ITERATIVE LEARNING CONTROL OF ROBOTIC MANIPULATORS BY HYBRID ADAPTATION SCHEMES

Application of 2-Dimensional Adaptive Control

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Abstract: This paper provides alternative approaches to solve iterative learning control (ILC) of robotic manipulators by introducing hybrid adaptation schemes and extended versions of those, that is, *2-dimensional adaptive control*. The 2-dimensional adaptive control strategies contain 2 types of adaptation processes, off-line tuning and on-line tuning, simultaneously, and provide more skillful learning properties where adaptive processes themselves are improved adaptively.

Keywords: learning control, adaptive control, robotic manipulators, adaptive algorithms, least squares identification, learning algoritms

1. INTRODUCTION

Iterative learning control (ILC) schemes have been one of the useful methods to achieve tracking control for uncertain processes with less prior information. Those generate desired control inputs for tracking through repetitions of the same tasks on the finite time interval, and have been applied to the control of processes which execute the same operations over and over again. The researches of ILC began with the original work (Arimoto *et al.*, 1984), and then there have been many investigations in those fields (for example, (Moore and Xu, 2000)). In the conventional ILC schemes, there are problems that the reference signals to be tracked should be identical in all iterations, and additionally, that the finite time interval on which each operation is defined, has the same length in all iterations. Those features of ILC are owing to the fact that ILC scheme is one of the servo-compensator which involves internal models of delayed-signals.

In order to relax those restrictions of ILC, alternative approaches to solve ILC of robotic manipulators by introducing hybrid adaptation schemes were provided (Miyasato, 2003). The hybrid adaptation schemes are adaptive control structures involving continuous-time control of processes and discrete-time updates of tuning parameters simultaneously (Ioannou and Sun, 1996). In the proposed methodology, the discrete-time updates of tuning parameters and the stabilizing control signals which are derived from certain $\mathcal{H}_\infty$ control problem, assure the boundedness of the overall control system, and attain convergence of tracking errors through the repetition of the operations on the finite time interval. The main advantage is that the reference signals and the time intervals of each operations, are not necessarily identical to the ones in the other operations.

In the present paper, an extended version of those results by applying *2-dimensional adaptive control strategy* is provided, which was originally proposed in adaptive control of 2-dimensional systems (Miyasato and Oshima, 1989). It contains 2 types of adaptation processes simultaneously, and one of those adaptation processes is improved adaptively by the other adaptation one. In ILC problems, those 2 types of adaptation processes are allocated appropriately into the hybrid adaptation schemes, and the convergence of both processes are assured, and one of those is improved by the other one. Therefore, the proposed composite adaptive systems provide more skillful learning properties where adaptive processes themselves are improved adaptively.

2. ROBOTIC MANIPULATORS AND TRACKING CONTROL

Basic preliminaries of robotic manipulators and tracking control of manipulators are summarized (Shen and Tamura, 1999), (Miyasato, 2002).

Consider a robotic manipulator with n degrees of freedom described by the following equation:

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) = \tau, \qquad (1)$$

where $\theta \in \mathbf{R}^n$ is a vector of joint angles, $M(\theta) \in \mathbf{R}^{n\times n}$ is a matrix of inertia, $C(\theta, \dot{\theta}) \in \mathbf{R}^{n\times n}$ is a matrix of Coriolis and centrifugal forces, $G(\theta) \in \mathbf{R}^n$ is a vector of gravitational torques, and τ is a vector of input torques (control input). It is assumed that the system parameters in $M(\theta)$, $C(\theta, \dot{\theta})$, and $G(\theta)$ are unknown. Robotic manipulators with rotational joints have the following properties (Spong and Vidyasagar, 1989).

Properties of Robotic Manipulators

(1) $M(\theta)$ is a bounded, positive definite, and symmetric matrix.
(2) $\dot{M}(\theta) - 2C(\theta, \dot{\theta})$ is a skew symmetric matrix.
(3) The left-hand side of (1) can be written into the following form,

$$M(\theta)a + C(\theta, \dot{\theta})b + G(\theta) = \Omega(\theta, \dot{\theta}, a, b)^T\Phi, \qquad (2)$$

where $\Omega(\theta, \dot{\theta}, a, b)$ is a known function of θ, $\dot{\theta}$, a, b, and Φ is an unknown system parameter.

The control objective is to determine a suitable control input τ such that the joint angle θ follows the desired reference angle θ_d (tracking control).

The basic structure of the tracking control for robotic manipulators is given as follows:

$$\tau = \Omega(\theta, \dot{\theta}, a, b)^T\hat{\Phi} - e + v, \qquad (3)$$

$$a \equiv \ddot{\theta}_d + \lambda^2 e - \lambda s, \qquad b \equiv \dot{\theta}_d - \lambda e, \qquad (4)$$

$$e \equiv \theta - \theta_d, \quad s \equiv \dot{e} + \lambda e, \qquad (\lambda > 0), \qquad (5)$$

where $\hat{\Phi}$ is an estimate of the unknown parameter Φ, and v is a stabilizing signal. Here, define a positive function V (energy function) by

$$V = \frac{1}{2}s^T M(\theta)s + \frac{1}{2}\|e\|^2, \qquad (6)$$

and take the time derivative of it along the trajectory of s, e, and θ.

$$\dot{V} = -\lambda\|e\|^2 + s^T v + s^T\Omega(\theta, \dot{\theta}, a, b)^T\tilde{\Phi}, \qquad (7)$$

$$\tilde{\Phi} \equiv \hat{\Phi} - \Phi. \qquad (8)$$

Hereafter, a and b are defined by (4).

In order to stabilize robotic manipulaors, the stabilizing control signal v in (3) is derived as a solution of certain $\mathcal{H}_\infty$ control problem, where the parameter error $\tilde{\Phi}$ is regarded as an external disturbance to the process. For that purpose, the following virtual process is introduced.

$$\dot{x} = f(x) + g_1(x)\tilde{\Phi} + g_2 v, \qquad (9)$$

$$x = \begin{bmatrix} e \\ s \end{bmatrix}, \quad f(x) = \begin{bmatrix} -\lambda e \\ -M^{-1}Cs \end{bmatrix},$$

$$g_1(x) = \begin{bmatrix} 0 \\ M^{-1}\Omega^{(a,b)T} \end{bmatrix}, \quad g_2(x) = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix}, \qquad (10)$$

$$\Omega^{(a,b)} = \Omega(\theta, \dot{\theta}, a, b). \qquad (11)$$

It should be noted that the time derivative of V (6) along the trajectory of the virtual system (9), (10), is the same as (7). That system (9), (10) is to be stabilized by utilizing $\mathcal{H}_\infty$ control strategy, where $\tilde{\Phi}$ is regarded as an external disturbance. For that, consider the next Hamilton-Jacobi-Isaacs equation (HJI equation), where the solution V is given by (6).

$$\frac{\partial V}{\partial t} + \mathcal{L}_f V + \frac{1}{4}\left\{\frac{\|\mathcal{L}_{g_1}V\|^2}{\gamma^2} - \mathcal{L}_{g_2}V\ R^{-1}(\mathcal{L}_{g_2}V)^T\right\} + q(x) = 0. \qquad (12)$$

The positive function $q(x)$ and positive definite symmetric matrix R are to be obtained from (12) based on inverse optimality, for the given solution V (6) and the positive constant γ. The substitution of V (6) into HJI equation (12) yields

$$-\lambda\|e\|^2 + \frac{s^T\Omega^{(a,b)T}\Omega^{(a,b)}s}{4\gamma^2} - \frac{1}{4}s^T R^{-1}s + q(x) = 0. \qquad (13)$$

Then $q(x)$ and R are given by

$$q = \lambda\|e\|^2 + \frac{1}{4}s^T\bar{K}s, \qquad (14)$$

$$R = \left(\frac{1}{\gamma^2}\Omega^{(a,b)T}\Omega^{(a,b)} + \bar{K}\right)^{-1}, \quad (\bar{K} = \bar{K}^T > 0), \qquad (15)$$

and the input signal (stabilizing control signal) v is obtained as a solution of the corresponding $\mathcal{H}_\infty$ control problem in the following way:

$$v = -\frac{1}{2}R^{-1}(\mathcal{L}_{g_2}V)^T = -\frac{1}{2}R^{-1}s = -\frac{1}{2}\left(\frac{1}{\gamma^2}\Omega^{(a,b)T}\Omega^{(a,b)} + \bar{K}\right)s. \qquad (16)$$

The next theorem is derived for the original robotic manipulators (1) (Miyasato, 2002).

Theorem 1 *The nonlinear control system of robotic manipulators (1) defined by (3), (4), (16) is uniformly bounded for arbitrary bounded* $\hat{\Phi}$. *Additionally, v (16) is an optimal control signal which minimizes the following cost functional J.*

$$J = \sup_{\tilde{\Phi}\in\mathcal{L}^2} \left\{ \int_0^t (q + v^T Rv)d\tau + V(t) - \gamma^2 \int_0^t \|\tilde{\Phi}\|^2 d\tau \right\}. \quad (17)$$

Furthermore, the next inequality holds.

$$\int_0^t (q + v^T Rv)d\tau + V(t) \leq \gamma^2 \int_0^t \|\tilde{\Phi}\|^2 d\tau + V(0). \quad (18)$$

3. ITERATIVE LEARNING CONTROL BY HYBRID ADAPTATION SCHEMES

The control task and the desired reference signals are defined on the finite time interval $[0, T]$. The operation is repeated on $[0, T]$. Hereafter, denote the signal $x(t)$ $(t \in [0, T])$ at the k-th iteration by $x_k(t)$. Then, ILC generates desired control inputs $\tau_k(t)$ $(0 \leq t \leq T), (k = 1, 2, 3, \cdots)$, and attains desired control performance $\lim_{k\to\infty} e_k(t) = \lim_{k\to\infty} s_k(t) = 0$ $(t \in [0, T])$ through repetitions of the operation on $[0, T]$. It is assumed that $s_k(0) = e_k(0) = 0$, which is a conventional assumption of many ILC schemes.

By considering (3), (16), the control at the k-th iteration $\tau_k(t)$ is determined as follows:

$$\tau_k(t) = \Omega_k^{(a,b)}(t)^T \hat{\Phi}(k) - e_k(t) + v_k(t), \quad (19)$$

where a_k and b_k are defined by (4), and

$$\Omega_k^{(a,b)}(t) \equiv \Omega(\theta_k(t), \dot{\theta}_k(t), a_k(t), b_k(t)). \quad (20)$$

$v_k(t)$ is a stabilizing signal given by

$$v_k(t) = -\left(K + \alpha \cdot \Omega_k^{(a,b)}(t)^T \Omega_k^{(a,b)}(t)\right) s_k(t), \quad (K = K^T > 0, \quad \alpha > 0). \quad (21)$$

$\hat{\Phi}(k)$ is a current estimate of Φ obtained from the operation in the $(k-1)$-th iteration.

In order to derive the update law of $\hat{\Phi}(k)$, for the manipulator dynamics described by

$$M(\theta_k)\ddot{\theta}_k + C(\theta_k, \dot{\theta}_k)\dot{\theta}_k + G(\theta_k) = \tau_k = \Omega(\theta_k, \dot{\theta}_k, \ddot{\theta}_k, \dot{\theta}_k)^T \Phi, \quad (22)$$

the next identification model $\hat{\tau}_k$ is introduced.

$$\hat{\tau}_k = \Omega(\theta_k, \dot{\theta}_k, \ddot{\theta}_k, \dot{\theta}_k)^T \hat{\Phi}(k). \quad (23)$$

Similarly to $\Omega_k^{(a,b)}(t)$, denote

$$\Omega_k^{(\ddot{\theta},\dot{\theta})}(t) \equiv \Omega(\theta_k(t), \dot{\theta}_k(t), \ddot{\theta}_k(t), \dot{\theta}_k(t)). \quad (24)$$

The update law of $\hat{\Phi}(k)$ is obtained as the hybrid adaptation scheme (least squares law).

$$\hat{\Phi}(k) = \hat{\Phi}(k-1) - \left\{ \Gamma(k-1)^{-1} + \int_0^T \Omega_{k-1}^{(\ddot{\theta},\dot{\theta})}(t) \Omega_{k-1}^{(\ddot{\theta},\dot{\theta})}(t)^T dt \right\}^{-1} \cdot \int_0^T \Omega_{k-1}^{(\ddot{\theta},\dot{\theta})}(t) \epsilon_{k-1}(t) dt, \quad (25)$$

$$\Gamma(k)^{-1} = \lambda_1(k)\Gamma(k-1)^{-1} + \lambda_2(k) \int_0^T \Omega_{k-1}^{(\ddot{\theta},\dot{\theta})}(t) \Omega_{k-1}^{(\ddot{\theta},\dot{\theta})}(t)^T dt, \quad (26)$$

$$\epsilon_k(t) = \hat{\tau}_k(t) - \tau_k(t), \quad (27)$$

$$0 < \lambda_1(k) \leq 1, \quad 0 \leq \lambda_2(k) < 2. \quad (28)$$

$\Gamma(k)^{-1}$ corresponds to an inverse of the covariance matrix in the conventional least squares estimates.

For stability analysis, a posterior identification error is introduced.

$$\bar{\epsilon}_k(t) = \tilde{\Phi}(k+1)^T \Omega_k^{(\ddot{\theta},\dot{\theta})}(t). \quad (29)$$

Define $W(k)$ by

$$W(k) = \tilde{\Phi}(k)^T \Gamma(k)^{-1} \tilde{\Phi}(k), \quad (30)$$

and obtain the following relation.

$$\begin{aligned} \Delta W(k) &\equiv W(k) - W(k-1) \\ &= -\{2 - \lambda_2(k)\} \int_0^T \bar{\epsilon}_{k-1}(t)^2 dt \\ &\quad -\{1 - \lambda_1(k)\}\tilde{\Phi}(k)^T \Gamma(k-1)^{-1} \tilde{\Phi}(k) \\ &\quad -\Delta\hat{\Phi}(k)^T \Gamma(k-1)^{-1} \Delta\hat{\Phi}(k) \leq 0, \end{aligned} \quad (31)$$

$$\Delta\hat{\Phi}(k) = \hat{\Phi}(k) - \hat{\Phi}(k-1). \quad (32)$$

Since $W(k) \geq 0$, $W(k)$ is uniformly bounded, and $\Delta W(k) \to 0$ as $k \to \infty$. Hereafter, $\lambda_1(k)$ and $\lambda_2(k)$ are chosen such that $\Gamma(k)^{-1} \geq \delta I > 0$ $(\delta > 0)$ holds (for example, this relation is satisfied, if $\lambda_1(k) = \lambda_2(k) = 1$, and $\Gamma(0)^{-1} > 0$). Then, $\hat{\Phi}(k)$ is uniformly bounded, and

$$\lim_{k\to\infty} \int_0^T \bar{\epsilon}_k(t)^2 dt = \lim_{k\to\infty} \Delta\hat{\Phi}(k) = 0. \quad (33)$$

By considering Theorem 1 and the stabilizing control signal $v_k(t)$, it is shown that $\Omega_k^{(a,b)}(t)$ and $\Omega_k^{(\ddot{\theta},\dot{\theta})}(t)$ are uniformly bounded, and

$$\lim_{k\to\infty} \bar{\epsilon}_k(t) = 0, \quad (0 \leq t \leq T). \quad (34)$$

As for $\epsilon_k(t)$, the next relation is derived.

$$\epsilon(k) = \hat{\tau}_k(t) - \tau_k(t) = \Omega_k^{(\ddot{\theta},\dot{\theta})}(t)^T \tilde{\Phi}(k)$$
$$= \bar{\epsilon}_k(t) - \Omega_k^{(\ddot{\theta},\dot{\theta})}(t)^T \Delta\hat{\Phi}(k). \quad (35)$$

Since $\bar{\epsilon}_k(t)$ and $\Delta\hat{\Phi}(k)$ converge to zero, and $\Omega_k^{(\ddot{\theta},\dot{\theta})}(t)$ is bounded (Theorem 1), it follows that

$$\lim_{k\to\infty} \epsilon_k(t) = \lim_{k\to\infty} \bar{\epsilon}_k(t) = 0, \quad (0 \le t \le T). \quad (36)$$

Next, the tracking errors $s_k(t)$, $e_k(t)$ are analyzed. From $\tau_k(t)$ and $\hat{\tau}_k(t)$, it follows that

$$\hat{M}(k,\theta_k)\dot{s}_k(t) + \hat{C}(k,\theta_k,\dot{\theta}_k)s_k(t) + e_k(t)$$
$$+ \left(K + \alpha \cdot \Omega_k^{(a,b)}(t)^T \Omega_k^{(a,b)}(t)\right) s_k(t)$$
$$= \hat{\tau}_k(t) - \tau_k(t) = \epsilon_k(t). \quad (37)$$

For that manipulator dynamics, define $\hat{V}_k(t)$ by

$$\hat{V}_k(t) = \frac{1}{2} s_k(t)^T \hat{M}(k,\theta_k) s_k(t) + \frac{1}{2}\|e_k(t)\|^2. \quad (38)$$

Here, consider that $\hat{\Phi}(k)$ is constant on the interval of the k-th iteration, and that $\hat{M}(k,\theta_k)$ and $\hat{C}(k,\theta_k,\dot{\theta}_k)$ have the same structures as $M(\theta_k)$ and $C(\theta_k,\dot{\theta}_k)$ respectively. Then, take the time derivative of $V_k(t)$ along the trajectory of s_k and e_k at the k-th iteration, and obtain

$$\dot{\hat{V}}_k(t) = -s_k(t)^T K s_k(t) - \alpha\|\Omega_k^{(a,b)}(t) s_k(t)\|^2$$
$$-\lambda\|e_k(t)\|^2 + \epsilon_k(t)^T s_k(t)$$
$$\le -\frac{1}{2}\lambda_{\min}(K)\|s_k(t)\|^2 - \lambda\|e_k(t)\|^2$$
$$+ \frac{1}{2\lambda_{\min}(K)}\|\epsilon_k(t)\|^2. \quad (39)$$

From (39), it follows that

$$\hat{V}_k(t) + \frac{1}{2}\lambda_{\min}(K)\int_0^t \|s_k(\tau)\|^2 d\tau$$
$$+\lambda \int_0^t \|e_k(\tau)\|^2 d\tau$$
$$\le \hat{V}_k(0) + \frac{1}{2\lambda_{\min}(K)}\int_0^t \|\epsilon_k(\tau)\|^2 d\tau. \quad (40)$$

It is assumed that $\hat{M}(k,\theta_k) > 0$, then the next relation is derived, since $\int_0^T \|\epsilon_k(t)\|^2 dt \to 0$.

$$\lim_{k\to\infty} \|s_k(t)\| = \lim_{k\to\infty} \|e_k(t)\| = 0, \ (0 \le t \le T), \ (41)$$

where the boundedness of tuning parameters and states is considered, and the assumption $\hat{V}_k(0) = 0$ ($e_k(0) = s_k(0) = 0$) is also considered.

Theorem 2 *It is assumed that $\hat{M}(k,\theta_k) > 0$ and $\hat{V}_k(0) = 0$ ($e_k(0) = s_k(0) = 0$). Then, the iterative learning control schemes composed of the least squares hybrid adaptation laws (25), (26), (27), control inputs $\tau_k(t)$ (19), and stabilizing control signals $v_k(t)$ (21), are uniformly bounded, and the tracking error converges to zero asymptotically through the repetition of the operation on the finite time interval* $[0,\,T]$ *(41).*

Remark **1.** Contrary to the conventional ILC schemes, it is seen that the desired trajectory θ_d does not need to be identical in all iterations, and additionally, the finite time interval in each operations does not need to have the same length. Even for such case, there need no major changes in the stability analysis.

2. The following hybrid adaptation laws can be also utilized to update $\hat{\Phi}(k)$ (Miyasato, 2003).

$$\hat{\Phi}(k) = \hat{\Phi}(k-1) - g(k)\frac{\int_0^T \Omega_{k-1}^{(\ddot{\theta},\dot{\theta})}(t)\epsilon_{k-1}(t)dt}{1 + \int_0^T \|\Omega_{k-1}^{(\ddot{\theta},\dot{\theta})}(t)\|^2 dt},$$
$$(0 < g(k) < 2). \quad (42)$$

3. In order to assure $\hat{M}(k,\theta_k) > 0$, the projection algorithm can be utilized.

4. $\ddot{\theta}_{k-1}(t)$ is needed in the computation of $\Phi(k)$. However, $\ddot{\theta}_{k-1}(t)$ can be removed by introducing $\hat{\tau}_{fk-1}$ for τ_{fk-1} of the following form ($\lambda_f > 0$)

$$\tau_{fk-1}(t) \equiv \frac{1}{s+\lambda_f}\tau_{k-1}(t) = \Omega_{fk-1}^{(\dot{\theta},\theta)}(t)^T \Phi, \quad (43)$$

where $\ddot{\theta}_{k-1}(t)$ is not included in $\Omega_{fk-1}^{(\dot{\theta},\theta)}(t)$.

4. 2-DIMENSIONAL ADAPTIVE CONTROL

In the hybrid adaptation schemes, $\hat{\Phi}(k)$ is updated at the end of each operation, and is constant during each operation (off-line tuning).

In the present section, the 2-dimensional adaptive control scheme is applied to ILC with hybrid adatation. The proposed control scheme contains two type of adaptation laws, that is, off-line tuning and on-line tuning of control parameters, simultaneously, and the adaptation process during each operation is improved adaptively by the other adaptation scheme.

By adding time-varying $\hat{\phi}_k(t)$, the control $\tau_k(t)$ at the k-th iteration is determined as follows:

$$\tau_k(t) = \Omega_k^{(a,b)}(t)^T\{\hat{\Phi}(k) + \hat{\phi}_k(t)\} - e_k(t) + v_k(t), \quad (44)$$

$$\dot{\hat{\phi}}_k(t) = -G_k \Omega_k^{(a,b)}(t) s_k(t), \quad (45)$$
$$(\hat{\phi}_k(0) = 0, \quad G_k = G_k^T > 0),$$

where $v_k(t)$ is defined by (21), and $\hat{\Phi}(k)$ is obtained by each of the hybrid adaptation laws (25), (26), (27), or (42). $\hat{\phi}_k(t)$ is reset to zero at the beginning of each operations ($\hat{\phi}_k(0) = 0$). Then, for $W_k(t)$ defined by

$$W_k(t) = V_k(t)$$
$$+\frac{1}{2}\{\hat{\phi}_k(t) - \phi_k\}^T G_k^{-1}\{\hat{\phi}_k(t) - \phi_k\}, \quad (46)$$

$$\phi_k = \Phi - \hat{\Phi}(k), \quad (47)$$

(where $V_k(t)$ is $V(t)$ (6) at the k-th iteration), the time derivative of $W_k(t)$ is evaluated as follows:

$$\begin{aligned}\dot{W}_k(t) &= -\lambda\|e_k(t)\|^2 \\ &\quad -s_k(t)^T\left(K+\alpha\cdot\Omega_k^{(a,b)}(t)^T\Omega_k^{(a,b)}(t)\right)s_k(t) \\ &\le 0 \end{aligned} \tag{48}$$

Therefore, it follows that $\hat{\phi}_k(t)$ is uniformly bounded ($\|\hat{\phi}_k(t)\|$ is bounded from above which does not depend on the time interval T) and that the control errors $s_k(t)$, $e_k(t)$ converge to zero along the direction of time t at each operation.

$$\lim_{t\to\infty} s_k(t) = \lim_{t\to\infty} e_k(t) = 0 \tag{49}$$

(That means that $\|s_k(T)\|$ and $\|e_k(T)\|$ can be made arbitrary small for sufficiently large T.) On the other hand, it is assured that $\epsilon_k(t) \to 0$ $(0 \le t \le T)$ as $k \to \infty$, and that $\hat{\Phi}(k)$ is uniformly bonded, because of the hybrid adaptation schemes of $\hat{\Phi}(k)$. Based on those properties, the converge of the output errors $s_k(t)$, $e_k(t)$ along the direction of repeated operations k $(k = 0, 1, \cdots)$ is investigated. For $\hat{V}_k(t)$ defined by (38), the next inequality is deduced, similarly to (40).

$$\begin{aligned}&\hat{V}_k(t) + \frac{1}{2}\lambda_{\min}(K)\int_0^t \|s_k(\tau)\|^2 d\tau \\ &+\lambda\int_0^t \|e_k(\tau)\|^2 d\tau \\ &+\frac{1}{2}\left\|G_k^{\frac{1}{2}}\int_0^t \Omega_k^{(a,b)}(\tau)s_k(\tau)d\tau\right\|^2 \\ &\le \hat{V}_k(0) + \frac{1}{2\lambda_{\min}(K)}\int_0^t \|\epsilon_k(\tau)\|^2 d\tau. \end{aligned} \tag{50}$$

Since $\lim_{k\to\infty}\int_0^T \|\epsilon_k(t)\|^2 dt = 0$, the following relation is derived,

$$\lim_{k\to\infty}\|s_k(t)\| = \lim_{k\to\infty}\|e_k(t)\| = 0,\ (0 \le t \le T), \tag{51}$$

where the boundedness of tuning parameters and states is considered, and it is assumed that $\hat{V}_k(0) = 0$ $(e_k(0) = s_k(0) = 0)$, and $\hat{M}(k, \theta_k) > 0$.

Theorem 3 *It is assumed that $\hat{M}(k,\theta_k) > 0$ and $\hat{V}_k(0) = 0$ $(e_k(0) = s_k(0) = 0)$. Then, the 2-dimensional adaptive control strategy ((44), (45), (19), (21), (25), (26), (27), or (42)) makes the overall adaptive system uniformly bounded. Furthermore, the tracking errors converge to zero asymptotically, along the directions of both time t and repeated operations k ((49), (51)).*

Remark 1. In the 2-dimensional adaptive control scheme, the true value $\phi_k = \Phi - \hat{\Phi}(k)$ which $\hat{\phi}_k(t)$ should estimate at each operation, is changing as the repeated operations go on. If internal signals are sufficiently rich, and $\hat{\Phi}(k) \to \Phi$ (as $k \to \infty$), then $\phi_k \to 0$, and the necessity of tuning of $\hat{\phi}_k(t)$ is made less and less. Even if $\hat{\Phi}(k)$ does not converge to its true value Φ, from the relation (50), it follows that

$$\lim_{k\to\infty}\left\|G_k^{\frac{1}{2}}\int_0^t \Omega_k^{(a,b)}(\tau)s_k(\tau)d\tau\right\|^2 = 0, \tag{52}$$

and the next equation holds.

$$\lim_{k\to\infty}\left\|\hat{\phi}_k(t)\right\|^2 = 0. \tag{53}$$

Therefore, anyway, the contribution of the term $\hat{\phi}_k(t)$ in $\tau_k(t)$, decreases as $k \to \infty$, and the main part of $\tau_k(t)$ becomes

$$\tau_k(t) \to \Omega_k^{(a,b)}(t)^T\hat{\Phi}(k) - e_k(t) + v_k(t).$$

Thus, as the repeated operations go on, the main part of the control input $\tau_k(t)$ shifts from the portion of on-line tuning ($\hat{\phi}_k(t)$) to the portion of off-line tuning ($\hat{\Phi}(k)$), and the contribution of $\hat{\phi}_k(t)$ decreases (or the contribution of $\hat{\phi}_k(t)$ is temporary). Hence, it is seen that the necessity of the tuning of $\hat{\phi}_k(t)$ grows less and less, and this means that the control situation itself is improved adaptively. As a result, the control performance such as convergence and transient property, is expected to be improved compared with the conventional adaptive control strategy. This property is also seen by comparing (50) with (40). Owing to the term $\frac{1}{2}\left\|G_k^{\frac{1}{2}}\int_0^t \Omega_k^{(a,b)}(\tau)s_k(\tau)d\tau\right\|^2$ in (50), the convergence of $s_k(t)$ and $e_k(t)$ in (50) is expected to be much better than those in (40).

2. It should be also noted that the two adaptation processes are driven by the different error signals; that is, the tuning of $\hat{\Phi}(k)$ along the direction of k is driven by the identification error $\epsilon_k(t)$, and the tuning of $\hat{\phi}_k(t)$ along the direction of t is driven by the output error $s_k(t)$. Hence, the on-line tuning of $\hat{\phi}_k(t)$ is affected by the off-line tuning of $\hat{\Phi}(k)$, but $\hat{\Phi}(k)$ is not affected by $\hat{\phi}_k(t)$ directly.

5. SIMULATION STUDIES

Numerical simulation studies are performed. A SICE-DD arm (the standard manipulator model in SICE) with two-degree of freedom is considered. Physical parameters are written in Table 1.

Table 1 Physical parameters.

Link (i)	1	2
m_i (kg)	12.27	2.083
I_i (kg $\cdot$ m^2)	0.1149	0.0144
l_i (m)	0.2	0.2
r_i (m)	0.063	0.080

The desired trajectory is given by

$$\theta_{d1}(t) = \frac{\pi}{2}\cdot\frac{6}{125}\cdot\left(\frac{5}{2}t^2 - \frac{1}{3}t^3\right),$$
$$\theta_{d2}(t) = \pi - 2\theta_{d1}(t), \qquad (0 \le t \le 5).$$

The time interval in each iteration is [0, 5] ($T = 5$). The hybrid adaptation scheme (Case 1: Theorem 2) and the 2-dimensional adaptive control scheme (Case 2: Theorem 3) are applied. The design parameters are chosen as follows:

$\underline{\text{Case 1 : hybrid adaptation scheme}}$

$$\lambda_1(k) = \lambda_2(k) = 1, \ \ K = I, \ \ \alpha = 1.$$

$\underline{\text{Case 2 : 2} - \text{dimensional adaptive control}}$

$$\text{Same as Case 1, and } G_k = 1000I.$$

The simulation results are shown, where $\|e_k\|^2 \equiv \int_0^5 \|e_k(t)\|^2 dt$, and $I_{PR}(k)$ (parameter ratio) is

$$I_{PR}(k) \equiv \frac{\|\hat{\phi}_k(T)\|^2}{\left(\|\hat{\phi}_k(T)\|^2 + \|\hat{\Phi}(k)\|^2\right)}. \qquad (54)$$

$I_{PR}(k)$ indicates the contribution of $\hat{\phi}_k(t)$, and $I_{PR}(k) \to 0$ in the 2-dimensional adaptive control (Case 2). In the simulation, Case 2 has much better convergence property than Case 1.

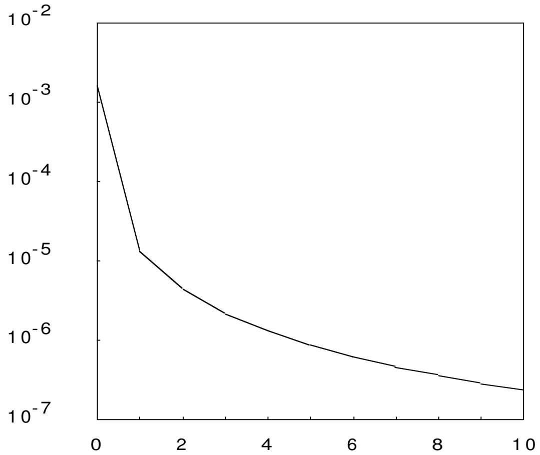

Fig. 1. Case 1 : $\|e_k\|^2$ vs. k.

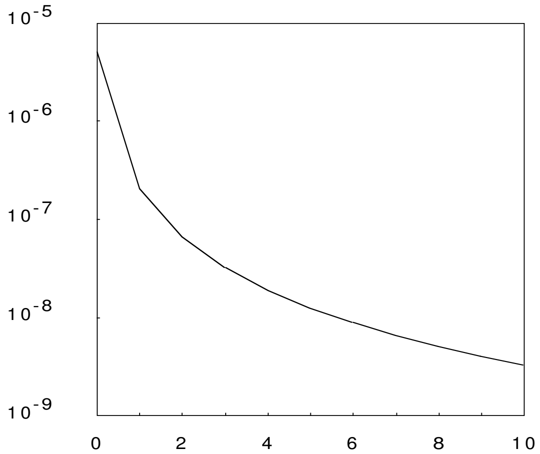

Fig. 2. Case 2 : $\|e_k\|^2$ vs. k.

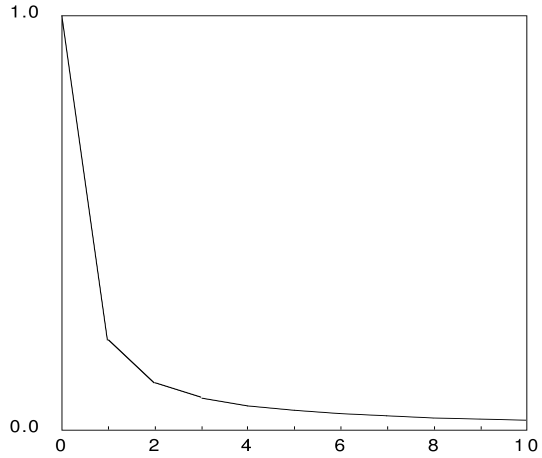

Fig. 3. Case 2 : $I_{PR}(k)$ vs. k.

6. CONCLUSION

Alternative approaches to solve iterative learning control (ILC) of robotic manipulators by introducing hybrid adaptation scheme and the extended version of those by applying 2-dimensional adaptive control strategy, are given. The 2-dimensional control schemes have composite adaptation structures, and those provide more skillful learning properties where adaptive processes themselves are improved adaptively. The same strategy can be applied to various adaptive control problems where repeated operations are involved.

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