RENEWABLE RESOURCES, CAPITAL ACCUMULATION AND SUSTAINABILITY

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Abstract: A profit maximizing firm producing a commodity with the aid of renewable resource and physical capital is considered to examine the existence and nature of steady state in the resource and capital accumulation. The model is then compared with an alternative resource management regime in which the social utility defined on the consumption and resource stock levels is to be maximized. For both social utility maximization and private profit motive regimes, the economy approaches a stationary state with larger resource stock than the stock level guaranteeing the maximum reproduction rate of resource. However, the sufficient conditions for supporting this result are different between the two regimes.

Keywords: renewable resource, capital accumulation, profit maximization, social welfare maximization, sustainability

1. INTRODUCTION

Let us consider an economy endowed with a natural resource as the only possible source of income. Is it possible for the economy to live out toward future forever? When the resource is of an exhaustible type, the problem may become more imminent than with other types. We should find a way on how to survive after depleting out the resource by establishing proper rules for resource and alternative asset management. This problem has been analyzed by Hartwick 1977) since the 1970s and later Katayama and Ohta (1999) have extended the analysis to care for uncertainty.

Even if the resource is renewable, the problem still lingers on our theoretical concern. Hartwick (1978) presents the optimal investment rule for the case of renewable resources. Namely, the resource reproduction rate must be employed to modify the rule of investing the resource margin to alternative capital accumulation process where the margin is calculated as the difference between resource price and marginal extraction cost multiplied by the amount extracted. We may consider any physical capital or monetary assets other than natural resources as the alternatives. Ohta and Katayama (2001) have extended the Hartwick (1977) rule further to incorporate monetary investment opportunities to overseas.

If we confine to physical capital as the alternative asset, we should ask ourselves what to do with this physical capital. One of the most natural ways to utilize the capital is to put it into a production process along with other production factors to obtain certain amount of products. After consuming a part of the production, we can invest the rest to accumulate capital. Then, the optimal time profiles of resource and physical capital are to be determined so as to compare the economy's total utility level at present and the consumption possibility sustained by the physical capital accumulation.

Now, to obtain the time profiles of two assets for the economy, we can think of two different regimes as social system, each of which has a specific objective function to maximize. As the first regime, Beltratti, Chichilnisky and Heal (1998) have analyzed characteristics of the steady state in the combination of renewable resource and physical capital. The resource stock changes by the difference between its reproduction rate and amount of harvest at each moment of time. The physical capital accumulation is governed by investment which equals savings of the economy obtained as the difference between production level and consumption. They assume that the economy has a social utility function on consumption level and resource stock existing at each moment of time. The discounted sum of instantaneous utility is maximized by controlling consumption and resource harvest. They have proved the existence and stability of a steady state under the utility maximizing regime. On the other hand, Benchekroun, Katayama and Long (2003) have drawn the condition for this economy to step on a path to steady state by specifying functional forms for utility and resource reproduction rates. Finally, Long and Katayama (2002) and Fujisaki, Ibuki and Katayama (2004) have extended the argument to a common property resource assuming several planning agents for utility maximization.

In the present paper we analyze the second regime in which the economy is characterized as a profit maximizing entity. The importance of profit maximizing formulation can be understood if we look back the series of analyses in the past to obtain optimal extraction or harvesting paths under profit maximization. Those attempts have made us equipped with well-known principles of resource economics, such as the Hotelling rule on the change per unit time in resource price net of marginal extraction cost being equal to time discount rate. In the first regime referred above, however, the extraction or harvesting cost is not analyzed at least explicitly. When we follow the utility maximizing procedure, any cost for obtaining resource is born by the society as a whole and thus it does not appear explicitly in the model. In the second regime, attention is paid directly to the behavior of resource holder and outcomes of profit maximizing behavior. The resource stock changes at each moment of time by the magnitude of reproduction less harvest as in the first regime, but the physical capital accumulates or decumulates by the amount of production less sales levels of product. In other word, the present paper is to focus on decentralized system of resource management, while the first regime is regarded as a centralized economy.

2. MODEL

We consider a continuous-time model. Let R denote the stock level of the resource and $S(R)$ its natural growth function.

For simplicity we assume that $S(R) \geq 0$ for $R \in [0, \bar{R}]$. Let us define $\widehat{R}$ by $S'(\widehat{R}) = 0$ and $\widehat{R} \in [0, \bar{R}]$. By assuming that $S(0) = 0$, $S'(0) > 0$ and $S''(R) < 0$, the function $S(R)$ is single peaked. Denoting $q(t)$ for the rate of extraction at time t, we have,

$$\dot{R} = S(R) - q. \tag{1}$$

The output F of the final good is a function of the stock of man-made capital K and the resource extracted q. Assume that $F_K > 0, F_q > 0, F_{KK} < 0, F_{qq} < 0$ and $F_{Kq} \geq 0$. Here the lower case letter expresses the partial derivate. And further assume that $F_K > \delta$, where δ denotes time discount rate and $0 < \delta < 0$.

A monopolist produces its output, and sells part of it at market and invests the rest of the amount for capital accumulation. The inverse demand function of the market is given by $p(y)$ and is assumed as $p'(y) < 0$. For simplicity we assume away the capital depreciation. Then, we have,

$$\begin{aligned} y &= F(K,q) - I = F(K,q) - \dot{K}, \\ \dot{K} &= F(K,q) - y. \end{aligned} \tag{2}$$

Here $\dot{K}$ is the monopolist's private capital accumulation.

Let $c(q, R)$ denote the extraction cost. The cost function is assumed to depend on the existing amount of the resource stock. It is called the stock externality in harvesting cost. It is assumed that $c_q > 0, c_R < 0, c_{qq} > 0, c_{RR} > 0$, and $c_{qR} \leq 0$.

The objective function of the monopolist is the present value of net cash flow from profit over the infinite planning period.

$$\max_{q,y} \int_0^\infty \left[p(y)y - c(q,R)\right] e^{-\delta t} dt. \tag{3}$$

The maximization is subject to (1) and (2) with boundary conditions $K(0) = K_0 > 0, R(0) = R_0 > 0$, and

$$\lim K(t) > 0, \lim S(t) \geq 0. \tag{4}$$

Note that the objective function contains factor cost for the capital input. The necessary conditions for maximization are,

$$\frac{\partial H}{\partial q} = -c_q - \lambda + \mu F_q = 0, \tag{5}$$

$$\frac{\partial H}{\partial y} = p'y + p - \mu = 0, \tag{6}$$

$$\dot{\lambda} = (\delta - S')\lambda + c_R, \tag{7}$$

$$\dot{\mu} = (\delta - F_K)\mu. \tag{8}$$

where H represents the current value Hamiltonian, λ and μ are conjugate variables for resource stock and physical capital, respectively. The basic structure of the model consists of these four equations and (1) and (2).

3. THE STEADY STATE

From (1), (2) and (8) we obtain that for a steady state,

$$q = S(R), \tag{9}$$
$$y = F(K, q), \tag{10}$$
$$\delta = F_K(K, q). \tag{11}$$

At the steady state equilibrium, the harvest is just the same amount of the increment of resource reproduced. It is possible to sell the whole of the produced product, since we assumed away the depletion of capital.

From (5) and (7), we have

$$-\frac{c_R}{c_q} = (\delta - S')\left\{\frac{(p'y+p)F_q}{c_q} - 1\right\}. \tag{12}$$

The left side of this equation is positive by the shape of harvesting cost function and the numerator of the first term in the parenthesis on the right is the marginal revenue from harvesting the resource by one unit, while the denominator is the marginal cost of harvesting.

The solution to the equations system (9)-(12), if exists, is a stationary state $\{F^*, K^*, q^*, y^*\}$.

From (12)

$$(p'y+p)F_q \gtrless c_q \Longrightarrow \delta - S' \gtrless 0. \tag{13}$$

Define $\widetilde{R}$ by $\delta = S'(\widetilde{R})$. Then, from (13), we have,

$$(p'y+p)F_q \gtrless c_q \Longrightarrow R^* \gtrless \widetilde{R}. \tag{14}$$

$\widetilde{R}$ is smaller than $\widehat{R}$ at which the maximal reproduction rate of the resource is attained. Therefore, it is possible that R^* is smaller than $\widehat{R}$ even when the bracket in the right hand side of (12) is positive. If the bracket in (12) is negative, in other words, when the marginal revenue is smaller than the marginal cost, the stationary stock level is smaller than $\widehat{R}$. Putting it differently, when the resource stock level at the stationary point is smaller, the marginal revenue can not cover the marginal cost of harvesting. When marginal cost equals marginal revenue,

$$\delta - S' = \infty. \tag{15}$$

Because, otherwise, the right hand side of (12) becomes zero. If the functional form of S satisfies $S'(\bar{R}) = -\infty$, it becomes that $R^* = \bar{R}$. Namely, the steady state level of the natural resource is the maximum stock level, and the harvest becomes zero and so are the production of man-made good and its sales. When $S'(\bar{R}) > -\infty$, the optimal solution to (9)-(12) does not exist.

Furthermore, when δ is small, $\widetilde{R} = \widehat{R}$ and the marginal revenue is larger (smaller) than the marginal cost, the stationary stock level R^* is larger (smaller) than $\widehat{R}$. When marginal cost equals marginal revenue, $R^* = \widetilde{R} = \widehat{R}$. Therefore,

$$(p'y+p)F_q \gtreqless c_q \Longrightarrow R^* \gtreqless \widetilde{R}. \tag{16}$$

What if $c_R = 0$, namely, when there is no stock externality? In this case, the left hand side of (12) is zero, and thus $\delta = S'$ as long as the marginal cost is not equal to marginal revenue, and

$$R^* = \widetilde{R} \leq \widehat{R}. \tag{17}$$

The inequality holds when $\delta = 0$. The result that R^* can not be larger than $\widehat{R}$ is obtained by Benchekroun, Katayama and Long (2003). In their model the objective function is social utility maximization, and they neglect harvesting cost and its stock externality.

The discussion concerning (14)-(17) is about the comparison between marginal revenue and marginal cost of extra harvesting. The first order condition for static optimality requires the equality among them in the monopoly case. Should it be confined to (15) in which there might be non existence of the stationary state of the natural resource stock?

In what follows we check the existence in more detail.

From (9) and (10),

$$y = F(K, q) = F(K, S(R)), \tag{18}$$

and at the stationary state the resource stock and the sale of the man-made product satisfy,

$$\frac{\partial y}{\partial R} = F_q S' \gtreqless 0 \iff R \lesseqgtr \widehat{R}. \tag{19}$$

Given $K = \bar{K}$, a constant, the relation between resource stock R and output y in (9) and (19) are depicted in Fig.1. See the first and the fourth quadrants.

At the second quadrant two different production functions are shown to the different stock levels of $\bar{K}$ and $\bar{K}'$.

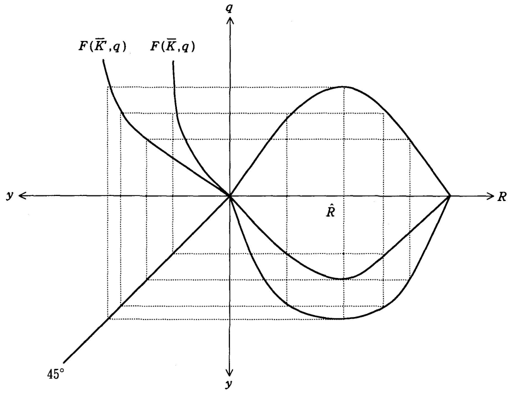

Figure 1: $R-y$ Relation

For given stock levels two different curves show $R-y$ relation in the fourth quadrant. When $F_{Kq} > 0$, the slope of $R-y$ curve gets steeper as $\bar{K}$ is larger.

However, K changes with q in (11), and thus

$$\frac{dK}{dq} = -\frac{F_{Kq}}{F_{KK}} \geq 0. \tag{20}$$

From (18),

$$\begin{aligned} \frac{dy}{dR} &= F_K \frac{dK}{dq}\frac{dq}{dR} + F_q S' \\ &= \left(F_q - F_K \frac{F_{Kq}}{F_{KK}}\right) S' \geq F_q S', \end{aligned} \tag{21}$$

with equality when $F_{Kq} = 0$.

As will be seen from the Fig.2, the slope of actual $R-y$ curve is steeper than the fixed capital stock case. When physical capital and natural resource are separable in the production process, the two curves have the same slope.

At the stationary state the capital stock is constant and it is shown at the points where two curves (19) and (21) cross each other in the Fig.2. There should be one which satisfies (19) and $\bar{K} = K^*$.

In Fig.2 there are two intersections noted x. Each point shows a combination (R^*, y^*) and (R^{**}, y^{**}). Both points locate on the same curve of stationary stock level. Does this imply multiple stationary states? To consider this, we depict an iso-harvesting cost curve on $R-q$ plane.

The slope of the iso-cost curve is given as

$$\left.\frac{dq}{dR}\right|_{\bar{c}} = -\frac{c_R}{c_q} > 0, \tag{22}$$

$$\left.\frac{d^2q}{dR^2}\right|_{\bar{c}} = -\frac{c_{qq}c_R^2 - 2c_{qR}c_q c_R + c_{RR}c_q^2}{c_q^3} < 0. \tag{23}$$

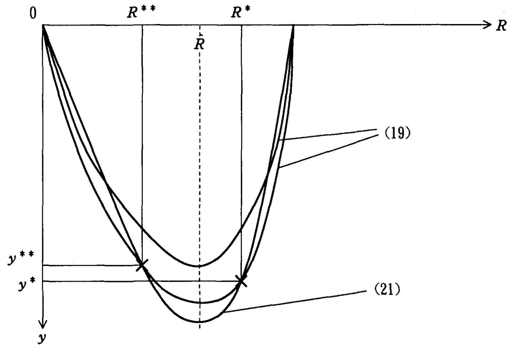

Figure 2: Steady State Equilibrium

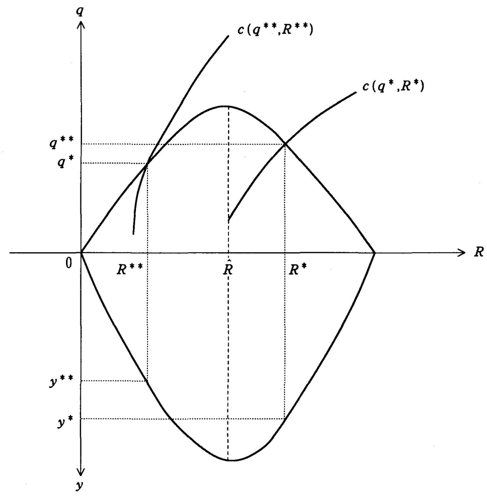

Figure 3: Iso-cost Curve and Steady State Solution

The last inequality holds when the cost function is quasi-concave, or when $c_{qR} = 0$, i.e. the harvesting cost is separable in flow q and stock R. Therefore, the iso-cost curve is upward sloping as the first quadrant in Fig.3. And the curve located in the further right direction expresses the lower cost. That is,

$$c\left(q^*, R^*\right) < c\left(q^{**}, R^{**}\right). \tag{24}$$

As Fig.3 shows there are two possible combinations (R^*, q^*) and (R^{**}, q^{**}) for the same K^*. However the point with lower cost should be chosen. Therefore, it is shown that there is a single stationary state $\{R^*, K^*, q^*, y^*\}$.

4. DYNAMIC PATH

Here we analyze the dynamic path. From (5), optimum harvest q is a function of R, K, λ and μ and the output y is a function of μ. Therefore,

$$\dot{R} = S(R) - q(R, K, \lambda, \mu), \tag{25}$$

$$\dot{K} = F(K, q(R, K, \lambda, \mu)) - y(\mu), \tag{26}$$

$$\dot{\lambda} = (\delta - S')\lambda + c_R(q(R, K, \lambda, \mu), R), \quad (27)$$

$$\dot{\mu} = (\delta - F_K(K, q(R, K, \lambda, \mu)))\mu. \quad (28)$$

And from (5),

$$(-c_{qq} + \mu F_{qq})\, dq = c_{qR}dR - \mu F_{qK}dK + d\lambda - F_q d\mu, \quad (29)$$

and

$$q_R > 0, q_K > 0, q_\lambda < 0, q_\mu > 0. \quad (30)$$

And from (6)

$$(p''y + 2p')\, dy = d\mu,$$

and usually it is assumed that the derivative of marginal revenue $p''y + 2p'$ is negative. Therefore,

$$y_\mu < 0.$$

The linearized system around the stationary state is

$$\begin{pmatrix} \dot{R} \\ \dot{K} \\ \dot{\lambda} \\ \dot{\mu} \end{pmatrix} = \begin{pmatrix} S' - q_R & -q_K & -q_\lambda & -q_\mu \\ F_q q_R & F_K + F_q q_K & F_q q_\lambda & F_q q_\mu - y_\mu \\ aS''\lambda + c_{qR}q_R + c_{RR} & c_{qK}q_K & (\delta - S') + c_{qR}q_\lambda & c_{qR}q_\mu \\ -F_{Kq}q_R\mu & -(F_{KK} + F_{Kq}q_K)\mu & -F_{Kq}q_\lambda & (\delta - F_K) - F_{Kq}q_\mu\mu \end{pmatrix} \times \begin{pmatrix} R - R^* \\ K - K^* \\ \lambda - \lambda^* \\ \mu - \mu^* \end{pmatrix}.$$

It seems to be difficult to obtain general characterization of the dynamic system, so assume that $c_{qR} = 0$ and $F_{Kq} = 0$. This implies that both cost function and production function are separable. Then, $q_R = 0$ and $q_K = 0$, from (29).

Let denote the matrix of the linearized system as A. Then,

$$\begin{aligned} |A - \gamma I| &= \begin{vmatrix} S' - \gamma & 0 & -q_\lambda & -q_\mu \\ 0 & F_K - \gamma & F_q q_\lambda & F_q q_\mu - y_\mu \\ -S''\lambda + c_{RR} & 0 & (\delta - S') - \gamma & 0 \\ 0 & -F_{KK}\mu & 0 & (\delta - F_K) - \gamma \end{vmatrix} \\ &= (S' - \gamma) \begin{vmatrix} F_K - \gamma & F_q q_\lambda & F_q q_\mu - y_\mu \\ 0 & (\delta - S') - \gamma & 0 \\ -F_{KK}\mu & 0 & (\delta - F_K) - \gamma \end{vmatrix} \\ &\quad + (-S''\lambda + c_{RR}) \\ &\quad \times \begin{vmatrix} 0 & -q_\lambda & -q_\mu \\ F_K - \gamma & F_q q_\lambda & F_q q_\mu - y_\mu \\ -F_{KK}\mu & 0 & (\delta - F_K) - \gamma \end{vmatrix} \\ &= (F_K - \gamma)\{(\delta - F_K) - \gamma\} \\ &\quad \times [(S' - \gamma)\{(\delta - S') - \gamma\} \\ &\qquad + (-S''\lambda + c_{RR})q_\lambda] \\ &\quad + F_{KK}\mu[(S' - \gamma)(F_q q_\mu - y_\mu) \\ &\qquad \times \{(\delta - S') - \gamma\} \\ &\qquad - (-S''\lambda + c_{RR})q_\lambda y_\mu] \\ &= (F_K - \gamma)\{(\delta - F_K) - \gamma\} \\ &\quad \times \{S'(\delta - S') + (-S''\lambda + c_{RR})q_\lambda \\ &\qquad - \delta\gamma + \gamma^2\} \\ &\quad + F_{KK}\mu[(S' - \gamma)(F_q q_\mu - y_\mu) \\ &\qquad \times \{(\delta - S') - \gamma\} \\ &\qquad - (-S''\lambda + c_{RR})q_{\lambda_{y_\mu}}] \\ &= 0 \end{aligned} \quad (31)$$

When $F_{KK} \cong 0$, (31) has four roots,

$$\gamma = F_K, \delta - F_K, \frac{\delta \pm \sqrt{D}}{2},$$
$$\text{where } D \equiv \delta^2 - 4[S'(\delta - S') + (-S'\lambda + c_{RR})q_\lambda] > \delta^2 > 0.$$

At the stationary state $\delta = F_K$, two characteristic roots are positive, one zero and one negative.

By properly choosing coefficient with exponential part with respect to t it is possible to approach to a stationary state. In other words by choosing extraction rate and production properly it is possible to take a trajectory converging to a stationary state.

5. COMPARISON WITH UTILITY MAXIMIZATION

Beltratti, Chichilnisky and Heal (1998) examine a utility maximization problem by assuming $u(y, R)e^{-\delta t}$ (in our notation) as the integrand in equation (3) of the present paper. As the result, they obtain two positive and another two negative characteristic roots when F_{qq} is very large, implying that the stationary point is a saddle. We have, in this paper, two positive roots and one negative under $F_{KK} \cong 0$. Even though the sufficient condition for stable trajectory is different from ours, their model also shows the possibility of approaching a steady state under appropriate policy for resource management. Common in both models is that the resource stock R^* at steady state is greater than $\hat{R}$ guaranteeing the maximum resource reproduction. Therefore, we can conclude that when an economy endowed with renewable resource endeavors physical capital accumulation as well, it will have a qualitatively similar resource stock level in the long-run, whether the resource is managed under social utility maximization or profit maximization by a firm having the right to dispose the resource harvest at hand to earn sales proceeds.

Benchekroun, Katayama and Ngo Long (2003) also use a utility maximization model. In order to obtain explicit solution path of variables, they assume a resource reproduction function of "tent type "with constant rate of marginal reproduction over stock level. Their utility function does not include resource stock level and is of fixed elasticity over consumption. Interesting result of their paper is that when the time discount rate is not equal to the constant slope of reproduction function, the steady state R^* becomes equal to $\hat{R}$ guaranteeing the maximum resource reproduction rate.

6. CONCLUSION

Assuming that a monopolist maximizes the discounted sum of profits toward future out of selling a product produced with the aid of a renewable resource and a physical capital in its own possession, we have analyzed the nature of steady state on resource and capital. Even if the firm does not have monopoly power on the product and the product price is constant, we have the same result. When $p = \mu$ (a constant) in equation (6), the coefficient matrix of the linearized differential equations system reduces to a 3×3 matrix. The harvesting cost function and production function being assumed separable respectively, it is easily shown that the trace of the coefficient matrix is positive and the determinant is negative. There are characteristic values of opposite signs and thus the dynamic path in the case assumes the same movement as in section 4.

Finally, let us remark on the "Green Golden Rule" examined by Beltratti, Chichilnisky and Heal (1998). As mentioned in the previous section, their model assumes a social utility on commodity consumption and natural resource stock and maximizes its discounted sum over time. They have called this model a "utilitarian" model. The "Green Golden Rule", on the contrary, is defined as to maintain the maximum possible and yet sustainable utility level and the stock level under the rule is shown to be greater than that under utilitarian model. We can present the "Green Golden Rule" in the present paper as the same notion as in their model. However, just like in their model, in order to prove the existence of green golden rule, we must assume a particular production function in which the marginal productivity of physical capital drops to zero for a certain finite level of the capital input. If the capital productivity continues to be positive until the input reaches infinity as in the Inada condition, the "Green Golden Rule" seems to be out of the nature.

REFERENCES

Beltratti, A., G. Chichilnisky, and G.M. Heal (1998), Sustainable Use of Renewable Resources, In: *Sustainability: Dynamics and Uncertainty*, (Chichilnisky, G., G. Heal and A. Vercelli Ed.), pp.49-76. Kluwer Academic Publishers.

Benchekroun, H., S. Katayama and Ngo V. Long (2003), Capital Resource Substitution, Overshooting, and Sustainable Development, paper presented at the Eighth Viennese Workshop on Optimal Control, Dynamic Games, Nonlinear Dynamics and Adaptive Systems, Vienna, May 14-16.

Fleming, W.H. and R.W. Rischel (1975), *Deterministic and Stochastic Optimal Control.* Springer.

Fujisaki, M., K. Ibuki and S. Katayama (2004), Renewable Common Property Resource and Private Capital Accumulation, *Journal of Kobe University of Commerce*, **55**, 1-6.

Hartwick, J.M. (1977), Intergenerational Equity and the Investing of Rents from Exhaustible Resources, *American Economic Review*, **67**, 972-974.

Hartwick, J.M. (1978), Investing Returns from Depleting Renewable Resource Stocks and Intergenerational Equity, *Economics Letters* **1**, 85-88.

Katayama, S. and H. Ohta (1999), Sustainability in Small Open Economies under Uncertainty, *Annals of Operations Research* **88**, 173-182.

Long, N.V. and S. Katayama (2002), Common Property Resource and Capital Accumulation, In: *Optimal Control and Differential Games*, (Zaccour, G. Ed.), pp.193-209. Kluwer Academic Publishers.

Ohta, H. and S. Katayama (2001), Consumption Sustainability in Resource Economies under Price Uncertainty, *Journal of International Cooperation Studies*, **8**, 67-81.