eng., Practical Training Center, Shanghai Second Polytechnic University, Shanghai, China

Abstract: This paper presents state space closed forms and transfer function recursive algorithms for evenly distributed RC interconnect models and their even length-order (ELO) model reduction. The closed-forms have a computation complexity *O*(1). The characteristics of the ELO model simplification to its original model are revealed. It is shown that extremely high-order RC interconnects can be accurately approximated only by a tenth or higher order ELO model. The order may be reduced further when external ports are included and dominated. The results are useful to VLSI interconnect model reduction and design, and to control systems with a distributed transmission line.

Keywords: Interconnect, Distributed, transmission lines, modelling, algorithms, model reduction.

I. INTRODUCTION

Modeling is important for systems and control. The rapid increase of integration level and operation speed makes IC interconnect one of the important limiting factors of today's VLSI circuit design performance. The advance of high-speed deep-submicron VLSI technology requires chip interconnect and packaging to be modeled by distributed circuits (Boese, *et al*., 1992; Chen and Wang, 1999; Freund, 1999; Liu, Pileggi and Stojwas, 1999; Pillage and Rohrer, 1990; Reed and Rohrer, 1999; Wang, Yu and Kuh, 2000; Wang, *et al*., 2002; Yuan, Wang and Wang, 2004; Zhou, Preparata and Kang, 1991). Such a detailed modeling level eventually results in large scale linear circuits to be analyzed. An effort of reducing the circuit order is then necessary in order to evaluate the circuit performance and characteristics in a reasonable time period, as required by real design practice. This modeling is also important for control systems with a distributed transmission line.

A lot of efforts have been done, such as widely used Asymptotic Waveform Evaluation for Timing Analysis (AWE) based on Padé approximation (Pillage and Rohrer, 1990), and recent error bounded Padé approximation via bilinear conformal transformation (Chen and Wang, 1999). The Balanced Truncation Method (BTM) is also useful for model reduction (Yuan, Wang and Wang, 2004) with a guaranteed model reduction performance over the whole frequency range. Some efforts for DC matching with a cost of high frequency mismatching and for fast computation with new algorithms have been emerged for the BTM (Freund, 1999). Other effort is via passive model order reduction algorithm based on Chebyshev expansion of impulse response of interconnect networks (Wang, Yu and Kuh, 2000).

Because the original distributed interconnect models have very high orders, it is important to have their closed forms and to further investigate model reduction approaches and features. A simple method seems to evenly divide interconnect into *m*-pieces and assume that each piece has its RC/RLC parameter values in proportion to its length if each has a same physical size. It is correct if *m* is sufficiently large in view of the infinitesimal pieces. This method is called an even length order (ELO) reduction or simplification method if *m* is small/low. This ELO model reduction method has a nature of simplicity and passive implementation. However, how large will the order *m* be sufficient in practice? What is its performance error? A thorough study is needed.

In order to offer a basis for evaluation of the ELO model reduction, a through and careful knowledge of original interconnect models are needed. Thus, this paper first derives precise state space model closed-forms and transfer function recursive algorithms of evenly distributed RC interconnect. They are also useful for future investigating performance robustness when uncertainties are considered (Wang, Lin and Shieh 1998). The conventional way to find this distributed linear model is bound to meet computation of high dimension matrix inverse and matrix multiplications. Recently, Wang et *al*. (2002)

have presented an approach for an exact closed-form of the state space model and an elegant recursive algorithm of the transfer function for the RC distributed interconnects. Wang's method (2002) avoids matrix inverse and reaches exact model accuracy. This paper follows Wang's approach to extend the models to include a source resistor and a load capacitor in addition to the load resistor.

Further, this paper develops the ELO model reduction via the closed-form in the time domain and the recursive algorithm in the frequency domain. Then, the performance, relationship and characteristics of the ELO model reduction for evenly distributed RC interconnect are investigated. Finally, two cases are studied and simulated.

The main features and contributions of this paper are the accuracy and effectiveness of the closed-forms and algorithms, and the simplicity and passivity of their ELO reduction models. The results are useful not only to VLSI interconnect but also to control systems with a long distributed transmission line.

II. Problem Formulation

In general, interconnect can be modeled as an RC interconnect or an RLC interconnect. Here, RC interconnect is considered first. The RLC interconnect will be discussed in a separate paper.

Two evenly distributed RC interconnect circuits are considered. One is an RC interconnect itself in Fig. 1. Another one is an RC interconnect with a source resistor R_s, a load resistor R_0 and a load capacitor C_0, in Fig. 2. The order of the distributed circuits is n (n>>1) as assumed. The input voltage is $v_{in}(t)$, then the output port has a voltage $v_o(t)$, where $v_{in}(t) = v_s(t)$ in Fig. 2. Its evenly distributed resistor is R and capacitor is C. The unevenly distributed interconnect model may refer to Wang *et al.* (2002). Because most interconnects can be considered as an evenly distributed interconnect, or consists of a set of evenly distributed interconnects. Thus, an evenly distributed RC interconnect is considered here.

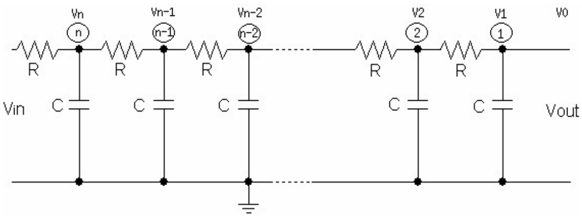

Fig. 1. Evenly distributed RC interconnect circuit

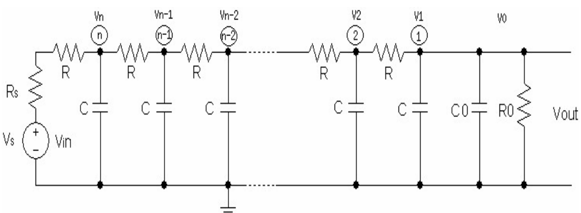

Fig. 2. Evenly distributed RC interconnect with connection

In the models, the node subscripts are ordered from the output terminal to the input terminal, different from a normal way. The state vector $\mathbf{x}(t)$ is selected as the node voltages of the interconnect circuit

$$\mathbf{x}(t) = [v_n(t), \cdots, v_1(t)]^T \quad (1)$$

in Figs.1-2. The distributed RC interconnect circuits can be described by a linear state-space model:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \quad y(t) = \mathbf{C}\mathbf{x}(t) + Du(t), \quad (2)$$

where the state variable $\mathbf{x}(t) \in R^n$, the input variable $u(t) \in R$ and the output variable $y(t) \in R$.

Its transfer function from $V_{in}(s)$ to $V_o(s)$ is

$$T(s) = V_o(s)/V_{in}(s) = N(s)/D(s) \quad (3)$$

where $N(s)$ and $D(s)$ are polynomials of s, and $V_{in}(s) = V_s(s)$ in the model of Fig.2.

Suppose an interconnect has its "total resister" R_t and "total capacitor" C_t, where the quote is used because it is really distributed, not "total". Thus, an accurate model of an evenly disturbed RC interconnect circuit should be a very high *n-th*-order model for its distribution nature. Then, its RC evenly distributed models in Figs. 1-2 have

$$R = R_t/n, \; C = C_t/n \quad (4)$$

The ELO reduction method divides an interconnect into m-even-length divisions with the division parameters proportional to the division length. Thus, the *m-th* ELO model has its RC parameters as

$$R_m = R_t/m = nR/m = rR, \; C_m = rC, \; r = n/m \quad (5)$$

where r is the reduction ratio of the original model order to the ELO reduction model order.

The problems are: (i) to find an accurate state space model and a transfer function recursive algorithm for the interconnect; (ii) to derive its ELO model and characteristic relationship to the original model; (iii) to execute simulation; and (iv) to analyze results.

The ELO model is notated as $\{\mathbf{A}_{em}, \mathbf{B}_{em}, \mathbf{C}_{em}, D\}$ and $T_{em}(s) = N_{em}(s)/D_{em}(s)$, where m is the reduced model order with $m<n$. It is noticed that the *m-th*-order ELO model has a same type of structure as in Figs. 1-2, but it replaces the order n by the order m.

The ELO model performance evaluation is executed by comparison with its corresponding original accurate *n-th*-order model for (i) the step response, (ii) the Bode plot, and (iii) the H_∞-norm of their transfer function difference, i.e., ELO reduction error,

$$E_{em} = \|T(s) - T_{em}(s)\|_\infty = \max_{s=j\omega,\, \omega\in[0,\infty)} |T(s) - T_{em}(s)| \quad (6)$$

The performance criterion (6) is the worst error of $E_{em}(j\omega) = T(j\omega) - T_{em}(j\omega)$ over the whole frequency range. From the below theorems and case study on $\{\mathbf{A}_{em}, \mathbf{B}_{em}, \mathbf{C}_{em}, D\}$ and $T_{em}(s)$ of the ELO model, a suitable order m in practical cases can be found. Because the ELO model has a similar structure to its original one, but with order m, therefore an obvious benefit is its inherent passive implementation, compared with other model reduction methods.

III. ELO MODEL OF EVEN RC INTERCONNECT

Wang, *et al.* (2002) presented the closed form of the state space model and the recursive algorithm of the transfer function model for the evenly distributed RC interconnect circuit shown in Fig.1. Here, its ELO model is derived as Theorem 3.1 as follows.

Theorem 3.1. Consider an *n-th*-order evenly distributed RC interconnect circuit in Fig.1 with its total length resistor R_t and total capacitor C_t in (4). Its *m-th*-order ELO model $\{\mathbf{A}_{em}, \mathbf{B}_{em}, \mathbf{C}_{em}, D\}$ is

$$\mathbf{A}_{em} = [1/(r^2RC)]\cdot \mathbf{A}_{om}\,,\ \mathbf{B}_{em} = [1/(r^2RC)]\cdot[1\ \ 0\ \ \cdots 0]^T\,,\quad \mathbf{C}_{em} = [0\ \ 0\ \ \cdots\ \ 1],\ D=0\,, \tag{7}$$

where $\mathbf{A}_{em} \in R^{m\times m}$, $\mathbf{B}_{em} \in R^{m\times 1}$, $\mathbf{C}_{em} \in R^{1\times m}$,

$$\mathbf{A}_{om} = \begin{bmatrix} -2 & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 \\ 0 & 1 & -2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2 & 1 \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \in R^{m\times m} \tag{8}$$

the state variable vector $\mathbf{x}(t)$, the input variable $u(t)$ and the output variable $y(t)$ are respectively as

$$\mathbf{x}(t) \in R^m,\ u(t) = v_{in}(t)\,,\ y(t) = v_o(t)\,. \tag{9}$$

Its transfer function is $T_{em}(s) = N_{em}(s)/D_{em}(s)$,

$$N_{em}(s) = 1/(r^{2m}C^mR^m)\,,$$

$$D_{e,j}(s) = \left[s + \frac{2}{r^2CR}\right]D_{e,j-1}(s) - \frac{1}{r^4C^2R^2}D_{e,j-2}(s)\,,\quad j = 2,\cdots,m \tag{10}$$

with its initial values

$$D_{e,0}(s) = 1\,,\quad D_{e,1}(s) = s + [1/(r^2CR)]\,. \tag{11}$$

When $m=1$, it leads to that $r=n$,

$$N_{e1}(s) = 1/(n^2CR) \text{ and } D_{e1}(s) = s + [1/(n^2CR)]\,. \tag{12}$$

Due to the page limit, all proofs are omitted hereafter.

From Theorem 3.1, it is clear that the *m-th*-order ELO model has a similar structure to its original model in Wang (2002), but with an order *m*, not *n*, and a model order reduction factor $r = n/m$. It coincides with the ELO principle.

Theorem 3.2. The ELO model reduction error of an evenly-distributed RC interconnect or transmission line in (6) is related to its reduction factor *r* in (5), but independent of its parasitic parameters RC.

Theorems 3.1-3.2 reveal the characteristics of the ELO model and its relationship to the original model.

IV. ELO MODEL OF EVEN RC INTERCONNECT WITH CONNECTION

Theorem 4.1. Consider the distributed RC interconnect circuit with the source and load parts as shown in Fig. 2. Take the state variable vector $\mathbf{x}(t)$ in (1), the input variable $u(t)$ and the output variable $y(t)$ respectively as

$$u(t) = v_s(t)\,,\ y(t) = v_o(t) = v_1(t)\,. \tag{13}$$

Then, its state space model $\{\mathbf{A}, \mathbf{B}, \mathbf{C}, D\}$ is:

$$\mathbf{A} = (1/RC)\cdot \begin{bmatrix} -1-\frac{1}{1+R_s/R} & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 \\ 0 & 1 & -2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2 & 1 \\ 0 & 0 & 0 & \cdots & \frac{1}{1+C_0/C} & -\frac{1}{1+C_0/C}(1+\frac{R}{R_0}) \end{bmatrix}, \tag{14}$$

$$\mathbf{B} = \frac{1}{RC}\begin{bmatrix} \frac{1}{1+R_s/R} & 0\cdots 0 \end{bmatrix}^T,\ \mathbf{C} = [0\cdots 0\ 1]\,,\ D = 0\,, \tag{15}$$

where $\mathbf{A} \in R^{n\times n}$, $\mathbf{B} \in R^{n\times 1}$ and $\mathbf{C} \in R^{1\times n}$.

Its transfer function $T_n(s) = N_n(s)/D_n(s)$ has a recursive algorithm:

$$N_n(s) = 1/[(1+C_0/C)(1+R_s/R)C^nR^n]\,,$$

$$D_j(s) = \left[s + \frac{2}{CR}\right]D_{j-1}(s) - \frac{1}{C^2R^2}D_{j-2}(s)\,,\quad j = 2,\cdots,n-1\,,\ (n > 2) \tag{16}$$

$$D_n(s) = \left[s + \frac{1}{CR}(\frac{1}{1+R_s/R}+1)\right]D_{n-1}(s) - \frac{1}{C^2R^2}D_{n-2}(s)\quad (n \ge 2) \tag{17}$$

with the initial values

$$D_1(s) = s + \frac{1}{CR}\cdot\frac{1}{1+C_0/C}(1+\frac{1}{R_0/R})\,,\ D_0(s) = 1\,. \tag{18}$$

Now let us consider its *m-th*-order ELO model.

Theorem 4.2. The *m-th*-order ELO model with its source and load as shown in Fig. 2 has its state space model $\{\mathbf{A}_{em}, \mathbf{B}_{em}, \mathbf{C}_{em}, D\}$: $D=0$,

$$\mathbf{A}_{em} = (1/r^2RC)\cdot \begin{bmatrix} -1-\frac{1}{1+R_s/rR} & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 \\ 0 & 1 & -2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2 & 1 \\ 0 & 0 & 0 & \cdots & \frac{1}{1+C_0/rC} & -\frac{1}{1+C_0/rC}(1+\frac{rR}{R_0}) \end{bmatrix}$$

$$\mathbf{B}_{em} = \frac{1}{r^2RC}\begin{bmatrix} \frac{1}{1+R_s/rR} & 0\cdots 0 \end{bmatrix}^T,\ \mathbf{C}_{em} = [0\cdots 0\ 1]\,. \tag{19}$$

Its transfer function is $T_{em}(s) = N_{em}(s)/D_{em}(s)$,

$$N_{em}(s) = 1/[r^{2m}(1+C_0/rC)(1+R_s/rR)C^mR^m]\,,$$

$$D_{e,j}(s) = \left[s + \frac{2}{r^2CR}\right]D_{e,j-1}(s) - \frac{1}{r^4C^2R^2}D_{e,j-2}(s)\,,\quad j = 2,\cdots,m-1\,,\ (m > 2) \tag{20}$$

$$D_{em}(s) = \left[s + \frac{1}{r^2CR}(\frac{1}{1+R_s/rR}+1)\right]D_{e,m-1}(s) - \frac{1}{r^4C^2R^2}D_{e,m-2}(s)\quad (m \ge 2) \tag{21}$$

with the initial values

$$D_{e,0}(s)=1\,,\; D_{e,1}(s)=s+\frac{1}{r^2RC}\cdot\frac{1}{1+C_0/rC}(1+\frac{rR}{R_0}) \quad (22)$$

where $r=n/m$. When $m=1$, it leads to that $r=n$,

$$N_{e1}=\frac{1}{(C_t+C_0)(R_t+R_s)}=\frac{1}{n^2CR(1+C_0/nC)(1+R_s/nR)}$$

$$D_{e1}(s)=s+\frac{1}{C_t+C_0}(\frac{1}{R_s+R_t}+\frac{1}{R_0}) \quad (23)$$

Remark 4.1. The above theorems show that the ELO model with the source and load parts depends on its parameter ratios of its distribution parameters to external parameters respectively as R/R_s, R/R_0, C/C_0, (R_t/R_s, R_t/R_0, C_t/C_0), and the order reduction ratio r.

Remark 4.2. The case of a distributed interconnect itself (Theorem 3.1) is really an extreme case of a distributed interconnect with its source and load parts (Theorem 4.2) as its source resistance $R_s \to 0$, load capacitance $C_0 \to 0$ and load resistance $R_0 \to \infty$, i.e., $1/R_0 \to 0$. This extreme case is an important case without any external distortion.

There are two extreme situations or cases: one is an interconnect itself without any distortion as mentioned above, which returns to the results in Theorem 3.1; another one is with large external parameters as discussed below. A regular case will be between these two extreme cases.

Corollary 4.3. Consider the distributed RC interconnect circuit in Fig. 2 with the dominant source and load parts

$$R/R_0 \approx 0\,,\; C/C_0 \approx 0\,,\; R/R_s \approx 0\,. \quad (24)$$

Then, its state space model { $\mathbf{A},\mathbf{B},\mathbf{C},D$ } is:

$$\mathbf{A}=\frac{1}{RC}\begin{bmatrix} -1 & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 \\ 0 & 1 & -2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2 & 1 \\ 0 & 0 & 0 & \cdots & C/C_0 & -C/C_0 \end{bmatrix},\; \mathbf{B}=\frac{1}{R_sC}\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\mathbf{C}=\begin{bmatrix}0 & 0 & \cdots & 1\end{bmatrix},\; D=0\,, \quad (25)$$

where $\mathbf{A}\in R^{n\times n}$, $\mathbf{B}\in R^{n\times 1}$ and $\mathbf{C}\in R^{1\times n}$. Its transfer function $T_n(s)=N_n(s)/D_n(s)$ has the following recursive algorithm:

$$N_n(s)=1/(C_0R_sC^{n-1}R^{n-1})\,,$$

$$D_j(s)=\left[s+\frac{2}{CR}\right]D_{j-1}(s)-\frac{1}{C^2R^2}D_{j-2}(s)\,,$$

$$j=2,\cdots,n-1\,,\; (n>2) \quad (26)$$

$$D_n(s)=\left[s+\frac{1}{CR}\right]D_{n-1}(s)-\frac{1}{C^2R^2}D_{n-2}(s)\,,\; n\ge 2 \quad (27)$$

with the initial values

$$D_0(s)=1\,,\quad D_1(s)=s+[1/(C_0R)]\,. \quad (28)$$

Corollary 4.4. The *m-th*-order ELO model in Fig. 2 with its dominant source and load

$$R_t/R_0 \approx 0\,,\; C_t/C_0 \approx 0\,,\; R_t/R_s \approx 0\,, \quad (29)$$

has its state space representative:

$$\mathbf{B}_{em}=(1/rR_sC)\cdot[1\;\; 0\cdots 0]^T\,,\; \mathbf{C}_{em}=[0\;0\cdots 1]\,,\; D=0\,,$$

$$\mathbf{A}_{em}=\frac{1}{r^2RC}\begin{bmatrix} -1 & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 \\ 0 & 1 & -2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2 & 1 \\ 0 & 0 & 0 & \cdots & \frac{rC}{C_0} & -\frac{rC}{C_0} \end{bmatrix}, \quad (30)$$

Its transfer function is $T_{em}(s)=N_{em}(s)/D_{em}(s)$,

$$N_{em}(s)=1/(r^{2m-2}C_0R_sC^{m-1}R^{m-1})\,,$$

$$D_{e,j}(s)=\left[s+\frac{2}{r^2CR}\right]D_{e,j-1}(s)-\frac{1}{r^4C^2R^2}D_{e,j-2}(s)\,,$$

$$j=2,\cdots,m-1\,,\; (m>2) \quad (31)$$

$$D_{em}(s)=\left[s+\frac{1}{r^2CR}\right]D_{e,m-1}(s)-\frac{1}{r^4C^2R^2}D_{e,m-2}(s)\,,$$

$$(m\ge 2) \quad (32)$$

with the initial values

$$D_{e,0}(s)=1\,,\; D_{e,1}(s)=s+[1/(rRC_0)] \text{ for } m>1 \quad (33)$$

where $r=n/m$. When $m=1$, it leads to that $r=n$,

$$N_{e1}=\frac{1}{C_0R_s} \text{ and } D_{e1}(s)=s+\frac{1}{C_0}(\frac{1}{R_s}+\frac{1}{R_0}) \quad (34)$$

Theorems 3.1, 3.2 and 4.2 reveal the characteristics of the ELO models and their respective relationship to their original distributed RC interconnect models. Corollaries 4.3 and 4.4 present the corresponding results for an extreme case of interconnect with dominant external parameters, while Theorem 3.1 is for the interconnect without any distortion of external parameters.

V. CASE STUDIES

In this section two cases are studied. One is an interconnect itself without the distortion to its characteristics. Another one is an interconnect with its dominant source and load parts.

Case 1: Consider an interconnect of 0.01cm long with the distribution characteristic data of resistor parameter 5.5kΩ/m and capacitor parameter 94.2pf/m. An 100th-order evenly distributed RC interconnect model is used as its original model with $R = 5.5\cdot 10^{-3}\ \Omega$ and $C = 9.42\cdot 10^{-5}$ pF. From Corollary 3.3 (Wang *et al.*, 2002) and Theorem 3.1, the original model { $\mathbf{A},\mathbf{B},\mathbf{C},D$ }, its *m-th*-order ELO model $\{\mathbf{A}_{em},\mathbf{B}_{em},\mathbf{C}_{em},D\}$ and transfer function $T_{em}(s)=N_{em}(s)/D_{em}(s)$, $m=1,2,3$, are as follows:

$$\mathbf{A}=1.9301\cdot 10^{18}\ \mathbf{A}_{o,100}\,,\mathbf{B}=1.9301\cdot 10^{18}[1\;0\cdots 0]^T\,,$$

$$\mathbf{C}=\begin{bmatrix}0\cdots 0 & 1\end{bmatrix},\; D=0\,,$$

where $\mathbf{A}_{o,100}$ is in (8) with $m=n=100$;

$$A_{e1}=-1.9301\cdot 10^{14}\,,\; B_{e1}=-1.9301\cdot 10^{14}\,,\; C_{e1}=1\,;$$

$$\mathbf{A}_{e2}=10^{14}\begin{bmatrix} -15.441 & 7.7205 \\ 7.7205 & -7.7205 \end{bmatrix},\; \mathbf{B}_{e2}=10^{14}\begin{bmatrix} 7.72057 \\ 0 \end{bmatrix},$$

$$\mathbf{C}_{e2}=[0\;\;1];$$

$$\mathbf{A}_{e3}=10^{15}\begin{bmatrix} -3.4742 & 1.7371 & 0 \\ 1.7371 & -3.4742 & 1.7371 \\ 0 & 1.7371 & -1.7371 \end{bmatrix},\; \mathbf{B}_{e3}=10^{15}\begin{bmatrix} 1.7371 \\ 0 \\ 0 \end{bmatrix},$$

$$\mathbf{C}_{e3} = [0 \quad 0 \quad 1];$$

$$N_{e1} = 1.9301 \cdot 10^{14}, \ D_{e1}(s) = s + 1.9301 \cdot 10^{14};$$

$$N_{e2} = 5.9606 \cdot 10^{29}$$

$$D_{e2}(s) = s^2 + 2.3162 \cdot 10^{14} \cdot s + 5.9606 \cdot 10^{29};$$

$$N_{e3} = 5.2419 \cdot 10^{45},$$

$$D_{e3}(s) = s^3 + 8.6856 \cdot 10^{15} s^2 + 1.8105 \cdot 10^{31} s + 5.2419 \cdot 10^{45}$$

Their approximation errors in (6) are as follows:

$$E_{e1} = -9.0114\ db, \ E_{e2} = -12.6492\ db,$$

$$E_{e3} = -15.5285\ db.$$

Figure 3 shows the step responses of the original model and its ELO models with $m = 1, 2, 3, 5, 10, 25$. The steepest curve is of the original model with the fastest rising time. The ELO models have slow step responses. The lower the ELO model order is, the larger the rising time is. The step response curves of the 1^{st}, 2^{nd} and 3^{rd} order ELO models are the far right three curves respectively. It is observed that all ELO models have a same final value as the original model has, i.e., their DC values are the same. The time at which the step response reaches 0.9 is listed as a rising time in Table 1.

Figures 4-5 display their Bode plots. It is seen from Fig.4 that the ELO models coincide well with the original one in a very large range of frequencies, especially in low frequencies. It is reflected in the time domain as that they have the same DC value (i.e., final value) in the step responses in Fig.3. However, their Bode plots are different in the high frequency range, and correspondingly they have different rising time in the time domain, i.e., their transient responses are different. The original model shows an increasing suppression as the frequency increases above 10^{16} Hz. However, the low-order ELO models can not follow this property well when the frequency is above a certain frequency, which is called the discord (or separate) frequency f_d of model approximation. The reason to have their f_d is that they have different roll-off rates as $-20m$ db/dec in high frequency range from the theory, where m is the ELO model order. Therefore, the higher the ELO model order is, the higher its discord frequency f_d is. It is also noticed that the lower order model has a narrower bandwidth, while the original model has the largest bandwidth of $7.39 \cdot 10^{13}$ Hz. The bandwidths and the discord frequencies are listed in Table 1 as shown in Figs. 4-5.

From Table 1 and Figs. 3-5, it is observed that at least the 5th order or a higher 10^{th} order model is required for a sufficiently good approximation to the original (100^{th}-order) model for any various source and load parts in view of the time domain responses and frequency domain characteristics.

Case 2: Consider the above interconnect with dominated external parameters: $R_s = 500\Omega$, $R_0 = 1M\Omega$ and $C_0 = 1pF$. Table 2 lists the performance of its ELO models. It shows that the 1^{st} or the 2^{nd} order ELO model can approximate the original model well. However, the required ELO model order for a very good model approximation depends on various detail source and load data. When the source resistance and load capacitance reduce and the load resistance increases, the ELO model order should be larger than one in order to reach a good approximation.

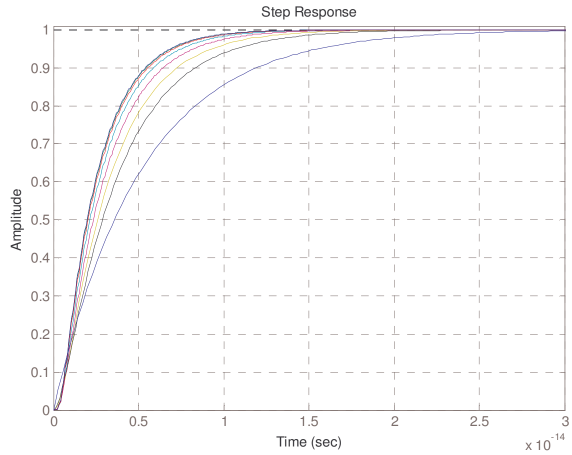

Fig. 3. Step responses of original and its ELO models

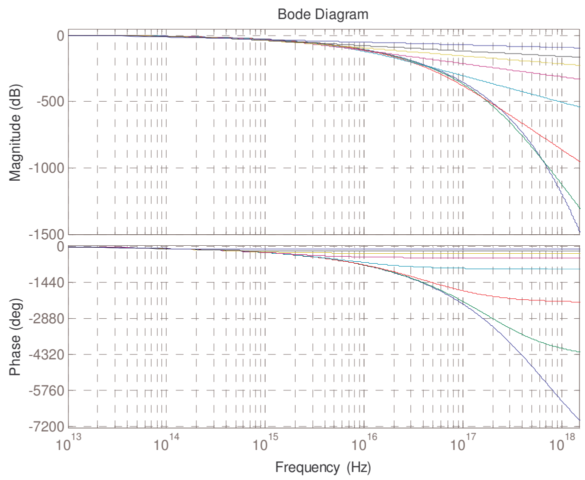

Fig.4. Bode plots of the original model and its ELO models

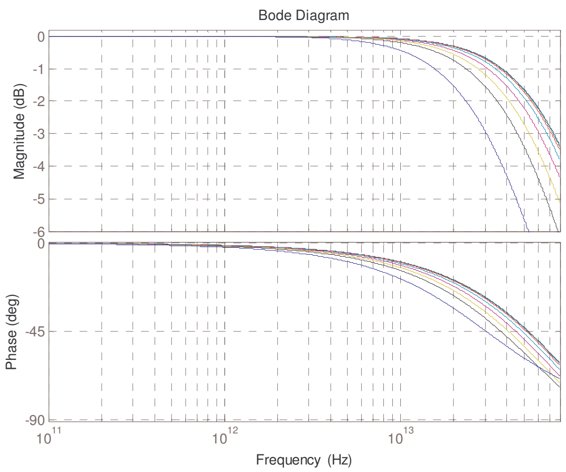

Fig. 5. Bandwidths of original model and its ELO models

The above cases reflect Theorem 3.1 and Corollary 4.4, i.e., two extreme cases of interconnects: without external distortion and with dominated external ports.

VI. CONCLUSIONS

This paper presents two evenly distributed RC interconnect models and their corresponding ELO (even length order) reduction models by the closed-forms of the state space models in the time domain and the recursive algorithms of the transfer function models in the frequency domain. One is an interconnect itself and another is an interconnect together with its source resistor, load resistor and capacitor. The new closed-form formulas and recursive algorithms are very efficient and powerful for model reduction in avoiding operation of large-dimension matrix inverse and matrix multiplication. The model closed-forms have only O(*1*) computation complexity. The theorems and corollaries reveal the characteristics of the ELO models and their relationship to the accurate high-order original model.

The ELO model reduction method of the evenly distributed RC interconnects is thoroughly investigated in the paper. It is revealed that ELO model reduction is independent of its parasitic parameters of RC.

The case studies are executed in both time domain and frequency domain. The ELO model approximation error, discord frequency and rising time are investigated. The results show that a very high order RC interconnect itself, such as 100th-order, can be accurately approximated by its only 5th–10th or higher order ELO model. When the source and load ports are included, the whole model can be well approximated by a lower order ELO model, e.g., only the first or second if the external parameters are dominant. However, for a good approximation, the ELO model order depends on their parameter ratios. More important is from Theorem 3.2 that this case study conclusion is really valid for any various parasitic parameters.

The ELO model is simple and can be used for the RC interconnect model reduction with the availability of the new model closed-forms and recursive algorithms. The ELO model has its inherent passive character. The results are useful not only for VLSI RC interconnect model reduction, but also for control systems with a (long) distributed transmission line.

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Table 1. Performance comparison of ELO model reduction in Case 1

Performance	Original	ELO order 1	ELO order 2	ELO order 3	ELO order 5	ELO order 10	ELO order 25	ELO order 50
Rising t (10^{-15} sec)	5.40	11.9	8.36	7.29	6.51	5.90	5.56	5.45
Discord f_d ($10^{15}Hz$)		1.20	3.12	6.36	15.1	46.0	274	732
Bandwidth (10^{13}Hz)	7.39	3.06	4.59	5.36	6.11	6.74	7.16	7.30
H-inf Error E_{em}		0.35435	0.23310	0.16733	0.10626	$8.8609 \cdot 10^{-15}$	$8.5840 \cdot 10^{-15}$	

Table 2. Performance comparison of ELO model reduction in Case 2

Performance	Original	ELO order1	ELO order 2	ELO order 3	ELO order 5	ELO order 10	ELO order 25	ELO order 50
Rising t (10^{-9} sec)	1.16	1.16	1.16	1.16	1.16	1.16	1.16	1.16
Discord f_d (10^{14}Hz)		1.68	4.54	13.8	40.4	243	$2.18 \cdot 10^3$	$6.99 \cdot 10^3$
Bandwidth (10^8 Hz)	3.14	3.14	3.14	3.14	3.14	3.14	3.14	3.14
H-inf Error (10^{-6})		5.0407	2.4905	1.6425	0.9647	0.45681	$4.3694 \cdot 10^{-5}$	