FRACTIONAL ROBUST CONTROL TO DELAY CHANGES IN MAIN IRRIGATION CANALS

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Abstract: This paper proposes a new strategy for control of open irrigation canals based on fractional order controllers. A methodology is developed to design PI fractional controllers combined with Smith predictors which are robust to changes in the time delay. This method is applied to solve the effective control problem of an open irrigation canal. Simulated results of a standard PI controller, PI plus Smith predictor controller, and the controller developed in this paper are compared when applied to the dynamical model of a real irrigation canal pool. Copyright 2005 IFAC.

Keywords: Fractional Order Controller; Time Delay Process, Open Irrigation Canal Control; Robust Control; Irrigation Systems Control.

I. INTRODUCTION

At present a lot of water is wasted in most irrigation canals because of lack of efficient control. In this context automatic control is considered as a powerful tool for improving efficiency in water distribution irrigation systems.

Irrigation canals are systems distributed over long distances, with significant time delays and dynamics that change with the operating conditions (Malaterre, 1998). A typical irrigation canal consists of several pools separated by gates that are used for regulating the water distribution from one pool to the next one (see Fig. 1).

The physical dynamics of an open canal has traditionally been modelled by the Saint-Venant equations, which are nonlinear hyperbolic partial differential equations (Saint-Venant, 1891).

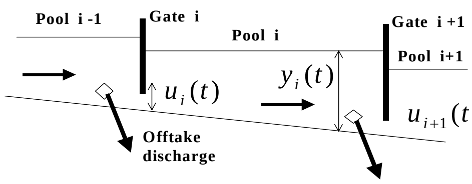

Fig. 1. Scheme of an open irrigation main canal with gates.

Different methods exist for the solution of Saint-Venant's equations, all of them exhibiting large mathematical complexities. These equations are also very difficult to use for prediction and control. Often, an equivalent first order system plus a delay is used to model the canal dynamic behavior (Weyer, 2001).

This model has the strong drawback that its parameters may experience large changes when the discharge regime varies. Then any controller to be designed for canals has to be robust to variations in some parameters of the linearized model.

Different strategies have been used for canal control. The most popular one is based on the classical PID controller. Many studies have shown that these controllers seem to be unsuitable to solve the problem of effective water distribution control in canals, due to the difficult dynamical behavior that characterizes these systems.

Fractional operators have been often applied in the last years by different authors, e.g. Podlubny, 1999, to model and control difficult dynamical behavior processes. An interesting feature of fractional-order controllers is that they exhibit some advantages when designing robust control systems in the frequency domain for processes whose parameters vary in a large range. These characteristics are explored in order to design robust controllers to solve the problem of effective water distribution control in irrigation canals whose dynamic parameters vary in a wide range. In particular this paper is focused on the design of a fractional controller combined with a Smith predictor, which shows to be very robust to changes in the time delay. The time delay is the parameter more determinant of the stability of the closed loop control of irrigation canals.

This paper is organized as follows. A model for the irrigation canal to be controlled is proposed in Section II. Section III develops the method for designing the PI fractional controller with the Smith predictor. Section IV compares the designed controller with other standard ones. Finally some conclusions are drawn in Section V.

II. IRRIGATION CANAL DYNAMIC MODEL

A linear model with concentrated parameters and a time delay can adequately characterize the dynamical behavior of irrigation canals in mensuration points (Rivas Perez et al., 2002). Experiments based on the response to a step like input were carried out in order to obtain a mathematical model that describes the dynamic behavior of a single canal pool. The experimental response of an irrigation canal pool to a step command is drawn in Fig. 2. Such response shows that the dynamic behavior of a single canal pool can be represented by expression:

$$G(s) = \frac{y(s)}{u(s)} = \frac{K}{(T_1 s + 1)(T_2 s + 1)} e^{-\tau s}, \quad (1)$$

where K is the static gain; T_1, T_2 are time constants; and τ is the time delay. Our canal model also includes disturbances *D(s)* caused by off-take discharges (see Fig. 3).

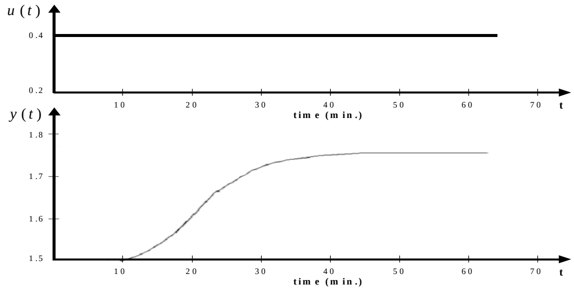

Fig. 2. Step response of an irrigation canal pool.

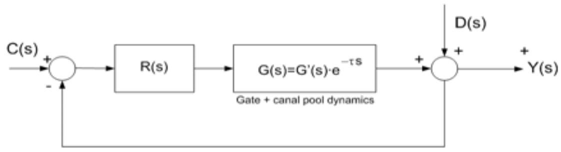

Fig. 3. Standard control scheme.

Experiments reported in previous works (Weyer, 2001, e.g.) showed that all these parameters exhibit strong variations. In our particular canal, we consider only variations in the time delay, $0 \le \tau \le \tau_{max}$. We denote as $K_0, \tau_0, T_{10}, T_{20}$ the nominal values of the model. We consider that T_1 is the dominant time constant (the larger one associated to the dynamics of the water), while T_2 is the smaller time constant that represents the motor + gate dynamics, which is much faster than the dynamics of the canal, and it is nearly invariant respecting to flow regimes.

III. PI FRACTIONAL CONTROLLER WITH SMITH PREDICTOR

Assume we want to design a controller for the system (1) with the next specifications: a) phase margin (ϕ_m), b) crossover frequency (ω_c), and c) zero steady state error. The last specification implies that the controller must include an integral term. Moreover the controller needs two parameters to be tuned in order to fulfill specifications a) and b). All this suggests that these three specifications can be attained by a PI controller of the form:

$$R(s) = K_p \left(1 + \frac{1}{T_i s}\right), \quad (2)$$

arranged according to the standard scheme of Fig. 3. The parameters of this controller can be obtained as follows.

Express model (2) as $G(j\omega) = G'(j\omega)\, e^{-j\omega\tau}$, where

$G'(j\omega)$ is the rational part of the model. Consider a general controller R(s) whose frequency characteristic is decomposed in the form $R(j\omega) = R_R(\omega) + jR_I(\omega)$ where R_R and R_I are real functions. In the particular case of the PI controller (3) these functions are:

$$R_R(\omega) = K_p\ , \quad R_I(\omega) = -\frac{K_p}{T_i\omega}\ . \qquad (3)$$

Then specifications a) and b) are accomplished if the following conditions are verified:

$$|G'(j\omega_c)||R(j\omega_c)| = 1\ ; \qquad (4)$$

$$\angle G'(j\omega_c) + \angle R(j\omega_c) - \tau\omega_c + \pi = \phi_m\ . \qquad (5)$$

which can be expressed as:

$$R_R(\omega_c) = \frac{\cos(\phi_m + \tau\omega_c - \pi - \angle G'(j\omega_c))}{|G'(j\omega_c)|}\ ; \qquad (6)$$

$$R_I(\omega_c) = \frac{\sin(\phi_m + \tau\omega_c - \pi - \angle G'(j\omega_c))}{|G'(j\omega_c)|}\ . \qquad (7)$$

Particularizing R_R and R_I by expressions (3), equations (6)-(7) are expressed in a compact form as:

$$\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -\dfrac{1}{\omega_c} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} K_p \\ \dfrac{K_p}{T_i} \end{pmatrix}}_{X} = -\underbrace{\begin{pmatrix} \cos(\phi_m + \tau\omega_c - \angle G'(j\omega_c)) / |G'(j\omega_c)| \\ \sin(\phi_m + \tau\omega_c - \angle G'(j\omega_c)) / |G'(j\omega_c)| \end{pmatrix}}_{B} \qquad (8)$$

and the vector X of controller parameters is given by $X = -A^{-1}B$.

The robustness of this controller to changes in the delay (maximum deviation of the delay from the nominal value that keeps stable the closed loop system) can be easily obtained:

$$\hat{\tau}_{\max} - \tau_0 = \frac{\phi_m}{\omega_c}\ . \qquad (9)$$

It has to be noticed that any controller R(s) of Fig. 3 that fulfills specifications a) and b) will exhibit the same time delay stability margin (9), independently of its particular form.

Then a different control structure has to be used in order to improve the robustness to changes in the delay. Next a structure based on the Smith predictor is proposed, which is shown in Fig.4. In this case the closed loop transfer function is:

$$Y(s) = M_c(s)C(s) + M_d(s)D(s)\ , \qquad (10)$$

where:

$$M_c(s) = \frac{R(s)G'(s)e^{-\tau s}}{1 + R(s)G'(s)(1 - e^{-\tau_0 s} + e^{-\tau s})}\ ; \qquad (11)$$

$$M_d(s) = \frac{1 + R(s)G'(s)(1 - e^{-\tau_0 s})}{1 + R(s)G'(s)(1 - e^{-\tau_0 s} + e^{-\tau s})}\ . \qquad (12)$$

Let us assume that now a fractional controller is used of the form:

$$R(s) = K_p\ (1 + \frac{1}{T_i s^{\alpha}})\ . \qquad (13)$$

where $0 \le \alpha \le 1$. The Final value theorem guarantees that the steady state error of the control system of Fig. 4 is zero if $0 \le \alpha$. Again parameters K_p and T_i are designed in order to achieve specifications a) and b). We denote (13) as the fractional PI controller, and the standard PI controller is obtained from this one by making $\alpha = 1$. The parameters of this controller can be obtained as follows.

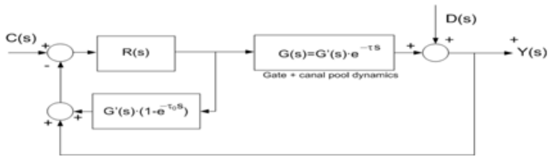

Fig. 4. Smith predictor based control scheme.

Denote $G_{eq}(j\omega) = G'(j\omega)\,(1 - e^{-j\tau_0\omega} + e^{-j\tau\omega})$, and decompose the frequency characteristics of R(s) in the form $R(j\omega) = R_R(\omega) + jR_I(\omega)$ where:

$$R_R(\omega) = K_p(1 + \frac{\cos(\frac{\pi}{2}\alpha)}{T_i\omega^{\alpha}})\ , \quad R_I(\omega) = -K_p\ \frac{\sin(\frac{\pi}{2}\alpha)}{T_i\omega^{\alpha}}\ . \qquad (14)$$

In this case conditions (4) and (5) yield to:

$$\underbrace{\begin{pmatrix} 1 & \dfrac{\cos(\frac{\pi}{2}\alpha)}{\omega_c^{\alpha}} \\ 0 & -\dfrac{\sin(\frac{\pi}{2}\alpha)}{\omega_c^{\alpha}} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} K_p \\ \dfrac{K_p}{T_i} \end{pmatrix}}_{X} = -\underbrace{\begin{pmatrix} \cos(\phi_m - \angle G_{eq}(j\omega_c)) / |G_{eq}(j\omega_c)| \\ \sin(\phi_m - \angle G_{eq}(j\omega_c)) / |G_{eq}(j\omega_c)| \end{pmatrix}}_{B} \qquad (15)$$

and the vector X of controller parameters is given again by: $X = -A^{-1}B$. In this case is not easy to obtain an analytical expression of the stability margin for the time delay but the next Section will demonstrate that, for processes like the one considered in this paper, this margin grows as fractional exponent α decreases.

IV. COMPARISON OF CONTROLLERS

In this Section we compare the robustness to changes in the time delay of different control schemes. Three control laws will be studied a) PI standard controller of Fig. 3, b) PI standard controller with Smith predictor (Fig. 4), c) Fractional PI controller with Smith predictor (Fig. 4). All these controllers are designed in order to exhibit the same dynamic behaviour (the same ω_c and ϕm) when the parameters of the canal take their nominal values.

Consider the canal described in Fig. 2, whose transfer function is (2). Its nominal parameters are $K_0 = 1.25$, $T_{10} = 300$ s, $T_{20} = 60$ s, and $\tau_0 = 600$ s. Typically the time delay may experience variations in the range $\tau_{min} = \tau_0/2 \le \tau \le 2\tau_0 = \tau_{max}$. But we will consider also that, under some special cases, the delay may be up to $4\tau_0 = \tau_{MAX}$. We compare the control systems from two points of view: a) maximum deviation of the time delay that makes the closed loop system unstable (in order to fulfill the limit case τ_{MAX}), b) degradation of the dynamic response in the range of normal work defined above $[\tau_{min}, \tau_{max}]$.

a) Design specifications

A crossover frequency $\omega_c = 0.0011$ rad/s is chosen. This implies a settling time of about 3600 s for the closed loop system (the settling time is approximately $t_s \approx 4/\omega_c$), which is considered adequate for this canal. A phase margin $\phi_m = 75°$ is chosen. Then from expression (9) we get that $\hat{\tau}_{\max} = 3\tau_0$.

b) Standard PI controller

Expression (8) leads to a controller of the form:

$$R(s) = 0.596\,(1 + \frac{1}{904.474s}). \qquad (16)$$

Fig. 5 shows the responses of this controller to unity step commands for different values of the time delay in the working range $[\tau_{min}, \tau_{max}]$. Moreover notice that $\tau_{\max} < \hat{\tau}_{\max} < \tau_{MAX}$, exhibiting thus this controller an unstable behavior in the before mentioned limit case.

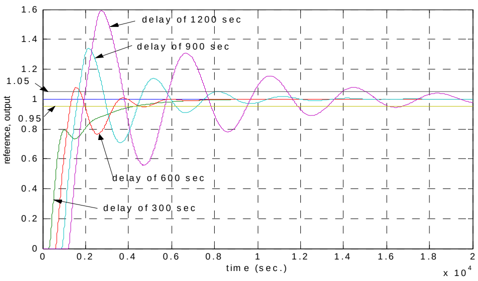

Fig. 5. Step response with a PI controller.

c) Standard PI controller with Smith predictor

Expression (15) particularized to $\alpha = 1$ leads to a PI controller of the form:

$$R(s) = 0.1035\,(1 + \frac{1}{112.25s}). \qquad (17)$$

Fig. 6 shows the responses of this controller to unity step commands for different values of the time delay in the working range. The maximum stable process delay allowed by this controller is $\hat{\tau}_{\max} = 3.767\tau_0 < \tau_{MAX}$. This stability limit for the delay is better than the one achieved with the previous controller, but it still needs some improvement in order to cope with the robustness condition τ_{MAX}.

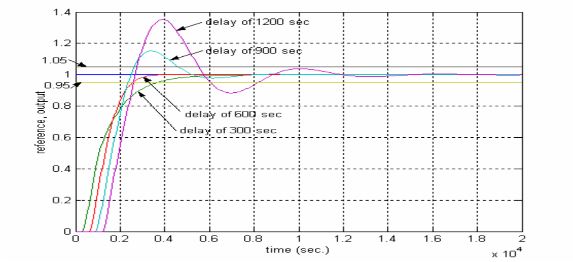

Fig. 6. Step response with a PI controller plus a Smith predictor.

d) Fractional PI controller with Smith predictor

Fig. 7 shows the maximum delay that the control system (13) with the Smith predictor can stabilize in function of the fractional parameter α. This plot has been determined by calculating (for a given value of α) the phase margin for different delay values. The maximum delay allowable in the case of the controller of Subsection IV.c is given by $\alpha = 1$ in Fig. 7: $\hat{\tau}_{\max} = 2260.5\,s$. Moreover we observe in this figure a maximum delay stability margin for $\alpha \approx 0.55$. Notice that the stability margin diminishes abruptly for values of α close but smaller than the optimum.

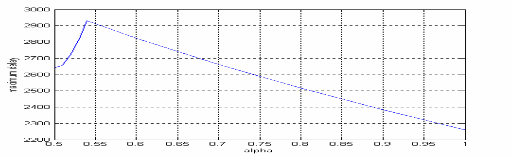

Fig. 7. Stability limit for the delay in function of α.

Taking into account that fractional controllers are approximated by difference equations (see Vinagre et al, 2000, e.g.), then a conservative election is to choose a value of α slightly larger than the optimum. This α would exhibit a stability margin close to the optimum, and large changes in the stability margin caused by the numerical errors produced by the discretization of the fractional operator would be prevented. Then we choose a value of $\alpha = 0.6$ for controller (13). From condition (15) we get now:

$$R(s) = -0.5053\ \left(1 - \frac{1}{29.073 s^{0.6}}\right) \cdot \qquad (18)$$

The responses to step inputs of our canal with this controller are shown in Fig. 8. In this figure and the next ones the response is negative at the beginning, which shows a non-minimum phase behavior. We mention that the output can take negative values because the considered dynamics of the plant is a linearization around a steady state of the canal (given by the water flow). Then variables are incremental.

In this case we get that (see Fig. 7): $\tau_{MAX} < \hat{\tau}_{\max} = 4.7\tau_0$, which fulfills the robustness specifications. But the settling time of the fractional controller with Smith predictor is much larger than the PI controller with Smith predictor. This is because the integral term of the fractional controller is of order α = 0.6. The Final Value Theorem (Ogata, 1993, e.g.) states that the fractional controller exhibits null steady state error if $\alpha > 0$, but the fact of being $\alpha < 1$ makes the output converge to its final value, in the case of the fractional controller, more slowly than in the case of a PI controller.

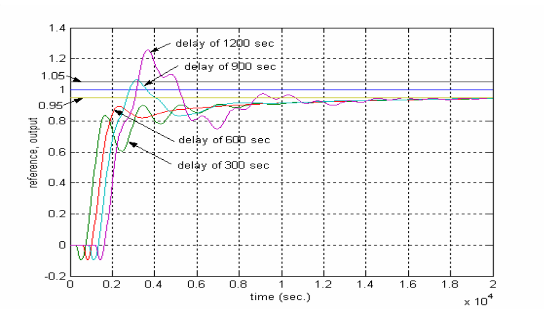

Fig.8. Step response with a PI fractional controller (α=0.6) plus a Smith predictor.

e) Design of a prefilter in order to speed up the output response

We propose in this Subsection the use of a prefilter in order to speed up the response of the process controlled by the fractional controller (18), and to reduce the settling time. Consider expression (11) and that a unity step command is applied. Then the output is of the form:

$$Y(s) = \frac{R(s)G'(s)e^{-\tau s}}{1 + R(s)G'(s)(1 - e^{-\tau_0 s} + e^{-\tau s})} \cdot \frac{1}{s} \cdot \qquad (19)$$

Assume that *R(s)* is of the form (13). If we neglect the fast dynamics components of response (19), we have that the remaining dynamics that dominate the output response at large times is given by:

$$Y(s) \approx \frac{\dfrac{KK_p}{T_i s^{\alpha}} e^{-\tau s}}{1 + \dfrac{KK_p}{T_i s^{\alpha}}} \cdot \frac{1}{s} = \frac{e^{-\tau s}}{1 + \dfrac{T_i}{KK_p} s^{\alpha}} \cdot \frac{1}{s} \qquad (20)$$

This suggests that in order to compensate this long-time dynamics (which is the responsible of the slow convergence of the output to its final value shown in Fig. 8) the step command *C(s)* can be passed through a filter of the form:

$$F(s) = \frac{1 + \dfrac{T_i}{KK_p} s^{\alpha}}{1 + \beta \dfrac{T_i}{KK_p} s} \cdot \qquad (21)$$

This filter is the unity transfer function in the case of the PI controller (α=1) and β=1; in any other case: (α<1) and/or β>1 this is a phase lag filter that smoothes the step reference. Responses in this last case with α=0.6 and β=18.86 are shown in Fig. 9. The modified reference formed by passing the step command through filter (21) is also shown. It can be observed that, compared to responses of the PI controller given in Fig. 6, the settling time of this controller is slightly smaller for values $\tau \le \tau_0$, while is slightly larger for values $\tau > \tau_0$. Overshooting is slightly larger in this controller than with the PI controller. Fig. 10 and 11 show the control signal for the two Smith predictor based control schemes, with PI controller and with fractional PI controller plus prefilter, respectively. Fig. 12 shows the temporal responses of the three control schemes when an off-take discharge is produced (approximately modeled by a step input in *D(s)*). Finally Fig. 13 shows the responses of the canal with controllers (17) and (18) and the prefilter, for delays out of the working range but smaller than the robustness requisite τ_{MAX}.

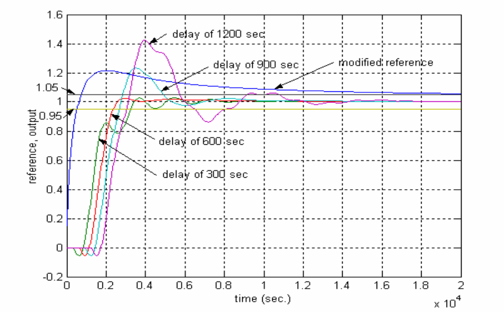

Fig. 9. Step response with a PI fractional controller (α=0.6) plus a Smith predictor and the preshaping filter (21).

V. CONCLUSIONS

- A methodology to design fractional PI controllers combined with Smith predictors for canals robust to changes in the time delay was proposed.

- Simulations show that this fractional PI controller behaves approximately the same as an standard PI controller in the working range of delays, but enlarges the stability range of the delays.
- A new command shaping fractional filter in order to achieve an acceptable settling time for the fractional PI controllers was proposed.
- Simulations showed also some drawbacks of our fractional controller: a) it exhibits a non-minimum phase behavior, b) compensates the effects of off-take discharges more slowly than the other considered control schemes.
- The response of our controller to off-take discharges becomes too slow for values $\alpha < 0.5$. Then practical values of α belong to the interval [0.5-1] considered in Fig. 7.
- Finally we mention that a canal is a process of distributed nature, and a linear dynamic model with fractional derivatives could be more accurate that the standard model used in this paper. We are now studying this possibility. In this case, our control scheme would remain valid with the proper tuning.

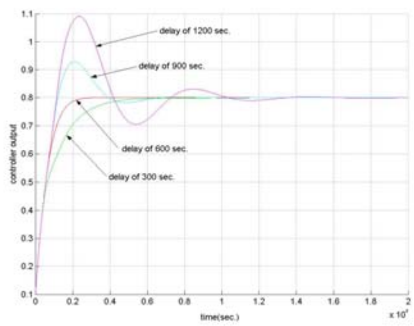

Fig. 10. Shape of the control signal with the PI controller plus Smith predictor.

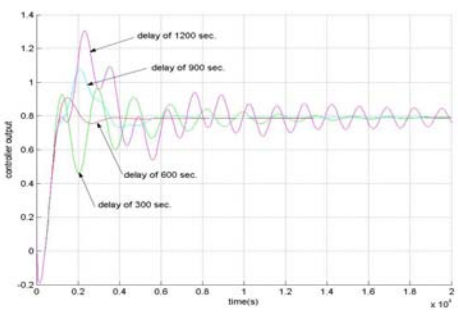

Fig. 11. Shape of the control signal with the PI fractional controller plus Smith predictor.

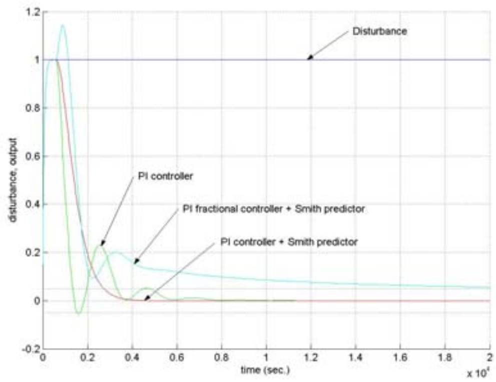

Fig. 12. Disturbance step response with the three control schemes.

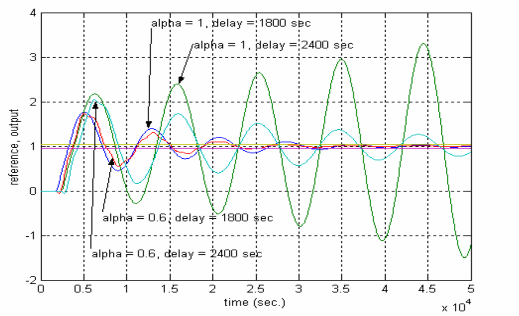

Fig. 13. Step responses with standard PI controller and PI fractional controller (α=0.6) plus a Smith predictor and the preshaping filter.

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