INTERVAL ANALYSIS AND NONLINEAR CONTROL OF WASTEWATER PLANTS WITH PARAMETER UNCERTAINTY

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Abstract: This paper presents the successful application of interval arithmetics to a simplified activated sludge model that describes the reduction of biodegradable substrate in biological wastewater treatment. Reliable analysis of the steady-state behaviour as well as plant control have to account for the dominant uncertain system parameter given by the maximum specific growth rate of biomass. The proposed control strategy consists of nonlinear control of oxygen concentration using desired trajectories derived from interval evaluations of the uncertain steady-state substrate concentration. By this, plant operating costs can be significantly reduced resulting in superior plant efficiency.

Keywords: Intervals, nonlinearity, uncertain dynamic systems, flatness-based control, water pollution

1. INTRODUCTION

Analysis and control of nonlinear systems with uncertain system parameters represent problems of intensive current research interest. Usually, guaranteed, i.e. conservative, bounds for all state variables of a nonlinear state space model are to be determined. Moreover, these conservative enclosures should be as tight as possible to the actual bounds of the state variables in order to minimize overestimation. Unfortunately, these bounds cannot be explicitly calculated in most cases. In this paper, the range of these uncertain parameters is only specified by their lower and upper bounds. Additional knowledge about the distribution of the parameters within the specified range is neither available nor required.

Traditionally, the analysis of nonlinear systems with uncertain parameters is performed either by Monte-Carlo simulations or by the application of other stochastic simulation techniques. In both cases, the main drawback is given by the fact that distributions of the uncertain parameters have to be provided. If only lower and upper bounds of the range of the uncertain parameters are known, a rectangular distribution of random numbers can be utilized as approximation for each of the system parameters. Alternatively, grids are often applied to determine various sets due to uncertain system parameters. Nevertheless, the efficiency of Monte-Carlo or other grid-based simulation techniques decreases rapidly for higher-dimensional systems while the computational effort often increases exponentially. Furthermore, most sim-

Table 1. Nominal values of the system parameters.

parameter	description	nominal value
V_A	volume of the aeration tank	$8000\ \mathrm{m}^3$
V_{Set}	volume of the settler	$4545\ \mathrm{m}^3$
Q_W	influent wastewater flow rate	$0.153\ \mathrm{m}^3/\mathrm{s}$
Q_{RS}	flow rate of return sludge	$0.0916\ \mathrm{m}^3/\mathrm{s}$
Q_{EX}	flow rate of excess sludge	$0.005\ \mathrm{m}^3/\mathrm{s}$
S_W	influent biodegradable substrate concentration	$0.616\ \mathrm{kg/m}^3$
S_{OW}	influent oxygen concentration in the wastewater	$0.5 \cdot 10^{-3}\ \mathrm{kg/m}^3$
$S_{O,sat}$	saturation concentration of dissolved oxygen	$5.3 \cdot 10^{-3}\ \mathrm{kg/m}^3$
Y	yield coefficient of heterotrophic biomass	0.67
$\hat{\mu}_{nom}$	max. specific growth rate of heterotrophic biomass	$1/14400\ 1/\mathrm{s}$
b	specific decay rate of heterotrophic biomass	$7.176 \cdot 10^{-6}\ 1/s$
K_S	half saturation coefficient for heterotrophic biomass	$0.02\ \mathrm{kg/m}^3$
K_{OS}	oxygen half saturation coefficient	$2 \cdot 10^{-4}\ \mathrm{kg/m}^3$
u_{O2}	influent oxygen flow rate (constant)	$1.487\ \mathrm{m}^3/\mathrm{s}$
ρ_{O2}	normal density of molecular oxygen	$1.428\ \mathrm{kg/m}^3$

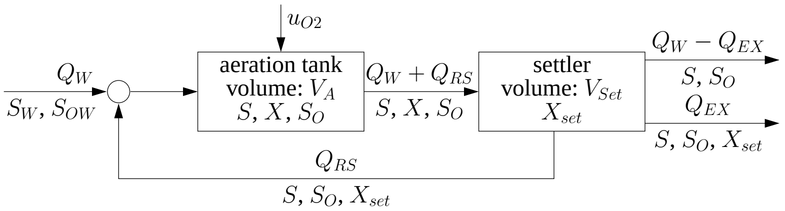

Fig. 1. Block diagram of the simplified wastewater treatment plant.

ulation algorithms applying these techniques cannot guarantee conservative enclosures of the state variables, i.e. in most cases it cannot be determined whether it is possible that state variables reach values outside the range determined by these algorithms.

On the contrary, the interval arithmetic approach allows for calculating guaranteed upper and lower bounds for all state variables of a nonlinear system. As compared to natural interval arithmetics, overestimation effects typically encountered in interval evaluations can be reduced by optimized interval arithmetics. The improvements stem from higher order methods for interval evaluation as well as efficient splitting and merging strategies of state and parameter intervals.

Calculation of guaranteed enclosures becomes indispensable if safety-critical or environmentally hazardous systems are to be analyzed. Here, an approach solely based on a set of nominal values of all system parameters may be deceptive. System design should provide the engineer with reliable results whether it is possible to guarantee specified limitations, especially to exclude hazardous operating conditions. In this article, interval arithmetics is applied to a subproblem of the activated sludge process in biological wastewater treatment (Köhne, 1998). In contrast to the widely used Activated Sludge Model ASM1 of the International Association on Water Quality (IAWQ), this subprocess only regards the reduction of organic matter (substrate). Nitrogen fractions of the wastewater and their removal are not adressed here. However, an extension of the analysis to the complete ASM1 model is straightforward and subject to future research.

The paper is structured as follows. First, the modelling of the simplified wastewater treatment plant is presented in detail. Second, a brief overview of natural and optimized interval methods for the analysis of nonlinear systems with uncertain parameters is given. Third, optimized interval arithmetics is applied to the steady-state analysis of the considered system with an uncertainty of the specific growth rate. Fourth, a flatness-based control of the oxygen concentration is described. Using the steady-state characteristic of the substrate concentration as well as the admissible substrate concentration at the plant output, desired trajectories for the oxygen concentration can be derived. The applicability and effectiveness of the interval approach is emphasized by a comparison of symbolic calculation and interval evaluation.

2. WASTEWATER PLANT MODELLING

As depicted in Fig. 1, the considered wastewater treatment plant consists of two tanks: an aeration tank (volume V_A) with activated sludge, where the biological reduction of organic matter takes place, and the settler tank (volume V_{Set}), where the cleaned water and the sludge are separated. The aeration tank is equipped with an oxygen supply u_{O2}, which represents the physical control input for the subsidiary control of the oxygen concentration in this tank. The system can be described by introducing four state variables. Three of them are related to the activated sludge tank: the concentration of biodegradable substrate S, the concentration of substrate consuming bacteria X and the oxygen concentration S_O in the sludge tank. The fourth state variable is given by the bacteria concentration X_{Set} in the settler.

The influent volume flow of the activated sludge tank is composed of two parts. The first part is given by a wastewater volume flow Q_W with substrate concentration S_W, oxygen concentration S_{OW} that does not contain any substrate consuming bacteria, i.e. $X = 0$. The second part stems from the returned volume flow Q_{RS} with substrate concentration S, oxygen concentration S_O, and bacteria concentration X_{Set}. The volume flow $Q_W + Q_{RS}$ at the exit of the sludge tank is of substrate concentration S, oxygen concentration S_O, and bacteria concentration X. The settler is modelled as an ideal separator. The volume flow $Q_W - Q_{EX}$ of separated cleaned water with substrate concentration S and oxygen concentration S_O represents the plant output. Here, the given substrate concentration has to be smaller than a specified limit value S_{lim} according to legal regulations. The volume flow Q_{EX} denotes the excess sludge of bacteria concentration X_{Set} that is continuously removed from the process.

Based on these volume flows, mass balance equations for each of the components result in a system of four non-linear first-order ordinary differential equations

$$\begin{aligned}
\dot{S} =& \frac{Q_W}{V_A}\left(S_W - S\right) - \mu\left(S, S_O\right)\frac{1}{Y}X, \\
\dot{X} =& -\frac{Q_W}{V_A}X + \frac{Q_{RS}}{V_A}\left(X_{Set} - X\right) \\
&+ \left(\mu\left(S, S_O\right) - b\right)X, \\
\dot{S}_O =& \frac{Q_W}{V_A}\left(S_{OW} - S_O\right) - \mu\left(S, S_O\right)\frac{1-Y}{Y}X \\
&+ \frac{\rho_{O2}}{V_A}\left(1 - \frac{S_O}{S_{O,sat}}\right)u_{O2}, \\
\dot{X}_{Set} =& \frac{\left(Q_W + Q_{RS}\right)X - \left(Q_{EX} + Q_{RS}\right)X_{Set}}{V_{Set}},
\end{aligned} \tag{1}$$

with the nonlinear specific growth rate

$$\mu\left(S, S_O\right) = \hat{\mu}\frac{S}{S + K_S}\frac{S_O}{S_O + K_{OS}} \quad . \tag{2}$$

As all concentrations must be positive from physical considerations, the inequalities

$$S \geq 0,\ X \geq 0,\ 0 \leq S_O \leq S_{O,sat},\ X_{Set} \geq 0 \tag{3}$$

for the state variables have to be satisfied. Moreover, the concentration S_O of dissolved oxygen is limited by the saturation concentration $S_{O,sat}$. The nominal values of all system parameters are stated in Tab. 1.

3. INTERVAL ARITHMETIC SYSTEM ANALYSIS

In this section, the major properties of interval arithmetic evaluation of nonlinear functions are summarized (Rump, 1996). For further details about the theory of interval arithmetics see e.g. (Moore, 1979).

3.1 Interval Evaluation of Nonlinear Functions

The intuitive extension of algebraic functions to interval arithmetics is referred to as natural interval evaluation. All basic algebraic operations like addition, subtraction, multiplication and division are replaced by their interval equivalents. The resulting lower and upper bounderies are expressions of the bounds of the corresponding operands.

Instead of applying natural interval evaluation directly to the function f, higher-order methods based on interval Taylor series of the nonlinear function f are often preferred (Rauh *et al.*, 2004*a*). In this paper, discussion is restricted to the midpoint-rule

$$f\left(x\right) \subseteq f_M\left(x\right) = f\left(x_m\right) + \frac{\partial f}{\partial x}\left(x\right)\left(x - x_m\right) \quad , \tag{4}$$

which is a zero-order Taylor series expansion of the function f at the midpoint $x_m = \frac{1}{2}\left(\underline{x} + \overline{x}\right)$ with an interval evaluation of the first-order remainder term.

Such higher-order methods often yield tighter approximations of the solution intervals than simple natural interval evaluation. However, this cannot always be guaranteed. Therefore, the algorithm proposed in this paper takes advantage of an intersection of the results of both natural interval evaluation and midpoint-rule.

3.2 Reduction of Overestimation

Two principle kinds of overestimation can be distinguished. First, the maximum and minimum values of the state intervals cannot always be determined exactly if just natural interval evaluation or higher-order methods are applied. Second, using only one axis-parallel interval box it is impossible to represent complexly shaped regions of the state variables in the state space even if the exact infimum and supremum of the sets can be determined. If this shape is to be approximated with higher accuracy, efficient splitting and merging techniques have to be introduced. At this, is becomes important that in interval arithmetics the property of subdistributivity

$$x\left(y + z\right) \subseteq xy + xz \tag{5}$$

is valid with $z \in [\underline{z}\ ;\ \overline{z}]$. Another very important property of interval arithmetics is inclusion monotonicity

$$\begin{aligned}
&x_i \subseteq y_i \quad (i = 1, \ldots, n) \\
&\qquad \Longrightarrow f\left(x_1, \ldots, x_n\right) \subseteq f\left(y_1, \ldots, y_n\right) \quad ,
\end{aligned} \tag{6}$$

where f is the interval extension of an analytical function. Splitting an interval x into l subintervals x_i, $i = 1, \ldots, l$,

$$\bigcup_{i=1}^{l} x_i = x, \quad x_i \cap x_j = \{\} \quad (i \neq j) \tag{7}$$

inclusion monotonicity directly yields

$$\bigcup_{i=1}^{l} f\left(x_i\right) \subseteq f\left(x\right) \quad . \tag{8}$$

3.3 Optimized Interval Evaluation

Since the exact infimum and supremum of the range of nonlinear functions cannot always be determined, optimized interval evaluation is employed to determine a single tight interval box enclosing possible function values.

If a single interval box is to be determined monotonicity properties of nonlinear functions should be exploited in order to reduce overestimation. Herefore, the sign of the entries of the interval Jacobian have to be checked. In case of monotonicity the input intervals of the function are replaced by infima or suprema of the corresponding intervals. For further reduction of overestimation and an improved approximation of complexly shaped regions of state variables in the state space splitting into subintervals has proven an effective means. At this, two effects can be exploited. First, the approximation of not-axis-parallel bounderies becomes possible. Second, overestimation involved with the evaluation of a subinterval is smaller as compared to the original interval. Due to inclusion monotonicity the hull of the subintervals provides a better approximation of the range as compared to the evaluation of a function using only one interval (Rauh *et al.*, 2004*b*).

If the monotonicity test is not successful, the input intervals can be split into several subintervals. The evaluation of all these subintervals will lead to tighter approximations of the exact set of state variables. If only the upper and lower bounds of the state variables are desired, the union of all considered subintervals is given by the smallest lower and largest upper bound w.r.t. these subintervals.

4. STEADY-STATE ANALYSIS

For the steady-state analysis the fact is exploited that all states S, S_O, X, X_{Set}, all system parameters Y, b, $\hat{\mu}$, K_S, K_{OS},V_A, V_{Set} as well as volume flow rates Q_A, Q_{RS}, Q_{EX} are positive. The nonlinear system of equations $\dot{x}(t) = 0 = f(x)$ is to be solved in order to obtain the steady-state state variables of the open-loop system. This analysis starts by eliminating X_{Set} according to

$$X_{Set} = \frac{Q_W + Q_{RS}}{Q_{EX} + Q_{RS}} \cdot X \tag{9}$$

in the remaining state equations. Consequently, the steady-state concentrations of X and X_{Set} are proportional to each other. The steady-state equation for the bacteria concentration becomes

$$\begin{aligned}\dot{X} = 0 = -(&\frac{Q_W}{V_A} + \frac{Q_{RS}}{V_A} - \mu + b \\ &- \frac{Q_{RS}}{V_A} \cdot \frac{Q_W + Q_{RS}}{Q_{EX} + Q_{RS}}) \cdot X \ .\end{aligned} \tag{10}$$

One solution is related to the case of minor technical interest, where all bacteria were flushed out of the sludge tank, i.e. $X = 0$. Hence, the second solution given by the expression in brackets is regarded further. A rearrangement leads directly to the following expression

$$\mu = \frac{Q_W}{V_A} + \frac{Q_{RS}}{V_A} - b - \frac{Q_{RS}}{V_A} \cdot \frac{Q_W + Q_{RS}}{Q_{EX} + Q_{RS}} \ . \tag{11}$$

Analogously, the steady-state equation for the substrate concentration follows from $\dot{S} = 0$ and can be brought into the form

$$\mu \cdot \frac{1}{Y} \cdot X = \frac{(S_W - S) \cdot Q_W}{V_A} \ . \tag{12}$$

Inserting this expression in $\dot{S}_O = 0$ results in a linear relationship between the steady-state values of S and S_O

$$\begin{aligned} S_O = &\underbrace{\frac{1}{\frac{Q_W}{V_A} + \frac{\rho_{O2} u_{O2}}{V_A S_{O,sat}}}}_{\alpha} \cdot \Bigg(\frac{Q_W}{V_A} \cdot S_{OW} \\ &+ \frac{\rho_{O2} u_{O2}}{V_A} - (1-Y) \cdot \frac{(S_W - S) \cdot Q_W}{V_A} \Bigg) \\ &= \alpha \cdot (1-Y) \cdot \frac{Q_W}{V_A} \cdot S \\ &+ \alpha \cdot \Bigg(\frac{Q_W}{V_A} \cdot S_{OW} + \frac{\rho_{O2} u_{O2}}{V_A} \\ &-(1-Y) \cdot \frac{Q_W}{V_A} \cdot S_W \Bigg) = \beta_1 \cdot S + \beta_2 \ . \end{aligned} \tag{13}$$

Both constants $\beta_1 \geq 0$ and $\beta_2 \geq 0$ are positive for the nominal values of the system parameters. Elimination of S_O in $\mu(S, S_O)$ yields a quadratic equation for S

$$\begin{aligned} S^2 + &\underbrace{\left(\frac{\beta_2}{\beta_1} + \frac{\beta_1 K_S + K_{OS}}{\beta_1 \left(1 - \frac{\hat{\mu}}{\mu}\right)} \right)}_{\gamma_1} S \\ &+ \underbrace{\frac{(\beta_2 + K_{OS}) K_S}{\beta_1 \left(1 - \frac{\hat{\mu}}{\mu}\right)}}_{\gamma_2} = 0 \ , \end{aligned}$$

which can be solved symbolically. Considering $\gamma_2 < 0$, the single positive solution is given by

$$S = -\frac{\gamma_1}{2} + \sqrt{\left(\frac{\gamma_1}{2}\right)^2 - \gamma_2} \ . \tag{14}$$

Finally, the steady-state expressions for the system states become

$$\begin{aligned} S^* &= -\frac{\gamma_1}{2} + \sqrt{\left(\frac{\gamma_1}{2}\right)^2 - \gamma_2} \ , \\ S_O^* &= \beta_1 \cdot S^* + \beta_2 \ , \\ X^* &= Y \frac{1}{\mu} \frac{(S_W - S^*) \, Q_W}{V_A} \ , \\ X_{Set}^* &= \frac{Q_W + Q_{RS}}{Q_{EX} + Q_{RS}} \cdot \left(Y \frac{1}{\mu} \frac{(S_W - S^*) \, Q_W}{V_A} \right) \ . \end{aligned} \tag{15}$$

By inserting the nominal values of Tab. 1 in these expressions, the steady-state values result in $S^* =$

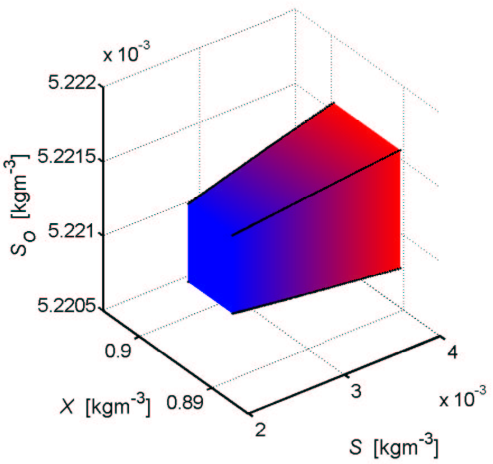

Fig. 2. Steady-state state variables as function of the uncertain growth rate $\hat{\mu} = [0.9\hat{\mu}_{nom}; 1.1\hat{\mu}_{nom}]$.

$0.3016 \cdot 10^{-2}\, kgm^{-3}$, $X^* = 0.8949\, kgm^{-3}$, $X^*_{Set} = 2.2633\ kgm^{-3}$, $S^*_O = 0.5221 \cdot 10^{-2}\ kgm^{-3}$. In the following, an uncertainty of the maximum growth rate of $\pm 10\%$ shall be taken into account. Fig. 2 depicts the interval approximation of the region in the three-dimensional state space calculated without direct evaluation of (15). This region represents a guaranteed and conservative inclusion of all steady-state state variables. In general, the solution of $\dot{x}(t) = 0 = f(x)$ is determined by Interval-Newton-Methods.

5. NONLINEAR CONTROL DESIGN

5.1 Proof of Differential Flatness

In order to account for the nonlinearities in the differential equation of the oxygen concentration S_O, a nonlinear control approach based on differential flatness is proposed. Differential flatness is a prerequisite for flatness-based control of non-linear systems, which are usually given in state space representation, i.e. $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$. A system is denoted as differentially flat (Fliess *et al.*, 1995) if appropriate flat outputs $\mathbf{y} = \mathbf{y}\left(\mathbf{x}, \mathbf{u}, \dot{\mathbf{u}}, \ldots, \mathbf{u}^{(\ell)}\right)$ exist that

(i) allow for expressing all system states $\mathbf{x}$ and all system inputs $\mathbf{u}$ as a function of these flat outputs $\mathbf{y}$ as well as their time derivatives, i.e. $\mathbf{x} = \mathbf{x}\left(\mathbf{y}, \dot{\mathbf{y}}, \ldots, \mathbf{y}^{(\beta)}\right)$ and $\mathbf{u} = \mathbf{u}\left(\mathbf{y}, \dot{\mathbf{y}}, \ldots, \mathbf{y}^{(\beta+1)}\right)$,

(ii) are differentially independent, i.e. they are not connected by differential equations.

If the first condition is fulfilled, the second condition is equivalent to $dim(\mathbf{u}) = dim(\mathbf{y})$. Here, the differential flatness of the first order differential equation for the oxygen concentration is obvious using the measurable state variable oxygen concentration as single flat output $y = S_O$ and the air supply rate u_{O2} as physical control input.

5.2 Flatness-based Control Design

The flatness-based control design relies on the state equation for the oxygen concentration that can be rearranged in the following form

$$\dot{S}_O = \underbrace{\left[-\frac{Q_W S_O}{V_A} - \hat{\mu}\frac{S}{S+K_S}\frac{1}{S_O+K_{OS}}\frac{1-Y}{Y}X\right]}_{-k_p(S,S_O,X)} \cdot S_O + \underbrace{\frac{\rho_{O2}}{V_A}\left(1-\frac{S_O}{S_{O,sat}}\right)}_{k_u(S_O)} u_{O2} + \underbrace{\frac{Q_W S_{OW}}{V_A}}_{z} . \tag{16}$$

Consequently, the inverse dynamics is obtained by solving for the physical control input

$$u_{O2} = \frac{1}{k_u(S_O)}\left[\dot{S}_O + k_p(S, S_O, X)S_O - z\right] . \tag{17}$$

At this, the required values for S and X can be obtained by either a reduced-order observer or a model-based interval algorithm. Defining $\upsilon = \dot{S}_O$ as new control input, the error dynamics can be asymptotically stabilized using the control law

$$\upsilon = \dot{S}_{Od} + \alpha_1(S_{Od} - S_O) + \alpha_0 \int_0^t (S_{Od} - S_O)d\tau, \tag{18}$$

which results in a second order error dynamics. Here, the coefficients α_1 and α_2 are determined by pole placement. As a result of the integral control part, steady-state accuracy w.r.t. the desired oxygen concentration S_{Od} is ensured. Transitions between two operating points can be described by a smooth desired trajectory, which must be at least C^1-continuous. This allows for employing the desired trajectory S_{Od} and its first time derivative $\dot{S}_{Od}$ as feedforward control part in the proposed control law.

Taking both subsidiary oxygen control and the uncertain maximum growth rate into account, the steady-state values for the substrate concentration $S = S(S_O, \hat{\mu})$ as well as the bacteria concentration $X = X(S_O, \hat{\mu})$ can be determined as depicted in Fig. 3 and Fig. 4.

Given these steady-state characteristics and a specified admissible substrate concentration at the plant output, the desired value S_{Od} in an operating point follows in two steps. First, the steady-state substrate concentration $S = S(S_O, \hat{\mu})$ is solved for the oxygen concentration $S_O = S_O(S, \hat{\mu})$. Second, the insertion of the maximum admissible substrate concentration $S = S_{lim}$ yields the desired value of the oxygen concentration, which is chosen as supremum of $S_{Od} = sup[S_O(S_{lim}, \hat{\mu})]$. The calculation of the desired oxygen concentration is illustrated in Fig. 5 for an uncertain growth rate $\hat{\mu} = [0.9\hat{\mu}_{nom}; 1.1\hat{\mu}_{nom}]$ according to an admissible user-specified substrate concentration $S_{lim} = 0.0035\ kgm^{-3}$ at the plant output. As ex-

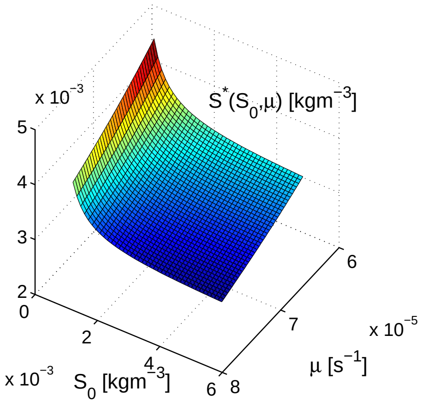

Fig. 3. Substrate concentration as function of oxygen concentration and uncertain growth rate $\hat{\mu} = [0.9\hat{\mu}_{nom}; 1.1\hat{\mu}_{nom}]$.

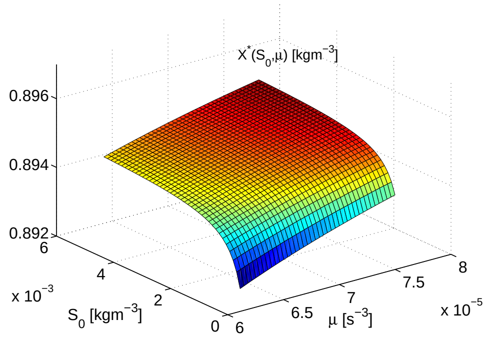

Fig. 4. Bacteria concentration as function of oxygen concentration and uncertain growth rate $\hat{\mu} = [0.9\hat{\mu}_{nom}; 1.1\hat{\mu}_{nom}]$.

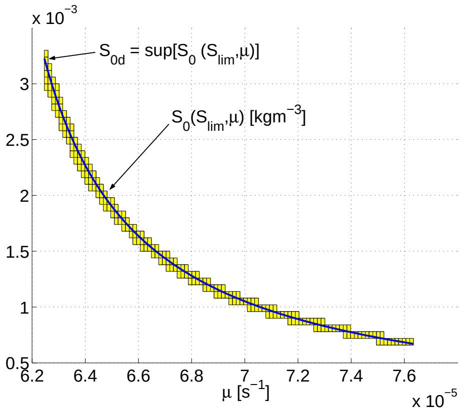

Fig. 5. Comparison of symbolic and interval calculation of the desired oxygen concentration in view of the uncertain growth rate $\hat{\mu} = [0.9\hat{\mu}_{nom}; 1.1\hat{\mu}_{nom}]$ for an substrate concentration limit of $S_{lim} = 0.0035 kgm^{-3}$.

pected the desired oxygen concentration is determined by the minimum growth rate.

By this, the subsidiary control of the oxygen concentration guarantees a steady-state substrate concentration in the plant output that is smaller than the specified limit value S_{lim} regarding the uncertain specific maximum growth rate $\hat{\mu}$.

Similarly, the second order error dynamics can be proven to be asymptotically stable for the uncertain maximum growth rate $\hat{\mu}$ using interval arithmetics at evaluating the coefficients of the characteristic polynomial, which must be positive according to the Hurwitz-criterion.

6. CONCLUSIONS

In this paper, interval arithmetics is applied to a simplified activated sludge model in biological wastewater treatment focussing on the reduction of biodegradable substrate. The corresponding nonlinear fourth-order state space model is subject to an uncertain maximum growth rate of heterotrophic bacteria. Thus, reliable analysis of the steady-state behaviour as well as plant control have to account for this dominant uncertainty. The proposed control strategy involves a subsidiary flatness-based control of oxygen concentration, where the desired trajectory is derived from interval evaluation of the uncertain steady-state substrate concentration. By this, guaranteed bounds for the oxygen required for an admissible substrate concentration in the plant output according to legal regulations can be calculated for the uncertain system. As a result, the plant operating costs can be significantly reduced by properly adjusted oxygen supply, leading to an increase in plant efficiency. Moreover, this strategy can be directly extended to the widely used Activated Sludge Model ASM1 of the IAWQ under consideration of uncertain system parameters.

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