COMPLETE SETS OF BASIC TYPES OF 2DOF TRACKING CONTROLLERS WITH FINITE LENGTH CONTROL SEQUENCES [1]

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Abstract: The paper deals with some specific feedback solutions to tracking problems for discrete linear SISO systems, characterized by a finite length control sequences (FLC) and by an infinite or a finite length error sequences. A complete set of all 2DoF FLC controllers for plants exposed to general disturbances (2DoF GFLC) is determined and it is shown that this set embeds the set of deadbeat ripple-free controllers (2DoF GDBRF), which moreover produce a finite length error sequences.

Keywords: Finite length control; Dead-beat ripple-free tracking; Disturbance rejection; parametrization; optimization

1. INTRODUCTION

In an algebraic approach to controller design, the plant is described by a transfer function and external signals usually by a known polynomial fractions characterizing a nominal situation. The main goal is to find a controller that guarantees the desired finite length sequences and internal BIBO stability of the closed-loop system. In state space formulation, this requirement corresponds to finding a finite length sequence only for one initial condition, though we usually require to solve the problem globally, i.e. for all possible initial conditions of the plant and generators of external signals (Grasselli *et al.*, 1995). This way formulated problem appears in literature rarely, e.g. in (Kučera and Kraus, 1995), where various initial conditions are modelled by an unknown finite length external signal effecting on generators. In the presented paper, various initial conditions are alternatively modelled by undetermined polynomials in denominators of the polynomial fractions of generators. Thus, respecting the globality, there can be formulated GFLC and GDBRF tasks. If state space models are used, there is no need to express the disturbance model explicitly, since it is the uncontrollable part of the controlled system. Generally, the whole surrounding of the controller can be modelled without explicit models of its parts by means of an augmented system with defined control input, error (controlled variable) and output (measuring variable) (Kučera and Kraus, 1995).

For a correct problem formulation, it is necessary to specify the sets of controllers capable to attain the desired goals. Causality, linearity and time invariance of controllers may be viewed as natural conditions. Leaving aside state controllers, two

[1] The authors gratefully acknowledge the financial support through the grant MSM 235200004 under the Ministry of Education of Czech Republic and the grant GAČR 102/02/0425 under the Grant Agency of the Czech Republic.

basic controller's structures 1DoF and 2DoF can be used to solve the formulated tasks.

Solution to 1DoF and 2DoF FLC problems was given in (Fikar and Kučera, 2000), however, without considering disturbances. They obtained necessary and sufficient conditions for the existence of the solution to 1DoF and 2DoF FLC problems, however, they were incorrect in case of a "non-causal" reference generator in the sence of (Kučera, 1979). Further, the complete set of 2DoF FLC controllers was not obtained and thus 1DoF controllers surprisingly were not a special case of 2DoF controllers. The complete set of 2DoF DB controllers for plants exposed to disturbances was specified by Chang in (Chang, 1998). The design of 1DoF DBRF controllers can be found e.g. in (Barbargires and Karybakas, 1994; Mošna *et al.*, 2001). The algebraic solution to 1DoF GDBRF problem is given in (Kučera and Kraus, 1995). In the state space setting, the design of GDBRF minimum step controllers with measured error was shown in (Grasselli *et al.*, 1995; Jetto, 1994), however, with some incorrectness in derived necessary and sufficient solvability conditions. Our paper specifies the complete set of all 2DoF FLC controllers and embeds into this set the parametrized sets of all 2DoF GFLC and GDBRF controllers.

2. NOTATION

The notation and notions used in the paper are standard and result from (Fikar and Kučera, 2000; Kučera, 1979). All systems are considered to be single input–single output, linear, time–invariant and discrete time, described by polynomial fractions in indeterminate z^{-1}, used in the $\mathcal{Z}$–transform as a delay operator. Polynomials in z^{-1} are denoted by capital letters without indication of their argument. A polynomial $X(z^{-1})$ is called "causal", if and only if $X(0) \neq 0$ and stable, if and only if the absolute values of all its roots are greater than one. The greatest stable factor of a polynomial X is denoted X^+ and totally unstable factor of a polynomial X is denoted X^-. Every stable polynomial X is a "causal" polynomial. The greatest common divisor of the polynomials X, Y is marked by the parenthesis (X, Y). Remind that a recurrent sequence, given by a polynomial fraction Y/X, $(X, Y) = 1$, is stable, if and only if the greatest causal factor of the polynomial X is a stable polynomial. If a finite length causal control sequence is desired, then the $\mathcal{Z}$–transform of u must be a polynomial U.

3. PROBLEM FORMULATION

Consider a plant with output y, input u and disturbance v described by

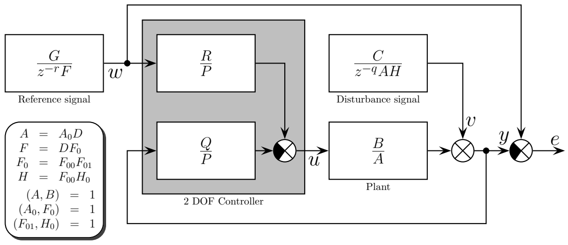

Fig. 1. Feedback closed-loop configuration

$$y = \frac{B}{A} \cdot u + v \tag{1}$$

where $(A, B) = 1$, $A(0) \neq 0$, $B(0) = 0$ and the generator of a general disturbance v, given by

$$v = \frac{C}{z^{-q} AH} \tag{2}$$

where q is a non-negative integer, $H(0) \neq 0$. In case $(A, C) = A$, the disturbance corresponds to an output plant disturbance and in case $(B, C) = B$ the disturbance is an input plant disturbance. In case $H = 0$, the disturbance can be viewed either as a finite length disturbance acting on the plant or as an infinite length disturbance, if its character complies with the internal model principle. The generator of a reference signal w is given by

$$w = \frac{G}{z^{-r} F} \tag{3}$$

where r is a non-negative integer, $F(0) \neq 0$. Hereafter we denote $D = (A, F)$ and polynomials A_0 and F_0, so that $A = A_0 D$ and $F = F_0 D$. Further we denote $(F_0, H) = F_{00}$ and polynomials F_{01} and H_0, so that $F_0 = F_{00} F_{01}$ and $H = F_{00} H_0$. Remark that the augmented system, obtained by aggregation of the plant(1) and generators (2), (3), is a complete model of controller's surrounding. Polynomials C resp. G of external signals generators can also be interpreted as the $\mathcal{Z}$–transforms of an initial state of the augmented system. An important point to be noticed is that the part of the state representation of the augmented system, which is observable by error $e = w - y$, is stabilizable resp. controllable if and only $H = H^+$, $F_0 = F_0^+$ resp. $H = 1$, $F_0 = 1$.

Now, consider the set of 2DoF causal controllers (Fig.1), described by $u = \boldsymbol{d}_f \cdot w - \boldsymbol{d}_b \cdot y$ where $\boldsymbol{d}_f$ and $\boldsymbol{d}_b$ are polynomial fractions. Note that the controller (4) is causal, if and only if there exists a causal polynomial P, $(P(0) \neq 0)$, such that

$$\boldsymbol{d}_f = \frac{R}{P} \qquad \boldsymbol{d}_b = \frac{Q}{P} \tag{4}$$

We will investigate three fundamental variants of a finite length stabilizing control problem, differing by requirements on closed–loop system behavior. The least demanding is a standard formulation of the problem (Fikar and Kučera, 2000). The solution to a standard finite length stabilizing control problem consists in finding a 2DoF causal controller such that the closed-loop system is internally BIBO stable, and, in a nominal situation, i.e. for a given $G = G_{nom}$ and $C = C_{nom} = 0$, the controller produces a finite control sequence $u = U$ and a stable error sequence $e = w - y$. Without loss of generality, we suppose that $(G_{nom}, z^{-r}F) = 1$. Such controller is denoted as the 2DoF FLC controller. A finite length control sequence and a stable error sequence are guaranteed just for a nominal situation, however, if reality differs from this situation, a generic fulfilment of the closed–loop system properties cannot be awaited. Thus a natural question appears, whether there exists a 2DoF FLC controller generating a finite length control sequence and a stable error sequence globally, i.e. for all G and C. Such controller will be denoted as a 2DoF GFLC controller.

The important subset of 2DoF GFLC controllers are such controllers which besides a finite length control sequence moreover produce also a finite length error sequence for arbitrary polynomials G and C. This version of dead–beat control plays an important role in sampled data control. Recall that in the discrete control of continuous systems, sampled with the frequency which is not in resonance with any of the damped frequencies of the plant, the ripples, caused by the controller at the controlled output, vanish in a finite time if and only if a finite length error sequence is guaranteed. Such controllers are called deadbeat ripple-free (DBRF) controllers and 2DoF GFLC controllers showing this properties globally for all G and C are called 2DoF GDBRF controllers.

The main goal is to find the set of all 2DoF FLC controllers and its practically important subsets of 2DoF GFLC and 2DoF GDBRF controllers. If the control sequence $u = U$ has the minimum length, we investigate the minimum step versions of these controllers.

Note that the accepted problem formulation admits that the denominator in (2), (3) can be chosen as a noncausal polynomial $(q, r > 0)$. In such cases the reference signal or a disturbance takes non-zero values even at the time instants with negative indices, in which, however, the requirement $u = U$ necessitates zero control. In this respect the paper (Fikar and Kučera, 2000) comes to a wrong conclusion that there exists no solution to the problem. This mistake is obviously caused by the simplification of the relationship between the stability of a polynomial and the stability of corresponding (noncausal) recurrent sequence. It is shown that the solution exists even in such a case, i.e. when a 2DoF controller responds with a r-step delay to a reference signal and with a q-step delay to a disturbance.

4. SET OF ALL 2DOF FLC CONTROLLERS

The following theorem specifies the set of all 2DoF FLC controllers, guarantees causality of controller and for internally BIBO stable closed-loop system a finite length control and stable error sequences in nominal situation, characterized by $G_{nom} = G$, $C_{nom} = 0$ and $(G_{nom}, z^{-r}F) = 1$.

Theorem 1: (2DoF FLC controller) A 2DoF controller (4) is a 2DoF FLC controller if and only if there exist polynomials L, X, Y, V and Z satisfying the relations

$$L = L^{+} \tag{5}$$

$$A_0^{-}DX + BY = G^{+}L \tag{6}$$

$$F^{-}V + z^{-r}BF_0Z = A_0^{+}G^{+} \tag{7}$$

and giving identities

$$\boldsymbol{d}_f = \frac{z^{-r}F_0LZ}{X} \qquad \boldsymbol{d}_b = \frac{A_0^{+}Y}{X} \tag{8}$$

The finite length control sequences and stable error sequences are then given in a nominal situation and for all arbitrary stable polynomials L by

$$U = A_0^{-}G^{-}Z \qquad e = \frac{G^{-}V}{z^{-r}A_0^{+}F^{+}} \tag{9}$$

Proof: Assume $(A, B) = 1$ and $B(0) = 0$. It is known that a 2DoF controller (4) is causal and the closed-loop system is internally BIBO stable if and only if there exist polynomials P, Q a M such that

$$AP + BQ = M, \quad M = M^{+} \tag{10}$$

Denoting $(A_0, M) = A_1$ and $(G, M) = G_1$, we may assume without loss of generality that

$$M = A_1G_1L \tag{11}$$

The polynomial M is stable if and only if

$$A_1 = A_1^{+}, \quad G_1 = G_1^{+}, \quad L = L^{+} \tag{12}$$

hold. It is obvious from (12) that there always exist polynomials A_2 resp. G_2 such that $A_0^{+} = A_1A_2$ resp. $G^{+} = G_1G_2$. Using the polynomial A_2G_2 for the extension of the controller's polynomial fractions (4) and for multiplication of the identity

(10), the necessary and sufficient conditions (10) can equivalently be expressed as

$$AP + BQ = A_0^+ G^+ L, \qquad L = L^+ \quad (13)$$

Since $(A, A_0^+) = A_0^+$ and $(A, B) = 1$ hold, the polynomial Q satisfies the identity (13) if and only if there exists a polynomial Y such that $Q = A_0^+ Y$. Substiting Q into (13), using $A = DA_0^- A_0^+$ and dividing the equation by the polynomial A_0^+, we obtain equivalently to (13) relations (5) and (6), where $X = P$. After the substitution of P and Q into (4), it is clear that a 2DoF controller is causal and the closed-loop system is internally BIBO stable if and only if there exist such polynomials X, Y, and L, satisfying (5) and (6), such that identities (8) are fulfilled.

Using (5), (6) for internally BIBO stable closed-loop system with a feedback resp. feedforward controller (8) resp. (4), we obtain the control sequence

$$u = \frac{A_0^- R G^-}{z^{-r} L F_0} \quad (14)$$

in a nominal situation. Since $A_0(0) \neq 0$ and $(A_0, F_0) = 1$, we deduce that $(A_0^-, z^{-r} F_0) = 1$. Since $L = L^+$, $(A_0^-, L) = 1$ and $(G^-, L) = 1$ hold. Finally, $(G, z^{-r} F) = 1$ implies that $(G^-, z^{-r} F_0) = 1$. Thus a finite length control sequence $u = U$ can be obtained if and only if $(R, z^{-r} F_0) = z^{-r} F_0$ hold and the controller (8) produces $u = U$ if and only if there exist polynomials R and Z such that

$$R = z^{-r} F_0 L Z \quad (15)$$

Substituting necessary and sufficient condition of a finite length nominal control sequence (15) into (14) and into feedforward controller (4), we obtain feedforward controller (8) and control sequence (9).

Using (5), (6) and (8), we find that each closed-loop internally BIBO stable system, producing a nominal finite length control sequence, gives rise to a nominal, though possibly unstable error sequence

$$e = \frac{A_0^+ G^+ - z^{-r} B F_0 Z}{A_0^+} \; \frac{G^-}{z^{-r} F} \quad (16)$$

Recalling that $(A_0^+, G^-) = 1$, $(G^-, z^{-r} F) = 1$, $F(0) \neq 0$, the error sequence e is stable if and only if there exists a polynomial V such that the nominator in the first polynomial fraction in (16) equals $F^- V$. This condition directly yields (7). We then obtain from (16) a nominal finite length control sequence (9). □

Corollary 1: A 2DoF FLC controller exists if and only if the polynomial F_0 is stable ($F_0 = F_0^+$). This contradicts the statement in (Fikar and Kučera, 2000) that the solution to FLC problem exists if and only if (in our notation) $z^{-r} F_0$ is stable. This statement is evidently false for $r > 0$, since this polynomial is non–causal and thus unstable in such case.

Our statement follows from Theorem 1. A FLC controller exists, if and only if there exist solutions to equations (6) and (7). The equation (6) has always solution for assumed $(A, B) = 1$. In order the equation (7) may have a solution, it must hold $F_0^- = 1$, since $(F^-, F_0) = F_0^-$ and $(F_0^-, A_0^+ G^+) = 1$. If this is the case, then $F^- = D^-$ and $(F^-, B) = 1$. Therefore, the solution to the equation (7) always exists for $F_0^- = 1$.

Remark 1: The 2DoF FLC controller guarantees a finite length control sequence and stable error sequence only for a nominal situation. If this supposition is not fulfilled, we obtain for the closed-loop system from (5)–(8)

$$u = \frac{A_0^- G Z}{G_{nom}^+} - \frac{C Y}{z^{-q} G_{nom}^+ H L^+} \quad (17)$$

$$e = \frac{G V}{z^{-r} A_0^+ F^+ G_{nom}^+} - \frac{C X}{z^{-q} A_0^+ G_{nom}^+ H L^+} \quad (18)$$

It follows from (17) and (18) that each 2DoF FLC controller guarantees stable error and control sequences globally for all G and C, if $H = H^+$.

5. SPECIFIC SUBSETS OF THE SET OF 2DOF FLC CONTROLLERS

In consequence of Theorem 1, we have obtained a tool for the specification of some specific subsets of the set of 2DoF FLC controllers. At first, we determine the subset of all 2DoF FLC controllers guaranteeing a finite length control sequence and stable error sequence globally for all polynomials G and C. These controllers are denoted 2DoF GFLC controllers.

Theorem 2: (2DoF GFLC controller) A 2DoF controller is a 2DoF GFLC controller, if and only if there exist polynomials X_1, Y_1, V_1 and Z_1 satisfying the relations

$$A^- H^- X_1 + z^{-q} B H Y_1 = 1 \quad (19)$$

$$F^- V_1 + z^{-r} B F_0 Z_1 = A_0^+ \quad (20)$$

and giving identities

$$\boldsymbol{d}_f = \frac{z^{-r} F_0^- F^+ Z_1}{H^- X_1} \qquad \boldsymbol{d}_b = \frac{z^{-q} A^+ H Y_1}{H^- X_1} \quad (21)$$

Thus, we obtain for all polynomials G and C

$$
\begin{aligned}
U &= A_0^- G Z_1 - C Y_1 \\
e &= \frac{G V_1}{z^{-r} A_0^+ F^+} - \frac{C X_1}{z^{-q} A^+ H^+}
\end{aligned} \tag{22}
$$

Proof: It follows from (17) that control sequence is finite and causal for all G and C if and only if there exist polynomials Z_1 and Y_1 (for arbitrary chosen G_{nom} and L) such that $Z = G_{nom}^+ Z_1$ and $Y = z^{-q} G_{nom}^+ H L Y_1$ and the relation (18) imply that error is a stable sequence if and only if $X = H^- X_h$.

Using Theorem 1, we can state that a 2DoF controller is a 2DoF GFLC controller if and only if it satisfies to Theorem 1, where polynomials Z and Y moreover satisfy relations $Z = G^+ Z_1$ and $Y = z^{-q} G^+ H L Y_1$.

Since $(G^+, F^-) = 1$ holds, it directly follows from (7) that $Z = G^+ Z_1$ holds for 2DoF FLC if and only if there exists also V_1 such that $V = G^+ V_1$. Substituting

$$
V = G^+ V_1, \quad Z = G^+ Z_1 \tag{23}
$$

into (7) and dividing this equation by the polynomial G^+, we obtain (20).

Now, we turn our attention to the conditions $Y = z^{-q} G^+ H L Y_1$ and $X = H^- X_h$. For future convenience, we denote $(D^+, L) = D_L^+$, $D^+ = D_L^+ D_0^+$ and $L = D_L^+ L_0$, where $L_0 = L_0^+$. After the substitution of Y, D^+ and L into (6) and using relations $D = D^- D^+$ and $A^- = D^- A_0^-$, we find that solutions of the equation always exist in the form $X = G^+ H^- L_0 X_0$, because $(D_0^+, L_0) = 1$, $(D_0^+, G^+) = 1$ and $(A_0^- H^-, G^- L_0) = 1$. Further, after the substitution of X, and division the equation by the polynomial $D_L^+ G^+ L_0$, we get the following identity for X_0 and Y_1

$$
A^- D_0^+ H^- X_0 + z^{-q} B H Y_1 = 1 \tag{24}
$$

$$
\begin{aligned}
X &= G^+ H^- L_0 X_0, \\
Y &= z^{-q} D_L^+ G^+ H L_0 Y_1, \\
L &= D_L^+ L_0
\end{aligned} \tag{25}
$$

where D_0^+ is an arbitrary factor of the polynomial D^+. Substituting L, X and Y into (8), we see that the factor L_0 of the polynomial L cancels out in these relations and therefore, its choice does not affect the behavior of the controller. From formal reasons, we choose it so that it holds $L_0 = D_0^+ L_1$ and thus $L = D^+ L_1$. Then $A_0^+ L = A^+ L_1$ and $F_0 L = F_0^- F^+ L_1$ hold, and relations (8) are given by

$$
\boldsymbol{d}_f = \frac{z^{-r} F_0^- F^+ Z_1}{H^- D_0^+ X_0} \qquad \boldsymbol{d}_b = \frac{z^{-q} A^+ H Y_1}{H^- D_0^+ X_0} \tag{26}
$$

Denoting $D_0^+ X_0 = X_1$ and substituting it into (24)and (26), we get (19) and (21). Then relations (27) have the form

$$
\begin{aligned}
X &= G^+ H^- L_1 X_1, \\
Y &= z^{-q} D^+ G^+ H L_1 Y_1, \\
L &= D^+ L_1
\end{aligned} \tag{27}
$$

Notice that each solution to the equation (24) is also given by the solution to the equation (19) for $X_1 = D_0^+ X_0$. If we put $D_0^+ = 1$, this equation corresponds to (24) and thus the set of solutions (19) corresponds to the unification of the sets of solutions (24), obtained for respective factors D_0^+.

Substituting $D_0^+ X_0 = X_1$ into (26), we get (21). Finally, we obtain the relation (22) from (17), (18) for $L = D^+ L_1$. □

Now, we determine the set of 2DoF GDBRF controllers guaranteeing globally both a finite length control and a finite length error sequence.

Theorem 3: (2DoF GDBRF controller) A 2DoF controller is a 2DoF GDBRF controller if and only if there exist polynomials X_2, Y_2, V_2 and Z_2 satisfying relations

$$
A H X_2 + z^{-q} B H Y_2 = 1 \tag{28}
$$

$$
F V_2 + z^{-r} B F_0 Z_2 = 1 \tag{29}
$$

and giving identities

$$
\boldsymbol{d}_f = \frac{z^{-r} F_0 Z_2}{H X_2} \qquad \boldsymbol{d}_b = \frac{z^{-q} H Y_2}{H X_2} \tag{30}
$$

The finite length sequences are then given by

$$
U = A_0 G Z_2 - C Y_2 \qquad e = \frac{G V_2}{z^{-r}} - \frac{C X_2}{z^{-q}} \tag{31}
$$

Proof: It follows from (22) that the 2DoF GFLC controller produces a finite error sequence for all C and G if and only if $X_1 = A^+ H^+ X_2$ and $V_1 = A_0^+ F^+ V_2 = A^+ F_0^+ V_2$, because $(C, A^+) = 1$ and $(A^+ F^+, G^-) = 1$. Substituting $X_1 = A^+ H^+ X_2$ into (19), we obtain (28) for $Y_1 = Y_2$. Substituting $V_1 = A_0^+ F^+ V_2 = A^+ F_0 V_2$ into (20), we obtain

$$
\begin{aligned}
A_0^+ F V_2 + z^{-r} B F_0 Z_1 &= A_0^+, \\
V_1 &= A^+ F_0 V_2
\end{aligned} \tag{32}
$$

Assuming that $(A_0^+, B F_0) = 1$, the equation (32) has solution if and only if $Z_1 = A_0^+ Z_2$. After the substitution into (32) we obtain (29). Relations (30) and (31) are obtained, when substituting

$$
\begin{aligned}
X_1 &= A^+ H^+ X_2, \quad Y_1 = Y_2, \\
V_1 &= A^+ F_0 V_2, \quad Z_1 = A_0^+ Z_2
\end{aligned} \tag{33}
$$

into (21) and (22). □

Table 1. Embedding of 2DoF GFLC and 2DoF GDBRF controllers into the complete set of 2DoF FLC controllers.

Type	L	F_0, H	X	Y	V	Z
GFLC	D^+L_1	*stable*	$G^+H^-L_1X_1$	$z^{-q}D^+G^+HL_1Y_1$	G^+V_1	G^+Z_1
GDBRF	D^+L_1	*unit*	$A^+G^+HL_1X_2$	$z^{-q}D^+G^+HL_1Y_2$	$A^+G^+V_2$	$A_0^+G^+Z_2$

Corollary 2: It follows from (23), (27) resp. (33) that both the sets of 2DoF GFLC and 2DoF GDBRF controllers are embedded in the set of 2DoF FLC controllers, if and only if polynomials F_0, H, L, X, Y, V and Z from Theorem 1 satisfies Table 1. The 2DoF GFLC resp. 2DoF GDBRF controllers exist if and only if the polynomials F_0 and H are stable polynomials resp. unit polynomials. This way specified sets of controllers are independent on the choice of the polynomial L_1, due to its cancellation after the substitution into relations (8). In case of GDBRF controllers, this is consistent with the internal model principle and guarantees that the model of uncontrollable external variables is contained in the controlled system and thus also in the open-loop. In the state space model this condition corresponds to the controllability of that part of the augmented system, which is observable by error e, which contradicts to (Grasselli *et al.*, 1995), where observability of the plant is a necessary condition.

6. CONCLUSION

The key idea of the paper was to determine the complete set of 2DoF FLC controllers. Newly performed analysis of behavior of FLC controllers in non–nominal situation, characterized by an eventual correction of reference signal w and by general plant disturbances, led to the formulation of GFLC and GDBRF tasks that have not yet been investigated in this context. The specification of the complete set of all 2DoF FLC controllers by Theorem 1 has created a theoretical background for the specification of several important subsets of controllers.

We have derived the necessary and sufficient conditions for the existence of 2DoF GFLC resp. GDBRF controllers and their minimum step variants. With regard to (Fikar and Kučera, 2000), this paper opposes the statement that solution to FLC problem does not exist for a noncausal polynomial in the denominator of the reference signal generator. Further, it has been proved for 2DoF controller that the solution to minimum step control is unique, if the plant is exposed to disturbances.

From a general point of view, 1DoF controller design versions are a special case of 2DoF versions. They are naturally less flexible, and, except the tradition, they have no visible advantages. The same goes for GFLC and GDBRF problems.

Finally remark, that the design of 2DoF controllers is always obtained by solution of two Diophantine equations. The set of solution of each them can be parametrized by a free parametrizing polynomial. The parametrizing polynomial for feed-back Diophantine equation linearly parametrizes only the response to disturbances, while the parametrizing polynomial for feed-forward Diophantine equation linearly parametrizes only the response to the reference signal. Contrary to (Rao and Rawlings, 2000; Fikar and Kučera, 2000; Mošna *et al.*, 2001) this new finding enables independent optimization of both mentioned responses with ℓ_1, ℓ_2 or ℓ_∞ norms.

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