PIECEWISE-CONSTANT CONTROL SIGNAL FOR PREDATOR-PREY SYSTEMS: APPLICATION TO ECOLOGICAL RECOVERY

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Abstract: A control strategy that determines piecewise-constant control signals for the tracking problem of predator-prey systems is proposed, since this type of signal is an idealized model of management policies adopted by environmental agencies. The reference trajectories are chosen as to restore the original dynamics of a disturbed system. Assuming that the system model is completely known, it is shown that accurate tracking can be achieved from periodic measurements of the sizes of both populations.

Keywords: Environmental engineering, Sliding-mode control, Tracking systems

1. INTRODUCTION

There are basically two main reasons for obtaining control strategies for predator-prey systems. The first one is to allow a sustainable exploitation of its resources. The other one is to restore ecological balance. These subjects have been treated by Cunha and Pagano (2002) and Meza *et al.* (2002). It is desirable to determine control signals that can be implemented by environmental agencies as management policies. The main objective of this paper [1] is to propose a control strategy that makes this possible. Its layout is as follows. In Section 2, the mathematical model of the considered predator-prey system is described. A situation of ecological recovery of a disturbed predator-prey systems is analyzed in Section 3. The proposed control strategy is presented in Section 4, and Section 5 shows the simulation results for the ecological recovery situation treated in Section 3.

2. THE PREDATOR-PREY SYSTEM MODEL

Consider the following *isolated* predator-prey system model, that is, free of human interference, (Kot, 2000)

$$\begin{aligned} \frac{d}{dT}N &= rN\left(1-\frac{N}{K}\right) - \frac{cNP}{\dfrac{N^2}{i}+N+a}, \\ \frac{d}{dT}P &= \frac{bcNP}{\dfrac{N^2}{i}+N+a} - mP. \end{aligned} \tag{1}$$

N and P represent the number of prey and predators, respectively, and T is time. The prey exhibit logistic growth in the absence of the predators with intrinsic rate of growth r and carrying capacity K. The *per capita* mortality of the predators is denoted by m and note that their *per capita* rate of consumption is described by a type IV functional

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response. With this kind of functional response, when the number of prey is sufficiently high, the predator's *per capita* rate of predation decreases due to either prey group defense or prey toxicity. The parameter i is viewed as a measure of the immunity of the predators from the prey or of their tolerance for the prey, and the parameters a and c as the half-saturation constant and maximum *per capita* predation rate, respectively, when there are no inhibitory effects. The conversion efficiency of the consumed prey into new predators is given by b. It is assumed that N and P are nonnegative and that the parameters a, b, c, i, K, m and r are positive and known.

With the aim of reducing the number of parameters of system (1), the following dimensionless variables are defined (Kot, 2000)

$$x_1 \triangleq \frac{N}{a}, \quad x_2 \triangleq \frac{c}{ra}P, \quad t \triangleq rT. \tag{2}$$

It is immediate to verify that (1) and (2), along with the chain rule, yields

$$\begin{aligned} \frac{d}{dt}x_1 &= f_1(\mathbf{x}) \triangleq x_1\left(1-\frac{x_1}{\gamma}\right) - \frac{x_1x_2}{\dfrac{x_1^2}{\alpha}+x_1+1}, \\ \frac{d}{dt}x_2 &= f_2(\mathbf{x}) \triangleq \frac{\beta\delta x_1x_2}{\dfrac{x_1^2}{\alpha}+x_1+1} - \delta x_2, \end{aligned} \tag{3}$$

where

$$\alpha \triangleq \frac{i}{a},\ \beta \triangleq \frac{bc}{m},\ \gamma \triangleq \frac{K}{a},\ \delta \triangleq \frac{m}{r}. \tag{4}$$

Note that x_1, x_2 and t are normalizations of N, P and T, respectively.

Fig. 1 shows the phase portraits for two sets of parameter values, differing only by the value of γ. As seen from Fig. 1.b, all trajectories with initial condition in the first quadrant converge to a stable equilibrium point of coexistence of the species. But if γ is increased, the phase portrait in Fig. 1.a shows that there are two regions of attraction separated by the stable manifolds of the non-trivial saddle point. One of them corresponds to the stable limit cycle and the other one to the stable equilibrium point at the x_1-axis. Hence, the initial condition determines if either the two species coexist oscillating with periodic trajectories or the extinction of the predators will be unavoidable in this isolated system.

3. ECOLOGICAL RECOVERY OF DISTURBED SYSTEMS

Suppose that (3) has the parameter values of Fig. 1.a, where $\alpha = 5.2$, $\beta = 2.0$, $\gamma = 4.1$ and $\delta = 2.5$, and that the state has been measured as

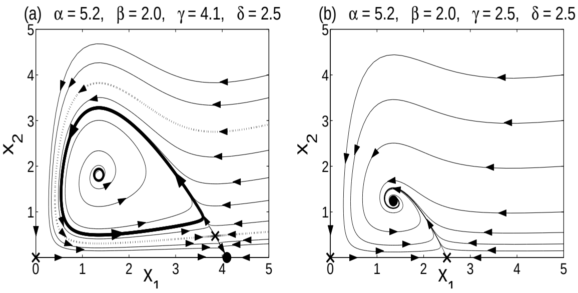

Fig. 1. Phase portraits (○ – unstable node or focus; • – stable node or focus; × – saddle point).

$\mathbf{x}_0 \triangleq \mathbf{x}(0) = [2.2304 \quad 0.1664]'$ at $t = 0$. For this initial condition, the state $\mathbf{x}_0$ is located outside the separatrices indicated in the phase portrait, and hence does not belong to the domain of attraction of the stable limit cycle. Consequently, the trajectories will converge to the stable equilibrium point $\mathbf{x}_e^p = [4.1 \ \ 0]'$ at the x_1-axis and the predators will not survive. It is also assumed that, prior to $t = 0$, the system was in ecological balance and that its trajectories were oscillating at the stable limit cycle shown in Fig. 1.a. Therefore, whenever some disturbance causes the state vector to leave the region of attraction of the limit cycle, human intervention is needed to restore the original dynamics of the isolated system (the limit cycle) and thus avoid an environmental catastrophe.

In order to take into account human interference, (3) is modified to obtain the *non-isolated* predator-prey system model

$$\begin{aligned} \frac{d}{dt}x_1 &= x_1\left(1-\frac{x_1}{\gamma}\right) - \frac{x_1x_2}{\dfrac{x_1^2}{\alpha}+x_1+1} + u_1, \\ \frac{d}{dt}x_2 &= \frac{\beta\delta x_1x_2}{\dfrac{x_1^2}{\alpha}+x_1+1} - \delta x_2 + u_2, \end{aligned} \tag{5}$$

or, equivalently, using vector notation

$$\frac{d}{dt}\mathbf{x} = \mathbf{f}(\mathbf{x}) + \mathbf{u}, \tag{6}$$

where $\mathbf{x} = [x_1 \ x_2]'$ is the state vector, $\mathbf{f} = [f_1 \ f_2]'$ (see (3)) and $\mathbf{u} = [u_1 \ u_2]'$ is the control vector representing human action.

Due to the foregoing discussion, an ecological catastrophe will be avoided if human action is capable of putting the state vector of the controlled system back into the referred basin of attraction, by means of an adequate environmental management policy. One alternative is to determine a control vector $\mathbf{u}$ that forces $\mathbf{x}$ to track a reference trajectory $\mathbf{r}$ that enters this region. As seen from the phase portrait of Fig. 1.b, all trajectories in the first quadrant converge to the stable equilibrium point $\mathbf{x}_e = [1.3510 \ 1.2418]'$ located inside the original limit cycle of the isolated system, and thus is inside its domain of attraction. Therefore, a reasonable choice for the reference trajectory $\mathbf{r}$ would be as a trajectory of the predator-prey system model (3) with the parameters of Fig. 1.b. Hence, the *reference system* is defined as (3)

$$\frac{d}{dt}r_1 = r_1\left(1 - \frac{r_1}{\gamma_r}\right) - \frac{r_1 r_2}{\frac{r_1^2}{\alpha_r} + r_1 + 1},$$
$$\frac{d}{dt}r_2 = \frac{\beta_r \delta_r r_1 r_2}{\frac{r_1^2}{\alpha_r} + r_1 + 1} - \delta_r r_2, \quad (7)$$

where $\mathbf{r} = [r_1 \; r_2]'$ is the *reference state vector*, and the *reference parameters* are specified as the ones of Fig. 1.b, where $\alpha_r = 5.2$, $\beta_r = 2.0$, $\gamma_r = 2.5$ and $\delta_r = 2.5$. The initial condition of the reference system is chosen as $\mathbf{r}_0 = \mathbf{x}_0$. In order to simplify future reference, the following abbreviations shall be used: *CS* for the controlled system (5); *IS* for the isolated system (3); and *RS* for the reference system (7). Fig. 2 shows the phase portraits and the trajectories in the normalized time domain t of RS and IS. Observe that the reference state $\mathbf{r}$ reaches the domain of attraction of the original limit cycle of the isolated system as it converges to the equilibrium point $\mathbf{r}_e = \mathbf{x}_e = [1.3510 \; 1.2418]'$.

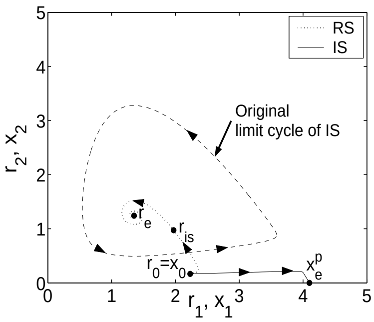

Fig. 2. Phase portraits of RS and IS.

It is important to remark that the implementation of a certain control signal $\mathbf{u}$ in the real predator-prey system might come across two major problems. Feedback control laws require the measurement of the population sizes of both species at every instant of time. However, this might not always be possible in numerous predator-prey interactions found in nature. The second problem is that the control signals should model the human action on the ecosystem. Note that, in (5), the control signals u_1 and u_2 represent the instantaneous rates of change (with respect to t) of x_1 and x_2, respectively, that have to be applied into the system by human action. Hence, if the control signals exhibit elevated rates of signal variation and/or assume a different value at every instant of time, one can argue that human action cannot respond in that manner in many ecosystems found in nature.

According to May and Beddington (1980), a constant harvest quota is an idealized model of real management policies adopted by environmental agencies. Hence, in order to adequate human actions to the control signals, this paper proposes a control strategy that determines piecewise-constant control signals from periodic measurements of the prey and predator population sizes. However, for this to be achieved, two assumptions on (5) are made: (A_1) the model (1) is completely known, i.e., the parameters a, b, c, i, K, m and r have been determined and do not vary with time (consequently, the same holds for α, β, γ and δ in (5)); and (A_2) the state $\mathbf{x}$ of (5) is measured with period Δ and $\mathbf{x}_0 \triangleq \mathbf{x}(0)$ is known.

4. A CONTROL STRATEGY USING PIECEWISE-CONSTANT SIGNALS

The control problem is to get the state vector $\mathbf{x}$ of (5) to track the reference state vector $\mathbf{r}$ of (7) in the normalized time interval $I = [0, \Delta]$ with a precision that satisfies a given performance specification, but without any further measurements of the state vector besides $\mathbf{x}_0$, and having as input a piecewise-constant signal $\mathbf{u}\colon I \to \mathbb{R}^2$. A solution to this tracking problem is shown in the sequel.

Consider the partition $P = (0, \lambda, 2\lambda, \ldots, p\lambda)$ of the interval $I = [0, \Delta]$, where

$$\lambda = \frac{\Delta}{p}, \; p \in \mathbb{Z}^+. \quad (8)$$

The control vector $\mathbf{u}$ is defined as a step map with respect to the partition P, that is,

$$\mathbf{u}(t) = \begin{cases} \mathbf{u}_k & = [u_{k,1} \; u_{k,2}]', \\ & \text{for } t \in I_k = [k\lambda, (k+1)\lambda), \\ & k = 0, \ldots, p-2, \\ \mathbf{u}_{p-1} & = [u_{p-1,1} \; u_{p-1,2}]', \\ & \text{for } t \in I_{p-1} = [(p-1)\lambda, \Delta], \end{cases} \quad (9)$$

where the constant vectors $\mathbf{u}_k$ are still to be specified, for $k = 0, \ldots, p-1$. Note that this is clearly a piecewise-constant signal.

For now, consider the system

$$\frac{d}{dt}\widetilde{\mathbf{x}} = \mathbf{f}(\widetilde{\mathbf{x}}) + \widetilde{\mathbf{u}}, \quad (10)$$

with $\widetilde{\mathbf{x}}(0) = \mathbf{x}(0)$. The vector $\widetilde{\mathbf{u}}\colon I \to \mathbb{R}^2$ is any continuous signal on the interval I, known at $t = 0$, that assures that the state vector $\widetilde{\mathbf{x}}$ tracks the reference state $\mathbf{r}$ within a given performance requirement. Here, $\widetilde{\mathbf{u}} = [\widetilde{u}_1 \; \widetilde{u}_2]'$ is defined according to the continuous control laws of the Sliding Mode Control (SMC) methodology developed by Slotine (1991), namely

$$\begin{aligned} \widetilde{u}_1 &= -f_1(\widetilde{\mathbf{x}}) + \dot{r}_1 - \eta_1 sat((\widetilde{x}_1 - r_1)/\epsilon_1), \\ \widetilde{u}_2 &= -f_2(\widetilde{\mathbf{x}}) + \dot{r}_2 - \eta_2 sat((\widetilde{x}_2 - r_2)/\epsilon_2), \end{aligned} \quad (11)$$

where sat: $\mathbb{R} \to \mathbb{R}$ is given by

$$sat(y) = \begin{cases} y & \text{for} \quad |y| \leq \quad 1, \\ 1 & \text{for} \quad y \;>\; \quad 1, \\ -1 & \text{for} \quad y \;<\; -1, \end{cases} \tag{12}$$

which satisfies the performance specification

$$\|\widetilde{\mathbf{x}}(t) - \mathbf{r}(t)\| \leq \max\{\epsilon_1, \epsilon_2\}, \text{ for } t \in [t_r, \Delta], \tag{13}$$

where

$$\begin{gathered} t_r = \max\{t_{r1}, t_{r2}\}, \\ t_{r1} = \frac{|\widetilde{x}_1(0) - r_1(0)|}{\eta_1}, \; t_{r2} = \frac{|\widetilde{x}_2(0) - r_2(0)|}{\eta_2}, \end{gathered} \tag{14}$$

and $\| \; \|$ denotes the *sup norm* on $\mathbb{R}^2$, for fixed positive real numbers η_1, η_2, ϵ_1 and ϵ_2. The *a priori* continuous signal $\widetilde{\mathbf{u}}$: $I \to \mathbb{R}^2$ is obtained numerically from the solution of the system of differential equations (7) and (10)–(11).

Since $\widetilde{\mathbf{x}}(0) = \mathbf{x}(0)$, it is to be remarked that if one could take $\mathbf{u} = \widetilde{\mathbf{u}}$ in (6), then, under the assumption that (6) is not disturbed, it is clear that $\mathbf{x}(t) = \widetilde{\mathbf{x}}(t)$ for all $t \in I$ and the tracking problem would be completely solved, since $\widetilde{\mathbf{u}}$ satisfies (13)–(14). But $\widetilde{\mathbf{u}}$ is not a piecewise-constant signal and one thus runs into the difficulties of the practical implementation of $\mathbf{u}$ as an environmental management policy mentioned in Section 3.

According to the proposed control strategy for the tracking problem of (6), the determination of the vectors $\mathbf{u}_k$, for $k = 0, \ldots, p-1$, consists in approximating the signal $\widetilde{\mathbf{u}}$: $I \to \mathbb{R}^2$ by a step map $\mathbf{u}$: $I \to \mathbb{R}^2$ with respect to the partition P of the interval I. From (9), it is seen that $\mathbf{u}(t) = \mathbf{u}_k$ for $t \in I_k$. Therefore, only the value $\mathbf{u}(k\lambda)$ at $t = k\lambda$ is required to completely determine $\mathbf{u}$ on the interval I_k. As will be shown, in order to obtain $\mathbf{u}(k\lambda)$ it is needed: (i) the signal $\widetilde{\mathbf{u}}|I_k$: $I_k \to \mathbb{R}^2$ of (10), where $\widetilde{\mathbf{u}}|I_k$ denotes the restriction of $\widetilde{\mathbf{u}}$: $I \to \mathbb{R}^2$ to the interval I_k; and (ii) an estimate $\widehat{\mathbf{x}}(k\lambda)$ of the state $\mathbf{x}(k\lambda)$. But before proceeding to determine the vectors $\mathbf{u}_k$, one last assumption on system (6) is made: (A_3) the dynamics of (6) is sufficiently slow, so that one may disregard the required computational time τ to obtain the continuous signal $\widetilde{\mathbf{u}}$: $I \to \mathbb{R}^2$ from the system of differential equations (7) and (10)–(11) by means of a chosen numerical method. In other words, it assumed that $\|\mathbf{x}(\tau) - \mathbf{x}(0)\| \approxeq \mathbf{0}$.

At this point, an *open-loop state estimator* of (6) is defined as

$$\frac{d}{dt}\widehat{\mathbf{x}} = \mathbf{f}(\widehat{\mathbf{x}}) + \mathbf{u}, \tag{15}$$

with $\widehat{\mathbf{x}}(0) = \mathbf{x}(0) = \widetilde{\mathbf{x}}(0)$. Hence, as long as (6) is not submitted to disturbances, $\widehat{\mathbf{x}}(t) = \mathbf{x}(t)$ for all $t \in I$. If this is the case, any investigation about the state $\mathbf{x}$ can be carried out by considering $\widehat{\mathbf{x}}$ instead. In what follows, a comparison is made between the trajectory of the state $\widehat{\mathbf{x}}$, which has as input the piecewise-constant signal $\mathbf{u}$ defined by (9), and the trajectory of the state $\widetilde{\mathbf{x}}$, which has as input the continuous signal $\widetilde{\mathbf{u}}$.

The *state deviation* is defined as

$$\Theta = \widehat{\mathbf{x}} - \widetilde{\mathbf{x}} \tag{16}$$

and the scalars $u_{k,j}$ in (9) as

$$u_{k,j} = \frac{1}{\lambda} \int_{k\lambda}^{(k+1)\lambda} \widetilde{u}_j(t)\,dt - \frac{1}{\lambda}(\widehat{x}_j(k\lambda) - \widetilde{x}_j(k\lambda)) \tag{17}$$

for $k = 0, \ldots, p-1$, $j = 1, 2$. Note that the first term on the right-hand side is nothing but the mean value of $\widetilde{u}_j|I_k$. It follows from the integral equations of (10) and (15), along with the control strategy (9) and (17), that

$$\Theta((k+1)\lambda) = \int_{k\lambda}^{(k+1)\lambda} (\mathbf{f}(\widehat{\mathbf{x}}(t)) - \mathbf{f}(\widetilde{\mathbf{x}}(t)))\,dt \tag{18}$$

at $t = (k+1)\lambda$, for $k = 0, \ldots, p-1$.

Although not presented here, simulations showed that if p in (8) is chosen sufficiently large,

$$\Phi \triangleq \sup_{t \in I} \|\Theta(t)\| = \sup_{t \in I} \|\widehat{\mathbf{x}}(t) - \widetilde{\mathbf{x}}(t)\| \tag{19}$$

will be less than an acceptable value according to practical interests. In other words, it was verified that p can be specified so that the estimated state vector $\widehat{\mathbf{x}}$ stays in a small neighborhood of the state vector $\widetilde{\mathbf{x}}$. Furthermore, simulations also showed that for a sufficiently large p,

$$\Psi \triangleq \sup_{t \in I} \|\mathbf{u}(t) - \widetilde{\mathbf{u}}(t)\| \tag{20}$$

will be less than an acceptable value, that is, $\widetilde{\mathbf{u}}$ can be approximated by $\mathbf{u}$ in the interval I in a reasonable manner.

In summary, the control strategy consists in determining a piecewise-constant control vector $\mathbf{u}$: $I \to \mathbb{R}^2$ from (9) and (17), based on the continuous signal $\widetilde{\mathbf{u}}$: $I \to \mathbb{R}^2$ in (11), known at $t = 0$, that gets the state $\widetilde{\mathbf{x}}$ to track the reference state $\mathbf{r}$ in the interval I within the performance specification (13)–(14). Here, the signal $\widetilde{\mathbf{u}}$ is obtained numerically from the system of differential equations (7) and (10)–(11), recalling that the SMC methodology was applied to define (11).

The block diagram of the proposed control strategy is shown in Fig. 3, where the block PCS (Piecewise-Constant Signal) represents (9) with (17) and ZOH is a zero-order hold. The initial condition of the integrator of the block PCS is $\int_{-\lambda}^{0} \widetilde{\mathbf{u}}(t+\lambda)\,dt$. It is emphasized that the state vector $\mathbf{x}$ of (6) is indeed only measured at $t = 0$. Furthermore, two important remarks about this methodology should be made. First, assumption (A_3) is indeed verified in many predator-prey interactions found in nature. Suppose, for instance, that the trajectories of (6) are oscillating at the stable limit cycle depicted in Fig. 1.a. If its period is in the order of years or months, the computational time τ required to obtain $\widetilde{\mathbf{u}}$ on the interval $[0, \Delta]$ can obviously be disregarded if it is in the order of days or hours. And second, since SMC is a general control scheme for nonlinear systems, the approach presented in this paper can be applied to predator-prey system models other than (5).

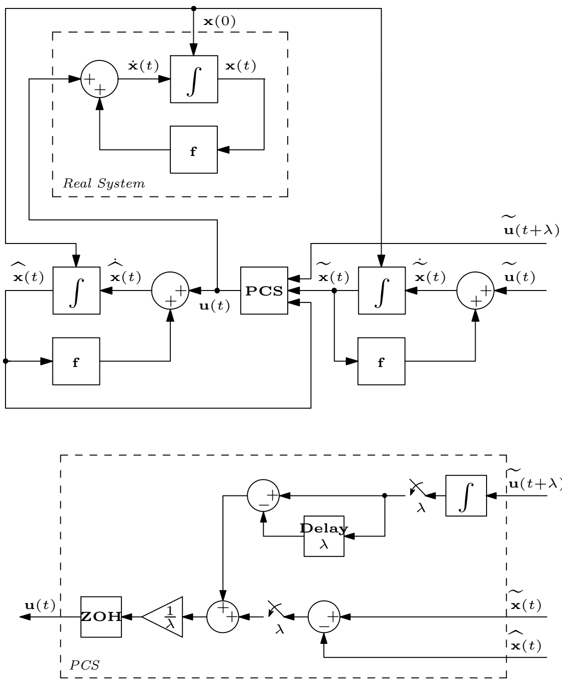

Fig. 3. Block diagram of the control strategy.

5. SIMULATION RESULTS

The period of the limit cycle of the isolated system (3) (with respect to the normalized time t) is approximately $W = 26.46$. Although t is a normalization of time T by (2), it is assumed for simplicity that 13.11 corresponds to a two-year period in the real ecosystem and that the state vector $\mathbf{x}$ is measured with a period corresponding to six years. Therefore, $W/12 = 13.11/12 \approx 1.09$ corresponds to two months in the real system. Define $\Delta = 36 \cdot 1.09 = 39.24$, so that the measurement period of $\mathbf{x}$ does correspond to six years. Choosing $p = 36$ in (8) yields $\lambda = \Delta/36 = 1.09$. The control problem is to get the state $\mathbf{x}$ to track the reference state $\mathbf{r}$ in the interval $I = [0, \Delta]$, without any further measurements of the state $\mathbf{x}$ besides $\mathbf{x}_0$, and by applying piecewise-constant input signals. The control strategy described in the previous section shall determine a control signal $\mathbf{u} = [u_1 \;\; u_2]'$ that is step map with respect to the partition $P = (0, \lambda, \ldots, \Delta)$ of the interval I. This means that the control signals u_1 and u_2 remain constant during every two months in the real system. Specify $\eta_1 = \eta_2 = 1$ and $\epsilon_1 = \epsilon_2 = 0.001$ in (11). Since $\mathbf{r}_0 = \mathbf{x}_0$, (13) and (14) give $\widetilde{\mathbf{x}}(t) = \mathbf{r}(t)$ for $t \in [0, 39.24]$. The *error vector* is defined as $\mathbf{e} = \mathbf{x} - \mathbf{r}$ or, in terms of its components, $\mathbf{e} = [e_1 \; e_2]'$, with $e_j = x_j - r_j$.

At $t_{is} = 10.9$, the value of the reference state is $\mathbf{r}_{is} \triangleq \mathbf{r}(t_{is}) = [1.9709 \;\;\; 0.9736]'$. From Fig. 2, it is seen that $\mathbf{r}_{is}$ is inside the basin of attraction of the original limit cycle of the isolated system. Therefore, the following control strategy for the system is proposed: (i) force the state $\mathbf{x}$ to track $\mathbf{r}$ during the *control interval* $I_c = [0, t_{is}] = [0, 10.9]$ by means of human action; and (ii) maintain system (6) free of human interference for all $t > t_{is} = 10.9$, since from the moment on human action is suspended, the resulting trajectories will converge to the original limit cycle.

By applying the proposed control scheme, a piecewise-constant control signal $\mathbf{u}$ is obtained for the control interval $I_c = [0, 10.9] \subset I$, recalling that $10\lambda = 10.9$. And, in order to maintain system (6) free of human action after $t = t_{is}$, define $\mathbf{u}(t) = \mathbf{0}$ for all $t > t_{is} = 10.9$. In other words, system (6) is then nothing but the isolate system (3). Fig. 4 shows the phase portraits and trajectories of RS and CS, obtained by simulation. It is also presented the resulting phase portrait and trajectories from the suspension of human action in system (6) after $t_{is} = 10.9$, that is, the dynamics of IS for all $t > t_{is}$. Note that the trajectories of the isolated system indeed return to oscillate at the original limit cycle. As seen from Fig. 5, a satisfactory tracking accuracy is obtained on I_c for the choice of $\lambda = 1.09$. Although the amplitudes of the components e_1 e e_2 are increasing on I_c, simulations showed that in case it was required $\mathbf{x}$ to track $\mathbf{r}$ in the interval $I = [0, \Delta] = [0, 39.24]$, one would obtain $\|\mathbf{e}\| \leq 0.0115$ for $t \in [0, 39.24]$, so that the error vector remains bounded by a reasonable value on I. The signals $\widetilde{\mathbf{u}}$ and $\mathbf{u}$ on the control interval I_c are presented in Fig. 6. Observe that the control signals u_1 and u_2 can be (ideally) implemented in the real system by means of an environmental management policy, thus allowing the restoration of the original dynamics of the isolated system and the ecological recovery of the predator-prey system. The policy consists in establishing constant harvesting quotas (if the control signals are negative) or constant rates of introduction of new individuals (if the control signals are positive), for both prey and predators, that should be complied by society on a two-month basis. Furthermore, the policy is obtained for the time interval $I_c = [0, 10.9]$, that is, for 20 months, by measuring only $\mathbf{x}_0$. Note that the prey are harvested at all times during the interval I_c, but for the predators harvesting is required as well as introduction of new individuals. However, as shown in Fig. 6, more predator individuals are harvested than new ones introduced.

It may not always be possible to introduce new individuals of a certain species into an ecosystem. If this is the case, the piecewise-constant control signals u_1 and u_2 have to be of nonpositive values at all times for the resulting management policy to be implemented. From Fig. 6, it is seen that only the u_2 assumes positive values. Since they are of relatively small amplitude, it is expected that by restricting $u_{k,j}$ in (9) and (17) to only assume nonpositive values, satisfactory results for the error $\mathbf{e}$ will be obtained. That is, $u_{k,j}$ is defined by (17), but redefined according to the condition $u_{k,j} > 0 \Rightarrow u_{k,j} = 0$, for $k = 0, \ldots, p-1$, $j = 1, 2$. This situation was simulated and although not shown here, the obtained tracking accuracy was almost identical to the one shown in Fig. 5.

Therefore, one concludes that, in the analyzed case, it was indeed possible to restore the original dynamics of the isolated system (3) by harvesting the prey and predators in the interval I_c and with no need of introducing new individuals, which might be counter-intuitive.

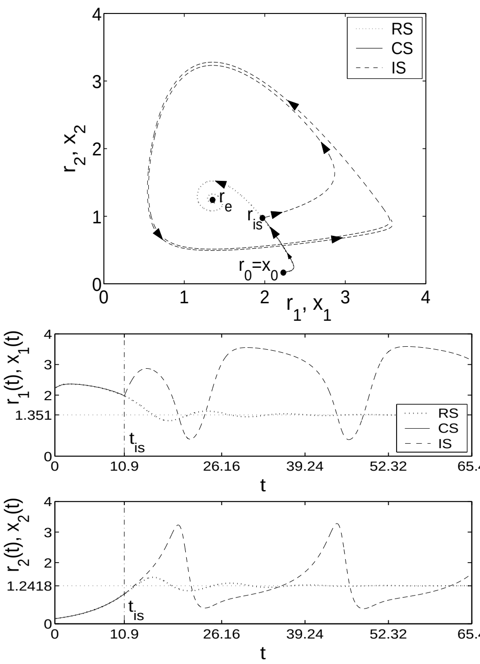

Fig. 4. Phase portraits and trajectories of RS, CS and IS.

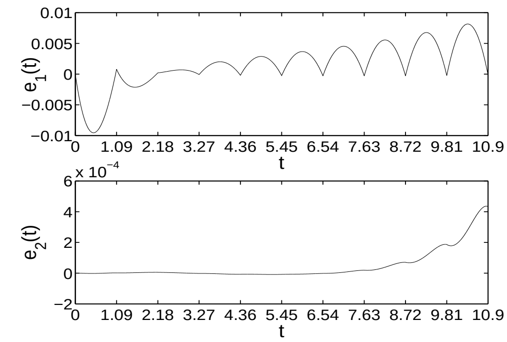

Fig. 5. Error vector $\mathbf{e}$ on $I_c = [0, 10.9]$.

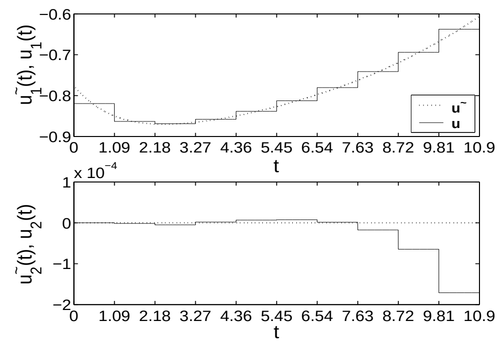

Fig. 6. Signals $\widetilde{\mathbf{u}}$ and $\mathbf{u}$ on $I_c = [0, 10.9]$.

6. CONCLUSION

The proposed control strategy determines piecewise-constant control signals that are possible to be (ideally) implemented by environmental agencies as management policies. Simulations showed that when the only difference between the reference parameters and the ones of the isolated system is $\gamma_r < \gamma$, it is indeed possible to obtain a satisfactory tracking accuracy in case the restriction that the control signals only assume nonpositive values is considered. This remains to be investigated. Furthermore, this methodology can be applied to ecological systems involving n species that are modelled by $\dot{\mathbf{x}} = \mathbf{h}(\mathbf{x}) + \mathbf{u}$, where $\mathbf{h}$: $\mathbb{R}^n \to \mathbb{R}^n$ is continuous on $\mathbb{R}_+ \times \ldots \times \mathbb{R}_+$ (n times) and exactly known.

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