STABILITY MARGINS FOR A RATE–BASED FLOW CONTROL PROBLEM IN MULTIPLE BOTTLENECK NETWORKS

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Abstract: In this paper, the lower bounds for the actual stability margins are derived for a rate–based flow control problem in multiple bottleneck communication networks. Considered stability margins are for the uncertainties in the multiple time–delays and for the rate of change of the time–delays. To observe the change in the actual stability margins with respect to the uncertainties in the time–delays and the rate of change of the time–delays, the lower bounds on the actual stability margins for different cases are depicted.

Keywords: Communication networks, flow control, robust control, stability margin, time-delay, multi-bottleneck networks

1. INRODUCTION

When the load into a data communication network is larger than the amount that the network can handle, congestion occurs and performance of the network degrades. Thus, to avoid congestion, flow control mechanisms are applied. In this way, the performance goals (e.g., satisfying the quality of service demands of the users) can be achieved. Flow control mechanisms avoid congestion by preventing users to send data at rates faster than the rates allowed by the network. The feedback information to be used to adjust the rates of the sources can be either the rate at which the user should transmit or the window size (credit) which is the number of packets that must be sent in a round trip time. Rate-based scheme is chosen as the standard feedback scheme for flow control in available bit rate (ABR) service in asynchronous transfer mode (ATM) networks by the ATM Forum (Bonomi and Fendick, 1995).

Many congestion and flow control algorithms and controller design methods are proposed for resource management in high speed communication networks. In (Altman *et al.*, 1998), a robust flow control algorithm is presented and a decentralized controller for the single bottleneck case for ABR service is designed using the stochastic control approach. The design of a controller for datagram networks with single bottleneck and the stability analysis of the proposed controller is given in (BenMohamed and Meerkov, 1993). The single bottleneck case considered in that work is generalized to the multiple bottleneck case in (BenMohamed and Meerkov, 1997). This work is the first to present the stability analysis of the closed loop system for the multiple bottleneck case. Besides, in (Mascolo, 1997), (Gomez-Stern *et al.*, 2002) and the references therein, Smith Predictor and other methods are used in the design of the controllers for ATM networks.

The design of a controller for the single bottleneck case using $\mathcal{H}_\infty$ control approach is given in (Özbay *et al.*, 1998), (Quet *et al.*, 2002). In these works, the stability and performance analysis for the designed controllers are also given. For the single bottleneck case, lower bounds for the actual stability margins have been derived in (Quet *et al.*, 2002). Depending on these papers, an $\mathcal{H}_\infty$ controller for the multiple bottleneck case is designed in (Biberoviç, 2001), (Biberoviç *et al.*, 2001). The implementation of the designed controller is given in (Munyas *et al.*, 2003). For the flow control problem in multiple bottleneck networks considered in these works, the stability margins for uncertainties in the multiple time–delays and for the rate of change of the time–delays are considered in the present work.

2. PROBLEM STATEMENT AND DERIVATION OF THE STABILITY BOUNDS

The network considered consists of n bottleneck nodes and n_i sources feeding the i^{th} bottleneck node (Biberoviç, 2001), (Biberoviç *et al.*, 2001). Note that, if any physical source sends data to more than one bottleneck node, this source may be considered as a different source for each bottleneck node for the purpose of controller design. It is also assumed that, besides the sources, each bottleneck can also send data through other bottlenecks; i.e., each bottleneck can also be a source for the other bottlenecks. The dynamics of the queue length at the i^{th} bottleneck node can be described as

$$\dot{q}_i(t) = \sum_{k=1}^{n_i} r^b_{i,k}(t) + \sum_{k=1,\ k\neq i}^{n} \rho^b_{k,i}(t) - c_i(t) - \sum_{k=1,\ k\neq i}^{n} \rho^s_{i,k}(t) \quad , \quad (1)$$

with $i = 1, \ldots, n$. Here, $q_i(t)$ is the queue length at the i^{th} bottleneck node at time t; $r^b_{i,k}(t)$ is the rate of data received at the i^{th} bottleneck node from the k^{th} source of the i^{th} bottleneck node at time t; $\rho^b_{k,i}(t)$ is the rate of data received at the i^{th} bottleneck node at time t from the k^{th} bottleneck node; $c_i(t)$ is the outgoing flow rate, except for the flow going to the other bottleneck nodes, of the i^{th} bottleneck node at time t; and $\rho^s_{i,k}(t)$ is the rate of data sent from the i^{th} to the k^{th} bottleneck node at time t. Since there is a time-varying backward delay, $\phi^b_{i,k}(t)$, between the k^{th} and the i^{th} bottleneck nodes, $\rho^s_{i,k}(t) = \rho_{i,k}(t - \phi^b_{i,k}(t))$, where $\rho_{i,k}(t)$ is the flow rate command at time t for the flow from the i^{th} to the k^{th} bottleneck node, which is to be computed (by the controller to be designed) at the k^{th} bottleneck node. Data receiving rates, $r^b_{i,k}(t)$ and $\rho^b_{k,i}(t)$, on the other hand, can be found as (see (Quet *et al.*, 2002), (Biberoviç, 2001))

$$r^b_{i,k}(t) = \begin{cases} (1 - \dot{\delta}^{rf}_{i,k}(t)) r_{i,k}(t - \tau_{i,k}(t)), & t - \tau^f_{i,k}(t) \geq 0 \\ 0, & t - \tau^f_{i,k}(t) < 0 \end{cases} \quad (2)$$

$$\rho^b_{i,k}(t) = \begin{cases} (1 - \dot{\delta}^{\rho f}_{i,k}(t)) \rho_{i,k}(t - \phi_{i,k}(t)), & t - \phi^f_{i,k}(t) \geq 0 \\ 0, & t - \phi^f_{i,k}(t) < 0 \end{cases} \quad (3)$$

where $r_{i,k}(t)$ is the flow rate command at time t for the flow from the k^{th} source of the i^{th} bottleneck node to the i^{th} bottleneck node, which is to be computed (by the controller to be designed) at the i^{th} bottleneck node; $\delta^{rf}_{i,k}(t)$ is the time-varying uncertain part of the forward time-delay, $\tau^f_{i,k}(t)$, from the k^{th} source of the i^{th} bottleneck node to the i^{th} bottleneck node at time t (i.e., $\tau^f_{i,k}(t) = h^{rf}_{i,k} + \delta^{rf}_{i,k}(t)$, where $h^{rf}_{i,k}$ is the time-invariant nominal part); and $\delta^{\rho f}_{i,k}(t)$ is the time-varying uncertain part of the forward time-delay, $\phi^f_{i,k}(t)$, from the i^{th} to the k^{th} bottleneck node at time t (i.e., $\phi^f_{i,k}(t) = h^{\rho f}_{i,k} + \delta^{\rho f}_{i,k}(t)$, where $h^{\rho f}_{i,k}$ is the time-invariant nominal part). Similarly, $\tau_{i,k}(t) = \tau^b_{i,k}(t) + \tau^f_{i,k}(t) = h^r_{i,k} + \delta^r_{i,k}(t)$ is the round-trip delay at time t for the flow from the k^{th} source of the i^{th} bottleneck node to the i^{th} bottleneck node and $\phi_{i,k}(t) = \phi^b_{i,k}(t) + \phi^f_{i,k}(t) = h^\rho_{i,k} + \delta^\rho_{i,k}(t)$ is the round-trip delay at time t for the flow from the i^{th} to the k^{th} bottleneck node. It is assumed that the uncertainties satisfy the following;

$$\begin{aligned} &\left|\delta^r_{i,j}(t)\right| < \delta^{r+}_{i,j}\ , && \left|\dot{\delta}^r_{i,j}(t)\right| < \beta^r_{i,j}\ , \\ &\left|\dot{\delta}^{rf}_{i,j}(t)\right| < \beta^{rf}_{i,j}\ , && \left|\delta^\rho_{i,k}(t)\right| < \delta^{\rho+}_{i,k}\ , \\ &\left|\delta^{\rho b}_{i,k}(t)\right| < \delta^{\rho b+}_{i,k}\ , && \left|\dot{\delta}^\rho_{i,k}(t)\right| < \beta^\rho_{i,k}\ , \\ &\left|\dot{\delta}^{\rho f}_{i,k}(t)\right| < \beta^{\rho f}_{i,k}\ , && \left|\dot{\delta}^{\rho b}_{i,k}(t)\right| < \beta^{\rho b}_{i,k}\ , \end{aligned} \quad (4)$$

for all t, for some *uncertainty bounds*, $\delta^{r+}_{i,j} > 0$, $0 < \beta^{rf}_{i,j} < \beta^r_{i,j} < 1$, $0 < \delta^{\rho b+}_{i,k} < \delta^{\rho+}_{i,k}$, $0 < \beta^{\rho f}_{i,k}, \beta^{\rho b}_{i,k} < \beta^\rho_{i,k} < 1$.

It can be shown that the system is captured by the fictitious system shown in Fig. 1, (Biberoviç, 2001) where $q_d := \left[\, q_{d,1} \cdots q_{d,n} \,\right]^T$, where $q_{d,i}$ is the desired queue length at the i^{th} bottleneck node ($i = 1, \ldots n$), $P_o(s)$ is the nominal plant, W_{22} and W_{21}, which depend on the uncertainty bounds introduced in (4), are weighting matrices, K is the controller to be designed, and Δ^o_{LTV} represents an arbitrary linear time-varying system with $\mathcal{L}^2$-induced norm less than 1 (for details see (Biberoviç, 2001), (Biberoviç *et al.*, 2001)).

For this system to be robustly stable for all $\|\Delta^o_{LTV}\|_\infty < 1$, K should stabilize P_o and,

$$\left\| W_{22} K \left(I + P_o K\right)^{-1} W_{21} \right\|_\infty \leq 1 \quad (5)$$

should be satisfied. Let us define $\bar{W} = diag\,(\xi_1, \ldots, \xi_n)$, where $\xi_i(s) := \frac{1}{s}\xi_{i,1} + \xi_{i,2}$ for all $i = 1, \ldots, n$, where $\xi_{i,1}$ and $\xi_{i,2}$ are given

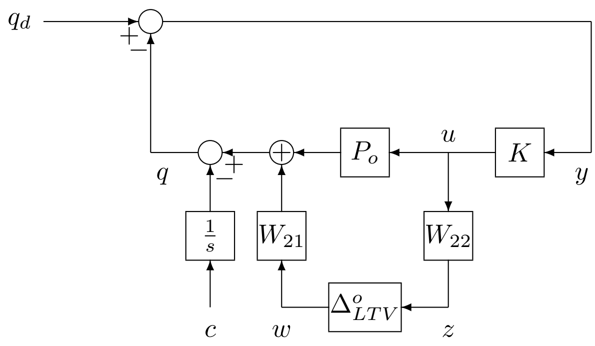

Fig. 1. Fictitious system, (Biberoviç *et al.*, 2001).

by the following expressions respectively,

$$\sqrt{\sum_{k=1}^{n_i}\left(e^r_{i,k,1}\right)^2 + 2\sum_{k=1,\ k\neq i}^{n}\left(e^\rho_{k,i,1}\right)^2 + 2\sum_{k=1,\ k\neq i}^{n}\left(e^{\rho b}_{i,k,1}\right)^2},$$

$$\sqrt{\sum_{k=1}^{n_i}\left(e^r_{i,k,2}\right)^2 + 2\sum_{k=1,\ k\neq i}^{n}\left(e^\rho_{k,i,2}\right)^2 + 2\sum_{k=1,\ k\neq i}^{n}\left(e^{\rho b}_{i,k,2}\right)^2}.$$

where

$$e^r_{i,j,1} = \frac{\sqrt{2}\left(\beta^r_{i,j} + \beta^{rf}_{i,j}\right)}{\sqrt{1-\beta^r_{i,j}}}\ , \qquad e^r_{i,j,2} = 2\sqrt{2}\delta^{r+}_{i,j}\ ,$$

$$e^\rho_{k,i,1} = \frac{\sqrt{2}\left(\beta^\rho_{k,i} + \beta^{\rho f}_{k,i}\right)}{\sqrt{1-\beta^\rho_{k,i}}}\ , \qquad e^\rho_{k,i,2} = 2\sqrt{2}\delta^{\rho+}_{k,i}\ ,$$

$$e^{\rho b}_{i,k,1} = \frac{\sqrt{2}\beta^{\rho b}_{i,k}}{\sqrt{1-\beta^{\rho b}_{i,k}}}\ , \qquad e^{\rho b}_{i,k,2} = 2\sqrt{2}\delta^{\rho b+}_{i,k}\ . \quad (6)$$

for all $i = 1,\ldots,n$, $j = 1,\ldots,n_i$ and $k = 1,\ldots,n, k \neq i$. Noting that $W_{21}W_{21}^* = \bar{W}\bar{W}^*$, it can be shown that (5) is satisfied if,

$$\left\|\hat{P}K\left(I+P_oK\right)^{-1}\bar{W}\right\|_\infty \leq 1 \qquad (7)$$

is satisfied, where $\hat{P} := JW_{22}$, where J is a signature matrix (see (Biberoviç, 2001)). Besides, to achieve good transient response and steady-state tracking goals, nominal performance problem is defined as the minimization of

$$\left\|W_1\ \left(I+P_oK\right)^{-1}\right\|_\infty \qquad (8)$$

with $W_1(s) = \frac{1}{s^2}$. Thus, combining the robust stability and nominal performance problems given in (7) and (8) a two–block $\mathcal{H}^\infty$ optimization problem can be defined as,

$$\inf_{K \text{ stabilizing } P_o}\left\|\begin{bmatrix} W_1\ \left(I+P_oK\right)^{-1} \\ \hat{P}K\left(I+P_oK\right)^{-1}\bar{W}\end{bmatrix}\right\|_\infty =: \gamma^{\text{opt}}\ . \qquad (9)$$

It can be shown that (see (Munyas *et al.*, 2003)) the above MIMO optimization problem is reduced to the following MISO problems;

$$\inf_{\hat{Q}_i\in\mathcal{H}_\infty}\left\|\sum_{j=1}^{n_i}\begin{bmatrix} W_1\alpha^r_{i,j}(1-\frac{1}{\alpha^r_{i,j}}P^r_{i,j}Q^r_{i,j}) \\ \xi_i Q^r_{i,j}\end{bmatrix} + \sum_{j=1,j\neq i}^{n}\begin{bmatrix} W_1\alpha^\rho_{j,i}(1-\frac{1}{\alpha^\rho_{j,i}}P^\rho_{j,i}Q^\rho_{j,i}) \\ \xi_i Q^\rho_{j,i}\end{bmatrix} + \sum_{j=1,j\neq i}^{n}\begin{bmatrix} W_1\alpha^{\rho b}_{i,j}(1-\frac{1}{\alpha^{\rho b}_{i,j}}\hat{P}^{\rho b}_{i,j}\hat{Q}^{\rho b}_{i,j}) \\ \xi_i \hat{Q}^{\rho b}_{i,j}\end{bmatrix}\right\|_\infty := \gamma^{\text{opt}}_i \qquad (10)$$

for $i = 1,\ldots,n$, resulting $\gamma^{\text{opt}} = \max_i \gamma^{\text{opt}}_i$. Here, $\alpha^r_{i,j}$, $\alpha^\rho_{k,i}$, and $\alpha^{\rho b}_{i,k}$ are positive scalars satisfying $\sum_{j=1}^{n_i}\alpha^r_{i,j} + \sum_{j=1,j\neq i}^{n}\alpha^\rho_{j,i} + \sum_{j=1,j\neq i}^{n}\alpha^{\rho b}_{i,j} = 1$, $i = 1,\ldots,n$, introduced to give different steady-state weights to different channels (Munyas *et al.*, 2003). Problems (10) can be decomposed into the following subproblems involving single delays,

$$\inf_{Q^r_{i,j}\in\mathcal{H}_\infty}\left\|\begin{bmatrix} W_1\alpha^r_{i,j}(1-\frac{1}{\alpha^r_{i,j}}P^r_{i,j}Q^r_{i,j}) \\ \xi_i Q^r_{i,j}\end{bmatrix}\right\|_\infty =: \gamma^r_{i,j} \quad (11)$$

$$\inf_{Q^\rho_{j,i}\in\mathcal{H}_\infty}\left\|\begin{bmatrix} W_1\alpha^\rho_{j,i}(1-\frac{1}{\alpha^\rho_{j,i}}P^\rho_{j,i}Q^\rho_{j,i}) \\ \xi_i Q^\rho_{j,i}\end{bmatrix}\right\|_\infty =: \gamma^\rho_{i,j} \quad (12)$$

$$\inf_{\hat{Q}^{\rho b}_{i,j}\in\mathcal{H}_\infty}\left\|\begin{bmatrix} W_1\alpha^{\rho b}_{i,j}(1-\frac{1}{\alpha^{\rho b}_{i,j}}\hat{P}^{\rho b}_{i,j}\hat{Q}^{\rho b}_{i,j}) \\ \xi_i \hat{Q}^{\rho b}_{i,j}\end{bmatrix}\right\|_\infty =: \gamma^{\rho b}_{i,j} \quad (13)$$

where $j = 1,\ldots,n_i$ for the problem defined in (11) and $j = 1,\ldots,n$, $j \neq i$ for the problems defined in (12) and (13). Note that, a suboptimal solution to (10) can be obtained by combining optimal solutions of (11)–(13), since

$$\gamma^{\text{opt}}_i \leq \sum_{j=1}^{n_i}\gamma^r_{i,j} + \sum_{j=1,j\neq i}^{n}\gamma^\rho_{j,i} + \sum_{j=1,j\neq i}^{n}\gamma^{\rho b}_{i,j} =: \gamma^{'}_i\ . \qquad (14)$$

Using the results of (Toker and Özbay, 1995), the optimal solution to each of the problems in (11)–(13) can be obtained. The rest of the design steps and implementation of the designed controller can be found in (Munyas *et al.*, 2003).

Considering the MISO problems in (10) and the inequality (14), the following inequality can be written,

$$\left\|\frac{\xi_i}{\gamma^{'}_i}\left[\sum_{j=1}^{n_i}Q^r_{i,j} + \sum_{j=1,j\neq i}^{n}Q^\rho_{j,i} + \sum_{j=1,j\neq i}^{n}\hat{Q}^{\rho b}_{i,j}\right]\right\|_\infty \leq 1$$

Let $\xi^{act}_i(s) := \frac{1}{s}\xi^{act}_{i,1} + \xi^{act}_{i,2}$, where $\xi^{act}_{i,1}$ and $\xi^{act}_{i,2}$ are defined as $\xi_{i,1}$ and $\xi_{i,2}$, respectively, except that the actual values of the uncertainty bounds, introduced in (4), are used instead of their design values. Assume that

$$\left(\xi_{i,1}^{act}\right)^2 \leq \frac{1}{\gamma_i^{'2}}\left(\xi_{i,1}\right)^2, \qquad \left(\xi_{i,2}^{act}\right)^2 \leq \frac{1}{\gamma_i^{'2}}\left(\xi_{i,2}\right)^2 \quad (15)$$

are satisfied for all $i = 1, \ldots, n$. Then,

$$\sup_{\omega\in\mathbb{R}}\left|\xi_i^{act}(j\omega)\right| \leq \sup_{\omega\in\mathbb{R}}\left|\frac{\xi_i(j\omega)}{\gamma_i'}\right| \quad (16)$$

is satisfied. This implies that the robust stability condition in (7) and in turn (5) is satisfied. Therefore, by re-writing (15), if the following inequalities are satisfied, robust stability of the system is guaranteed,

$$\left\{\sum_{j=1}^{n_i}\left(e_{i,j,1}^{r,act}\right)^2 + \sum_{j=1,j\neq i}^{n}\left(e_{j,i,1}^{\rho,act}\right)^2 + \sum_{j=1,j\neq i}^{n}\left(e_{i,j,1}^{\rho b,act}\right)^2\right\} \leq \frac{1}{\gamma_i^{'2}}\left\{\sum_{j=1}^{n_i}\left(e_{i,j,1}^{r}\right)^2 + \sum_{j=1,j\neq i}^{n}\left(e_{j,i,1}^{\rho}\right)^2 + \sum_{j=1,j\neq i}^{n}\left(e_{i,j,1}^{\rho b}\right)^2\right\} \quad (17)$$

and

$$\left\{\sum_{j=1}^{n_i}\left(e_{i,j,2}^{r,act}\right)^2 + \sum_{j=1,j\neq i}^{n}\left(e_{j,i,2}^{\rho,act}\right)^2 + \sum_{j=1,j\neq i}^{n}\left(e_{i,j,2}^{\rho b,act}\right)^2\right\} \leq \frac{1}{\gamma_i^{'2}}\left\{\sum_{j=1}^{n_i}\left(e_{i,j,2}^{r}\right)^2 + \sum_{j=1,j\neq i}^{n}\left(e_{j,i,2}^{\rho}\right)^2 + \sum_{j=1,j\neq i}^{n}\left(e_{i,j,2}^{\rho b}\right)^2\right\} \quad (18)$$

for $i = 1, \ldots, n$, where the actual stability margins for $e_{i,j,1}^{\cdot}$ and for $e_{i,j,2}^{\cdot}$ are denoted by $e_{i,j,1}^{\cdot,act}$ and $e_{i,j,2}^{\cdot,act}$, respectively. Here, the superscript $\cdot$ represents r, ρ or ρb, $i = 1, \ldots, n$ and $j = 1, \ldots, n_i$ for the sources and $j = 1, \ldots, n, j \neq i$ for the nodes. It can be seen that the lower bounds for the actual stability margins for each node can be calculated independent from the other nodes. Since the number of sources connected to a node may be greater than 1, then, the inequalities in (17) and (18) lead to infinitely many solutions for the lower bounds and any of the solutions will provide robust stability of the system.

3. RESULTS

To observe the effects of the uncertainty bounds chosen to design the controller, the lower bounds on the actual stability margins satisfying (17) and (18) are depicted for a number of example cases. When these inequalities are considered, it is seen that summations of three terms exist on both sides of both inequalities. For both inequalities, for the summation on the left hand side to be less than or equal to the one on the right hand side a sufficient condition is that $\sum_j\left(e_{i,j,k}^{\cdot,act}\right)^2 \leq \frac{1}{\gamma_i^{'2}}\sum_j\left(e_{i,j,k}^{\cdot}\right)^2$ where the superscript $\cdot$ represents r, ρ or ρb, $i = 1, \ldots, n$ and $j = 1, \ldots, n_i$ for the sources and $j = 1, \ldots, n, j \neq i$ for the nodes and $k = 1, 2$. Then, the following terms are defined,

$$e_{i,k}^{r,act} := \sqrt{\sum_{j=1}^{n_i}\left(e_{i,j,k}^{r,act}\right)^2}, \quad e_{i,k}^{\rho,act} := \sqrt{\sum_{j=1,j\neq i}^{n}\left(e_{j,i,k}^{\rho,act}\right)^2},$$

$$e_{i,k}^{\rho b,act} := \sqrt{\sum_{j=1,j\neq i}^{n}\left(e_{i,j,k}^{\rho b,act}\right)^2}, \qquad k = 1, 2.$$

Here, $e_{i,1}^{r,act}$ gives a measure for the actual stability margin relating to the rate of change of $\delta_{i,j}^{r}(t)$ and $\delta_{i,j}^{rf}(t)$, $j = 1, \ldots, n_i$; $e_{i,1}^{\rho,act}$ gives a measure for the actual stability margin relating to the rate of change of $\delta_{i,j}^{\rho}(t)$ and $\delta_{i,j}^{\rho f}(t)$, $j = 1, \ldots, n, j \neq i$, and $e_{i,1}^{\rho b,act}$ gives a measure for the actual stability margin relating to the rate of change of $\delta_{i,j}^{\rho b}(t)$, $j = 1, \ldots, n, j \neq i$. Similarly, $e_{i,2}^{\cdot,act}$ gives a measure for the actual stability margin relating to the magnitude of the same variables. Thus, to observe the effect of the uncertainty bounds on the actual stability margins, $e_{i,k}^{\cdot,act}$ is calculated and depicted.

Due to space limitations only one example case is included here. Further cases may be found in (Munyas and İftar, 2004). The example network consists of 3 nodes and there are 2, 3 and 4 sources connected to the 1^{st}, 2^{nd} and 3^{rd} node, respectively. The nominal time-delays and design parameters used are given in Table 1 and Table 2, respectively. While $\beta_{1,1}^{r}$, $\beta_{1,2}^{\rho}$ and $\beta_{3,2}^{\rho b}$ are changed from 0.001 to 0.999, $\delta_{1,1}^{r+}$ is changed from 0.001 to 3.5, $\delta_{1,2}^{\rho+}$ is changed from 0.001 to 4 and $\delta_{1,1}^{\rho b+}$ is changed from 0.001 to 3. For this network, in the calculation of the actual stability margins, $\beta_{1,1}^{r}$ and $\delta_{1,1}^{r+}$ are changed for the 1^{st} node; $\beta_{1,2}^{\rho}$ and $\delta_{1,2}^{\rho+}$ are changed for the 2^{nd} node and $\beta_{3,2}^{\rho b}$ and $\delta_{3,2}^{\rho b+}$ are changed for the 3^{rd} node. Meanwhile, $\beta_{1,1}^{rf} = \beta_{1,2}^{\rho f} = 0$ and all the other design parameters for the three nodes are held constant at their design values. For cases in which $\beta_{i,j}^{\cdot f}$ is taken as $\frac{1}{2}\beta_{i,j}^{\cdot}$ or $\beta_{i,j}^{\cdot}$ and for cases where different network conditions and parameter values are considered, see (Munyas and İftar, 2004).

The results are given in Figures 2 $\sim$ 10. Fig. 2 indicates that, as $\beta_{1,1}^{r}$, the design bound on $\dot{\delta}_{1,1}^{r}(t)$, is increased, the stability margin on $\dot{\delta}_{1,1}^{r}(t)$ is also increased, indicated by the increase in $e_{1,1}^{r,act}$. Figures 2 $\sim$ 4 also indicate that, when $\beta_{1,1}^{r}$ is changed and all other uncertainty bounds are kept constant, the values of $e_{1,2}^{r,act}$, $e_{1,k}^{\rho,act}$ and $e_{1,k}^{\rho b,act}$, $k = 1, 2$, remain almost constant except when $\beta_{1,1}^{r}$ is made too close to 1. This indicates that the stability margins on $\delta_{1,j}^{r}(t)$, $\delta_{1,j}^{\rho}(t)$, $\delta_{1,j}^{\rho b}(t)$, $\dot{\delta}_{1,j}^{\rho}(t)$ and $\dot{\delta}_{1,j}^{\rho b}(t)$ are insensitive to changes in $\beta_{1,1}^{r}$ except when $\beta_{1,1}^{r}$ is too close to 1. As $\beta_{1,1}^{r}$ gets close to 1, γ_1' increases without bounds, driving

Table 1. Nominal time delays.

j	$h^r_{1,j}$	$h^\rho_{1,j}$	$h^{\rho b}_{1,j}$	$h^r_{2,j}$	$h^\rho_{2,j}$	$h^{\rho b}_{2,j}$	$h^r_{3,j}$	$h^\rho_{3,j}$	$h^{\rho b}_{3,j}$
1	1	−−	−−	2	3	2	2.5	2	1
2	1	2	1	2	−−	−−	2.5	1	0.5
3	−−	3	1.5	2	3.5	3	2.5	−−	−−
4	−−	−−	−−	−−	−−	−−	2.5	−−	−−

$e^{\cdot,act}_{1,k}$, except $e^{r,act}_{1,1}$, to zero. Figures 2 ∼ 4 further indicate that as $\delta^{r+}_{1,1}$, the design bound on $\delta^r_{1,1}(t)$, is increased, $e^{r,act}_{1,2}$ increases, but $e^{r,act}_{1,1}$, $e^{\rho,act}_{1,k}$ and $e^{\rho b,act}_{1,k}$, $k = 1, 2$, remain almost constant as long as the other uncertainty bounds are kept constant. Similar conclusions are drawn from Figures 5 ∼ 7 when r is replaced by ρ and from Figures 8 ∼ 10 when r is replaced by ρb. In (Munyas and İftar, 2004), it is also shown that the effect of larger $\beta^{rf}_{i,j}$ and of $\beta^{\rho f}_{i,j}$ is to increase $e^{r,act}_{i,1}$ and $e^{\rho,act}_{i,1}$, respectively. The effects of changing the uncertainty bounds are summarized in Table 3, where 1 means that the stability margin increases with increasing bound, 0 means that the stability margin is insensitive to changes in the bound and −1 means that the stability margin is insensitive to changes in the bound except when the bound gets too close to 1.

4. CONCLUSION

In this work, stability margins for uncertainties in the multiple time–delays and for the rate of change of the time–delays are considered. The lower bounds for the stability margins have been derived for a rate–feedback flow control problem in multiple bottleneck networks. According to the sufficient conditions obtained, the lower bounds on the actual stability margins are depicted with respect to the bounds on the uncertainties for various cases. The results show that when the bounds on the magnitude of the uncertainties and the rate of change of the uncertainties in the time–delays are increased, the corresponding stability margins also increase.

However, the results of (Munyas *et al.*, 2003) indicate that, the controller designed with high uncertainty levels will be conservative, resulting in a smooth but slow response. When these bounds are decreased, the response becomes more oscillatory but faster (Munyas *et al.*, 2003). In addition, the bound on the rate of change of the delay uncertainty should not be chosen close to 1, since this may reduce some margins.

Table 2. Design parameters.

i,j	1,1	1,2	1,3	2,1	2,2	2,3	3,1	3,2	3,3	3,4
$\alpha^r_{i,j}$	0.1	0.15	−−	0.2	0.15	0.05	0.08	0.12	0.06	0.09
$\alpha^\rho_{i,j}$	−−	0.2	0.2	0.25	−−	0.3	0.35	0.25	−−	−−
$\alpha^{\rho b}_{i,j}$	−−	0.05	0.1	0.08	−−	0.07	0.05	0.1	−−	−−
$\beta^r_{i,j}$	0.1	0.1	−−	0.15	0.15	0.15	0.2	0.2	0.2	0.2
$\beta^{rf}_{i,j}$	0.02	0.02	−−	0.03	0.03	0.03	0.04	0.04	0.04	0.04
$\beta^\rho_{i,j}$	−−	0.25	0.3	0.3	−−	0.33	0.4	0.1	−−	−−
$\beta^{\rho b}_{i,j}$	−−	0.1	0.15	0.15	−−	0.2	0.25	0.05	−−	−−
$\beta^{\rho f}_{i,j}$	−−	0.15	0.15	0.15	−−	0.13	0.15	0.05	−−	−−
$\delta^{r+}_{i,j}$	1	1	−−	2	2	2	2.5	2.5	2.5	2.5
$\delta^{\rho+}_{i,j}$	−−	2	3	3	−−	3.5	2	1	−−	−−
$\delta^{\rho b+}_{i,j}$	−−	1	1.5	2	−−	3	1	0.5	−−	−−

Table 3. Effects of the uncertainty bounds on the stability margins

	$\delta^r_{i,j}(t)$	$\dot\delta^r_{i,j}(t)$	$\delta^\rho_{i,j}(t)$	$\dot\delta^\rho_{i,j}(t)$	$\delta^{\rho b}_{i,j}(t)$	$\dot\delta^{\rho b}_{i,j}(t)$
$\beta^r_{i,j}$	−1	1	−1	−1	−1	−1
$\beta^{rf}_{i,j}$	−1	1	−1	−1	−1	−1
$\beta^\rho_{i,j}$	−1	−1	−1	1	−1	−1
$\beta^{\rho f}_{i,j}$	−1	−1	−1	1	−1	−1
$\beta^{\rho b}_{i,j}$	−1	−1	−1	−1	−1	1
$\delta^{r+}_{i,j}$	1	0	0	0	0	0
$\delta^{\rho+}_{i,j}$	0	0	1	0	0	0
$\delta^{\rho b+}_{i,j}$	0	0	0	0	1	0

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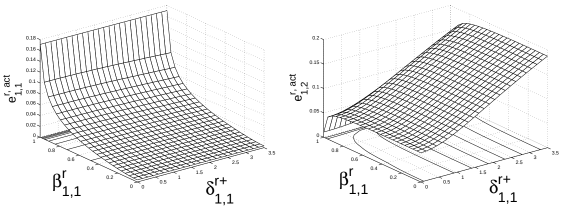

Fig. 2. Stability margins $e^{r,act}_{1,1}$ and $e^{r,act}_{1,2}$.

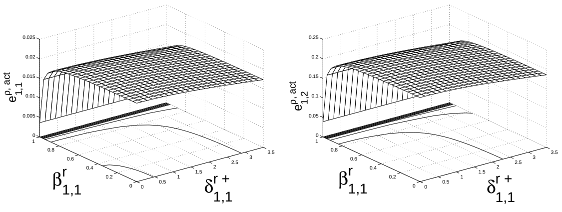

Fig. 3. Stability margins $e^{\rho,act}_{1,1}$ and $e^{\rho,act}_{1,2}$.

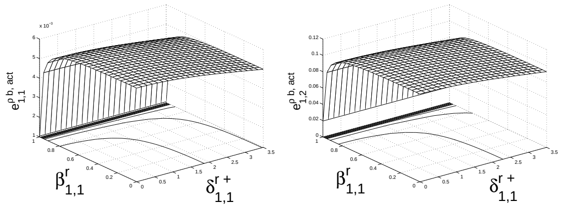

Fig. 4. Stability margins $e^{\rho b,act}_{1,1}$ and $e^{\rho b,act}_{1,2}$.

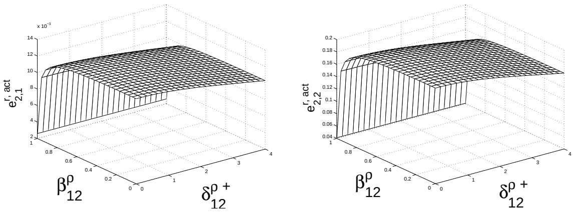

Fig. 5. Stability margins $e^{r,act}_{2,1}$ and $e^{r,act}_{2,2}$.

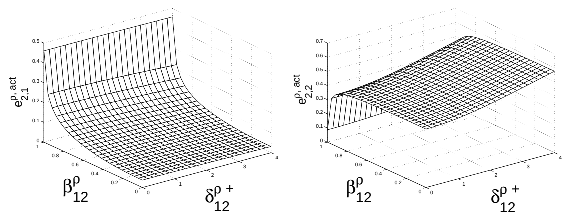

Fig. 6. Stability margins $e^{\rho,act}_{2,1}$ and $e^{\rho,act}_{2,2}$.

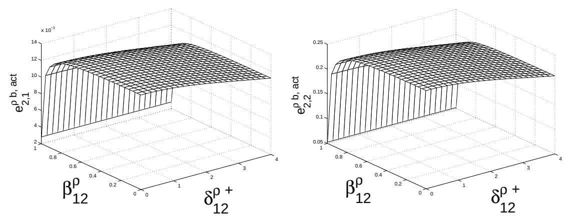

Fig. 7. Stability margins $e^{\rho b,act}_{2,1}$ and $e^{\rho b,act}_{2,2}$.

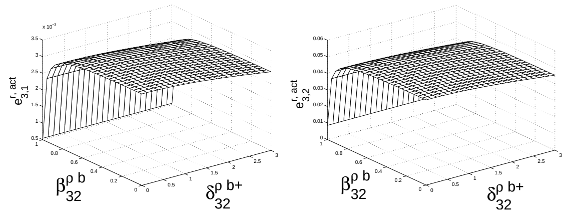

Fig. 8. Stability margins $e^{r,act}_{3,1}$ and $e^{r,act}_{3,2}$.

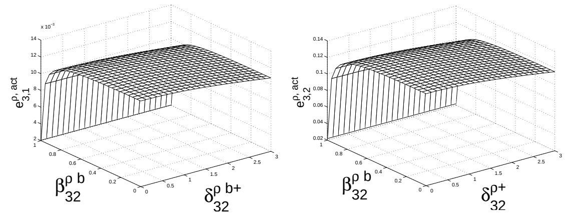

Fig. 9. Stability margins $e^{\rho,act}_{3,1}$ and $e^{\rho,act}_{3,2}$.

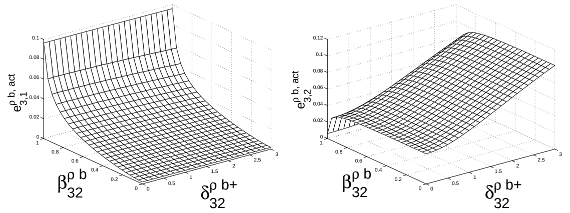

Fig. 10. Stability margins $e^{\rho b,act}_{3,1}$ and $e^{\rho b,act}_{3,2}$.