FLOW CONTROL FOR CONTINUOUS PETRI NET MODELS OF HDS : STABILITY ISSUES

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Abstract: This paper is about the flow control design of hybrid dynamical systems modelled with continuous Petri nets and described as a set of bilinear state space representations. The marking vector stands for the state space vector, where some places are observable and other are not. The transitions are divided into controllable ones and uncontrollable ones. Gradient-based controllers are investigated in order to adapt the firing speeds of the controllable transitions according to a desired trajectory of the marking. The equilibriums and stability of the controlled system are discussed according to the controller time parameter.

Keywords: Control, stability criteria, Lyapunov functions, Petri nets, gradient methods

1 INTRODUCTION

Continuous Petri nets are useful for the study of hybrid dynamical systems (HDS) (Zaytoon *et al.,* 1998) because they combine discrete and structural aspects (Diaz *et al.,* 2001; Murata, 1989) with continuous evolution (David *et al.,* 1992). A lot of results based on continuous Petri nets have been established for the control design of HDS (Silva *et al.,* 2003). The motivation to use continuous Petri nets is either to model the continuous part of HDS, or to work out a continuous approximation of the discrete part in order to avoid the complexity associated to the exponential growth of states. Flow control design has been developed with different classes of controllers: constrained state feedback (Amrah *et al.,* 1996; Krogh *et al.,* 1996; Lefebvre, 1999), linear programming (Hanzalek, 2003), optimal control (Egilmez *et al.,* 1994) and also gradient based control (Lefebvre *et al.,* 2003; 2004). The obtained results show that the continuous approach is an interesting alternative to the supervisory control (Giua *et al.,* 1994; 2004; Ramadge *et al.,* 1987; Uzam *et al.,* 1999) and to the max plus algebra (Cohen *et al.*, 1999) developed in the context of discrete event systems theory.

This paper focuses on the design of flow controllers suitable to drive the continuous part of HDS. Properties of gradient based controllers (Lefebvre *et al.,* 2003; 2004) are further investigated. The main contribution is to discuss the equilibriums and stability of the controlled system according to the controller parameters. For this purpose, HDS are totally or partially modelled with continuous Petri nets with variable speeds (VCPN) (David *et al.,* 1992) described as a set of bilinear state space representations. The marking vector is considered as a state space vector. Outputs are defined by the marking of places subsets, and inputs correspond to the maximal firing frequencies of the controllable transitions. The controllers are based on the minimisation of the instantaneous quadratic error between desired and measured outputs (Thomas, 1997; Widrow *et al.,* 1990) and calculate on line the maximal firing frequencies of the controllable transitions.

The paper is divided into 6 sections. Section 2 is about Petri nets, VCPN and the modelling and control of HDS. Section 3 concerns the gradient based flow control of VCPN. Equilibriums of the controlled system are investigated in section 4. Stability issues are discussed in section 5. A manufacturing line example is studied throughout the paper.

2 PETRI NETS

A Petri net (PN) with n places and p transitions is defined as $< P,\ T,\ \text{Pre},\ \text{Post},\ M_0 >$ where $P=\{P_i\}_{i=1,\dots,n}$ is a not empty finite set of places, $T=\{T_j\}_{j=1,\dots,p}$ is a not empty finite set of transitions, such that $P \cap T = \emptyset$ (David *et al.*, 1992; Murata, 1989). IN is defined as the set of integer numbers. Pre : $P \times T \rightarrow IN$ is the pre-incidence application: Pre (P_i, T_j) is the weight of the arc from place P_i to transition T_j and $W_{PR} = (w^{PR}_{ij})_{i=1,\dots,n,\ j=1,\dots,p} \in IN^{n \times p}$ with w^{PR}_{ij} = Pre (P_i, T_j) is the pre-incidence matrix. Post : $P \times T \rightarrow IN$ is the post-incidence application: Post (P_i, T_j) is the weight of the arc from transition T_j to the place P_i and $W_{PO} = (w^{PO}_{ij})_{i=1,\dots,n,\ j=1,\dots,p} \in IN^{n \times p}$ with w^{PO}_{ij} = Post (P_i, T_j) is the post-incidence matrix. The PN incidence matrix W is defined as $W = W_{PO} - W_{PR} \in IN^{n \times p}$. Let us also defined $M(t) = (m_i(t))_{i=1,\dots,n} \in IN^n$ as the marking vector at time t and $M_0 \in IN^n$ as the initial marking vector. ${}^{o}T_j$ (resp. $T_j{}^{o}$) stands for the preset (resp. post-set) places of T_j. A firing sequence is defined as an ordered series of transitions that are successively fired from marking M to marking M'. Such a sequence is represented by its characteristic vector $X = (x_j)_{j=1,\dots,p} \in IN^p$ where x_j stands for the number of T_j firings.

$$M' = M + W.X. \tag{1}$$

2.1 Continuous Petri nets

Continuous Petri nets have been developed in order to provide a continuous approximation of the discrete behaviours of discrete event systems (DES) (David *et al.,* 1992). A continuous Petri net with n places and p transitions is defined as < PN, X_{max} > where PN is a Petri nets and $X_{max} = (x_{max\,j})_{j=1,\dots,p} \in IR^{+p}$ is the vector of maximal firing frequencies (IR^+ is the set of non-negative real numbers). The marking $m_i(t) \in IR^+$ of each place P_i, $i = 1,\dots,n,$ has a non-negative real value and each transition firing is a flow of marks in continuous PN. Let us define $X(t) = (x_j(t))_{j=1,\dots,p} \in IR^{+p}$ as the firing frequencies vector at time t. The marking evolution is given by (2):

$$\dot{M}(t) = W.X(t)\,. \tag{2}$$

Among the existing models of continuous PN, continuous PN with variable speeds (VCPN) are well known approximations of timed PN (David *et al.,* 1992) where vector $X(t)$ depends continuously on the marking of the places according to (3) :

$$x_j(t) = x_{max\,j}.\mu_j(t)\,, \qquad \mu_j(t) = \min_{P_i \in {}^{o}T_j} (m_i(t)) \tag{3}$$

A self loop is usually attached to each transition T_j in order to limit the effective firing frequencies $x_j(t)$. Other specifications of the firing frequencies can also be considered as in the next example.

2.2 HDS modelling and control

Continuous PN are suitable to model the continuous part of HDS in a very intuitive way thanks to the underlying digraph structure. Combined with TPN they are useful to describe HDS with a single formalism (Balduzzi *et al.,* 2000; Zaytoon *et al.,* 1998) as illustrated with the example in figure 1, modelled with the hybrid PN in figure 2.

The places P_1 and P_2 are continuous and the markings m_1 and m_2 stand respectively for the height of liquid in tank 1 and tank 2 given by (4):

$$S_1.\dot{m}_1 = x_1 - x_2 - x_3\,, \qquad S_2.\dot{m}_2 = x_2 + x_3 - x_4 \tag{4}$$

where S_1 and S_2 stand for the sections of tank 1 and tank 2. The transitions T_1 to T_4 are also continuous whose firing represents respectively the input flow (T_1), the output flow (T_4) and the flows through the pipes A (T_2) and B (T_3) according to (5):

$$\begin{aligned} x_1 &= D & x_3 &= \alpha_3.\sqrt{\sup(m_1,h) - \sup(m_2,h)} \\ x_2 &= \alpha_2.\sqrt{m_1 - m_2} & x_4 &= \alpha_4.\sqrt{m_2} \end{aligned} \tag{5}$$

where D, α_2, α_3 and α_4 are related to the system specifications and it is assumed that $m_1 \geq m_2$.

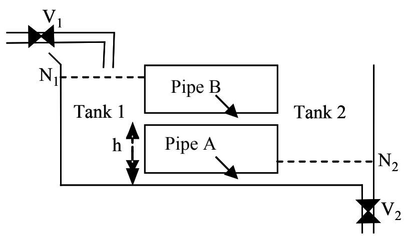

Figure 1: Two tanks system

The discrete part of the PN (places P_3 and P_4 and transitions T_5 and T_6) stands for the controller. A mark in P_3 means that valve V_1 is open and V_2 is closed. On the contrary a mark in P_4 means that valve V_2 is open and V_1 is closed. The arcs from P_1 to T_5 and from P_2 to P_6 are test arcs (the value of the places P_1 and P_2 is not changed by firing the transitions T_5 and T_6). The goal of the controller is to open V_1 and close V_2 when $m_2 < N_2$ and to open V_2 and close V_1 when $m_1 > N_1$.

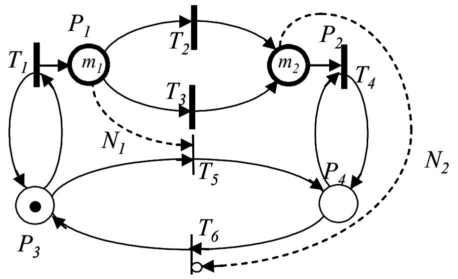

Figure 2: Hybrid PN of the two tanks system

Another example to motivate the use of continuous PN to model and control HDS will be considered in the next section (figure 3). But, instead of using discrete PN as controllers, flows are controlled with gradient–based algorithms.

2.3 State space representation

Equation (3) has commutations and shows products between state space and inputs vectors. So, VCPN are piecewise linear models that will be described with a set of bilinear state space representations. The set T is divided into 2 disjoint subsets T_C, and T_{NC} such that $T = T_C \cup T_{NC}$. T_C is the subset of the controllable transitions, and T_{NC} is the uncontrollable transitions subset. Let us define $X_C(t) = (x_j(t))_{Tj \in TC} \in IR^{+d}$ and $X_{NC}(t) = (x_j(t))_{Tj \in TNC} \in IR^{+p-d}$:

$$D^{-1}.X(t) = \begin{pmatrix} X_C(t) \\ X_{NC}(t) \end{pmatrix}, \qquad (6)$$

with $D \in IR^{p \times p}$. The controllable inputs vector $U(t) = X_{max\ C}(t) \in IR^{+d}$ corresponds to the maximal firing frequencies to be controlled. The input vector is constrained $0 \leq U(t) \leq U_{max}$ in order to limit the firing frequencies in a non negative bounded interval. The uncontrollable maximal firing frequencies $X_{max\ NC}$ are supposed to be constant according to the VCPN models. The output vector $Y(t) = Q.M(t) \in IR^{+e}$ is composed of a selection of subnets marking that are observable, with $Q = (q_{ki}) \in IR^{e \times n}$. The goal of the controller is to drive $Y(t)$ according to some reference trajectories in the output space. Equation (2) can be rewritten as:

$$\begin{aligned} \dot{M}(t) &= W_C.X_C(t) + W_{NC}.X_{NC}(t) \\ Y(t) &= Q.M(t) \end{aligned} \qquad (7)$$

with $W_C = (w_{C\ ij}) \in IR^{n \times d}$ and $W_{NC} = (w_{NC\ ij}) \in IR^{n \times (p-d)}$ such that $(W_C \mid W_{NC}) = W.D$.

Several phases occur in the VCPN behaviour. Each phase φ is active between two successive commutations of the "min" operators in (3) and corresponds to a particular configuration of these operators characterised by the p functions of classification f_j:

$$\forall T_j \in T: \qquad f_j: \quad IR^{+n} \rightarrow \{1,\ldots,n\}$$
$$M(t) \rightarrow m_{fj}(t) = f_j(M(t)) \qquad (8)$$

such that $m_{fj}(t) = \mu_j(t)$. Each function f_j specifies the place in the preset of T_j which has the minimal marking. During each phase φ, a constant relation between the components of vectors $X_C(t)$ and $M(t)$ and also between $X_{NC}(t)$ and $M(t)$ occurs. This relation can be expressed with a vectorial form by using the set of vectors $A_j(\varphi) \in \{0,1\}^{1 \times n}$ and $B_j(\varphi) \in \{0,1\}^{1 \times n}$ which are constant during each phase but which may varied from one phase to another:

$$\begin{aligned} x_{C\,j}(t) &= u_j(t).m_{f_j}(t) \\ &= u_j(t).A_j(\varphi).M(t), \qquad j = 1,\ldots d \\ x_{NC\,j}(t) &= x_{\max NCj}.m_{f_j}(t) \\ &= x_{\max NCj}.B_j(\varphi).M(t) \qquad j = 1,\ldots p-d \end{aligned} \qquad (9)$$

Equation (7) can be rewritten as (10):

$$\begin{aligned} \dot{M}(t) &= \left(\sum_{j=1}^{d} u_j(t).W_{Cj}.A_j(\varphi) + \sum_{j=1}^{p-d} x_{\max NC\,j}.W_{NCj}.B_j(\varphi) \right).M(t) \\ Y(t) &= Q.M(t), \end{aligned} \qquad (10)$$

where W_{Cj} denote the j^{th} column of matrix W_C and W_{NCj} denote the j^{th} column of matrix W_{NC}.

3 FLOW CONTROL FOR VCPN

Flow control for VCPN was investigated by several authors (Amrah *et al.,* 1996; Hanzalek, 2003; Egilmez *et al.,* 1994; Krogh *et al.,* 1996; Lefebvre, 1999; Silva *et al.* 2003). Such methods have provided interesting but local results attached to a specific phase in the VCPN behaviour. Moreover, they require strong conditions concerning the transitions to control and the places to observe. This paper focuses on another approach that takes advantage of the gradient propagation through the PN nodes. Our previous works have shown that they are also suitable to control the continuous part of HDS using VCPN models (Lefebvre *et al.,* 2003).

3.1 Gradient – based controllers

For simplicity, let us consider single iteration controllers in discrete time and focus on single output case. The instantaneous error is defined by $\varepsilon(k) = y_d(k) - y(k)$, where $y_d(k)$ stands for the desired output at time $t = k.\Delta t$, and $y(k)$ stands for the measured output of the VCPN at same time. By using a first order numerical integration method:

$$\begin{aligned} y(k) = y(k-1) + \Delta t.(&\sum_{j=1}^{d} u_j(k-1).Q.W_{Cj}.A_j(\varphi) \\ &+ \sum_{j=1}^{p-d} x_{\max NC\,j}.Q.W_{NCj}.B_j(\varphi)).M(k-1) \end{aligned} \qquad (11)$$

The updating to each new pattern of the input vector $U(k)$ is performed by writing a Taylor series expansion of the cost function $v(k) = 1/2.\varepsilon^2(k) \in IR$. The second order terms are usually neglected in the evaluation of the Hessian matrix but a small positive term $\lambda.I$ is added to avoid ill conditioned matrices (Hagan *et al.,* 1995). The actualisation of controllable inputs is given by (12):

$$\delta U(k) = -(S(k).S(k)^T + \lambda.I)^{-1}.S(k).\varepsilon(k), \qquad (12)$$

with $\delta U(k) = U(k) - U(k-1)$. $S(k) = (s_\gamma(k)) \in IR^d$ is the gradient of the output with respect to the input variation $\delta U(k) = (\delta u_\gamma(k))$ performed at time $t = k.\Delta t$. The gradient $s_\gamma(k)$ is computed with a first order method by using the functions f_β defined by (8):

$$\delta s_\gamma(k) = \sum_{i=1}^{n} q_i \cdot \left(w_{i\gamma}.\mu_\gamma(k) + \sum_{\substack{j=1 \\ j\neq\gamma}}^{p} w_{ij}.x_{\max j}.s_{f\beta\gamma}(k) \right).\Delta t \qquad (13)$$

with $\delta s_\gamma(k) = s_\gamma(k) - s_\gamma(k-1)$ and $s_\gamma(0) = 0$.

The previous control algorithm can be extended to the multi output case, and to multi iterations updating (Lefebvre *et al.,* 2004).

3.2 Example

The VCPN with the marking vector $M(t) = (m''_0(t), m''_1(t), m''_2(t), m_1(t), m_2(t), m'_1(t), m'_2(t))^T$ shown in figure 3 is the model of a manufacturing process with 2 machines M_1 and M_2 corresponding to the 2 transitions T_1 and T_2 and 2 buffers with limited capacities corresponding to the subsets of places $\{P_1, P'_1\}$ and $\{P_2, P'_2\}$ (Amrah *et al.,* 1996). The maximal capacities C_1 and C_2 of the places P_1 and P_2 correspond to the initial marking $m_1(0) + m'_1(0) = C_1$ and $m_2(0) + m'_2(0) = C_2$. Pieces enter in the system by firing T_0. The firing frequencies are bounded by the marking of the places P''_0, P''_1, and P''_2. In the sequel, $M_0 = (1,1,1,0,0,3,3)^T$, $\lambda = 1$ and $\Delta t = 0.06$. When the system is not controlled, it evolves freely as expressed in (14). For example, with a vector of maximal firing frequencies $X_{max} = (5, 4, 3)^T$, the system reaches a stationary point after 3s and the marking tends to $(m_1, m_2) = (12/5, 9/4)$.

$$\begin{aligned} \dot m_1(t) &= x_{\max 0}.\min(m''_0(t), m'_1(t)) \\ &\quad - x_{\max 1}.\min(m''_1(t), m_1(t), m'_2(t)) \\ \dot m_2(t) &= x_{\max 1}.\min(m''_1(t), m_1(t), m'_2(t)) \\ &\quad - x_{\max 2}.\min(m''_2(t), m_2(t)) \end{aligned}$$

$$\begin{aligned} m'_i(t) &= C_i - m_i(t) = 3 - m_i(t) \qquad & i = 1,2 \\ m''_j(t) &= m''_j(0) = 1 & j = 1,2,3 \end{aligned} \qquad (14)$$

Equation (14) is a piecewise bilinear model with constrained inputs $0 \le U(t) \le 10$. The set of controllable transitions is $T_C = \{T_0\}$, $T_C = \{T_1\}$, $T_C = \{T_2\}$, $T_C = \{T_0, T_1\}$, $T_C = \{T_1, T_2\}$, $T_C = \{T_0, T_2\}$ or $T_C = \{T_0, T_1, T_2\}$ depending on the system specifications. For example, if the manufacturing system is considered as a single input, multi outputs system, with $T_C = \{T_0\}$ and Q = ((0 0 0 1 0 0 0)^T; (0 0 0 0 1 0 0)^T)^T, then $U(t) = x_{max\ 0}(t)$, $y_1(t) = m_1(t)$, and $y_2(t) = m_2(t)$. The controller should adapt the maximal firing frequency of the input transition T_0 in order to drive buffers level to some desired values (for example $y_{d1} = 1/2$ and $y_{d2} = 2/3$). A vectorial form of (14) is given by (15):

$$\begin{aligned} \dot M(t) &= (u(t).W_{C1}.A_1(\varphi) + x_{\max 1}.W_{NC1}.B_1(\varphi) \\ &\quad + x_{\max 2}.W_{NC2}.B_2(\varphi)).M(t), \\ Y(t) &= \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.M(t) \end{aligned} \qquad (15)$$

with W_{C1}.= (0, 0, 0, 1, 0, -1, 0)^T, W_{NC1}.= (0, 0, 0, -1, 1, 1, -1)^T, W_{NC2}.= (0, 0, 0, 0, -1, 0, 1)^T. The row vectors $A_1(\varphi)$, $B_1(\varphi)$, and $B_2(\varphi)$ depend of the current phase.

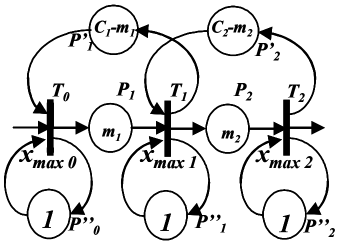

Figure 3: VCPN model of a manufacturing process

The single iteration and also multi-iterations controllers reach both the goal (figure 4) because the desired marking is included in the set of the reachable equilibriums for controller $u(t) = x_{max\ 0}(t)$.

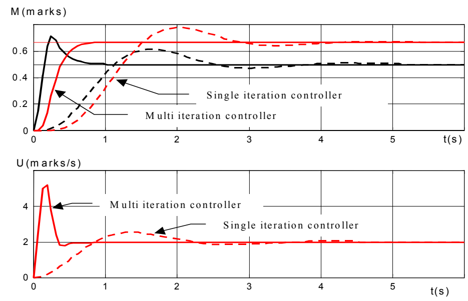

Figure 4: Single input - multi output system

4 SET OF EQUILIBRIUMS

According bilinear state space representation of VCPN (2), the set of equilibriums for the system under control is given by (16):-

$$\sum_{j=1}^{d} w_{Cij}.u_j(t).m_{f_j}(t) + \sum_{j=1}^{p-d} w_{NCij}.x_{\max NCj}.m_{f_j}(t) = 0 \qquad (16)$$

Let us define the matrices $A(M) = (a_{ij}(M))_{i=1,\dots,n,\ j=1,\dots,d} \in IR^{+\ n \times d}$, and $B(M) = (b_i(M))_{i=1,\dots,n} \in IR^{+\ n}$, with $a_{ij}(M) = w_{Cij}.m_{fj}(t)$ and $b_{ij}(M) = w_{NCij}.x_{max\ NCj}.m_{fj}(t)$. Thus, (17) can be rewritten as $A(M(t)).U(t) = B(M(t))$. A marking vector M_e is an equilibrium for a constant control vector U if $A(M_e).U = B(M_e)$ holds. In order to describe the set of solutions of this equation in terms of control vector U, let us introduce also the following notations r = rank($A(M_e)$) and h = rank($A(M_e) \mid B(M_e)$). If $r = h$, there exists at least one constant control vector U such that M_e is an equilibrium. The set of admissible constant control vectors is given by table 1, according to the Moore-Penrose inverse of matrix A. For constrained control vectors, one has to verify that the resulting control vectors satisfy the constraints. In case $r < h$, no constant control vector exists.

Table 1 : Set of admissible constant control vector

Parameters		r	A^+	Solutions
$r=h$	$d=n$	$r=n$	A^{-1}	Unique
		$r<n$	Max. rank fact.	Several
	$d<n$	$r=d$	$(A^T A)^{-1} A^T$	Unique
		$r<d$	Max. rank fact.	Several
	$n<d$	$r=n$	$A^T(A.A^T)^{-1}$	Several
		$r<n$	Max. rank fact.	Several

A systematic investigation of the set of equilibriums for the VCPN in figure 3 results in figures 5 to 7 (in black: without control, in light grey: with constant control, in dark grey: reachable points from the origin with constant control).

When all transitions are controllable, the region of equilibriums fills the complete area [0, 3] x [0, 3]. Let us mention that the region of reachable points from the origin with constant control is always strictly included in the region of equilibriums.

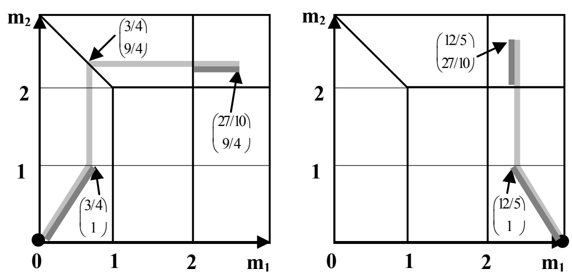

Figure 5: a) $U = x_{max0}$, b) $U = x_{max1}$

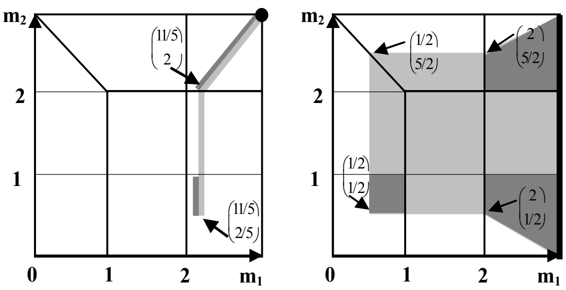

Figure 6: a) $U = x_{max2}$, b) $U = (x_{max1}, x_{max2})^T$

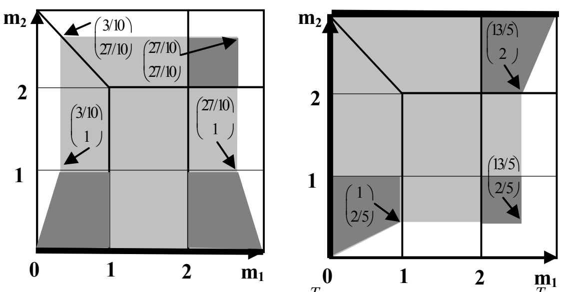

Figure 7: a) $U = (x_{max0}, x_{max1})^T$, b) $U = (x_{max0}, x_{max2})^T$

5 STABILITY ISSUES

Let us consider in this section the case of a single output VCPN in discrete time as given by (11). In order to study the stability of the closed loop system, $v(k)$ is considered as a Lyapunov function and $\delta\varepsilon(k) = \varepsilon(k+1) - \varepsilon(k)$ can be rewritten as:

$$\delta v(k) = 1/2.(\varepsilon^2(k+1) - \varepsilon^2(k)) = \delta\varepsilon(k).(\varepsilon(k) + 1/2.\delta\varepsilon(k)) \quad (17)$$

Let us rewrite $\delta\varepsilon(k)$ as :

$$\delta\varepsilon(k) = \sum_{j=1}^{d} \left(\frac{\partial y}{\partial u_j}\right)_k .\delta u_j(k) = S^T(k).\delta U(k) \quad (18)$$

Thus (17) can be rewritten as :

$$\delta v(k) = S^T(k).\delta U(k).(\varepsilon(k) + 1/2.S^T(k).\delta U(k)) \quad (19)$$

When the term $S(k).S(k)^T$ is neglected in (12), the updating rule of the controller corresponds to the gradient method: $\delta u(k) = \eta.S(k).\varepsilon(k)$ and (19) can be rewritten as :

$$\delta v(k) = -\eta.S^T(k).S(k).\varepsilon^2(k).(1 - 1/2.\eta.S^T(k).S(k)) \quad (20)$$

Thus a sufficient condition for stability is:

$$0 < \eta < 2/(S^T(k).S(k)) \quad (21)$$

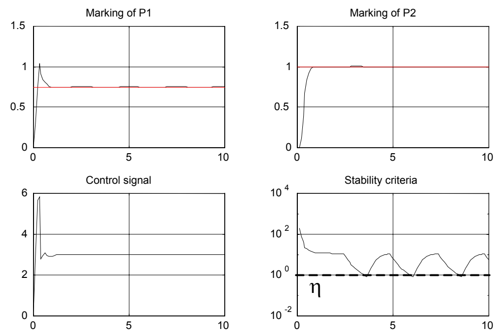

Figure 8: Stable control to reach $M_e = (3/4, 1)^T$

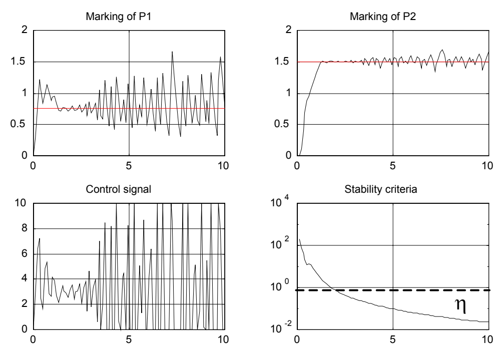

Figure 9 : Unstable control to reach $M_e = (3/4, 3/2)^T$

Illustration of the stability criteria (21) on the example in figure 3 is shown in figures 8 and 9. Both simulations are obtained with a 50–iterations gradient–based controller of transition T_0: $U(t) = x_{max\,0}(t)$ and the learning rate $\eta = 1$. The outputs $y_1(t) = m_1(t)$ and $y_2(t) = m_2(t)$, the control signal $U(t)$ and the stability criteria 2 / $(S(k)^T.S(k))$ are reported for each simulation. In figure 8, the equilibrium $M_e = (3/4,\ 1)^T$ is reached according to a stable control design (the stability criteria is satisfied). Let us mention small amplitude variations around the equilibrium due to numerical instabilities. In figure 9, the controller cannot stabilize the trajectory around the desired output $M_e = (3/4,\ 3/2)^T$. One can also notice that the stability criteria is not satisfied.

6 CONCLUSIONS

In this paper, gradient-based controllers have been investigated for the flow control of HDS modelled by VCPN. The control inputs are the maximal firing frequencies of the controllable transitions, and the goal of the proposed controllers is to reach reference values in the outputs space. The existence and uniqueness of equilibriums was discussed in a systematic way. A sufficient condition for stability of the closed loop system was also provided. Let us notice that the discrete dynamics due to the phase commutations are implicitly taken into account with the iterative calculation of the gradient. Nevertheless, at this time, we have no result to analyse separately the implications of the discrete and continuous dynamics for stability purposes. Such a “two levels” analysis will be further investigated. The next questions will be also studied in future works. How to calculate and to locate the minimal number of controllable transitions depending on the PN model and the desired outputs? How to modify the proposed algorithm in order to use it with discrete Petri nets? The proposed method should be also efficient to track trajectories and also to learn complex behaviours. So the use of learning algorithms with PN models will be pursued in order to develop “learning PN” for the control design and diagnosis of HDS.

REFERENCES

Amrah A., Zerhouni N., El Moudni A., (1996), Constrained state feedback control of a class of discrete event systems modelled by continuous Petri nets, *Proc. ICARCV96*, pp. 979-984, Singapore.

Balduzzi F., Giua A., Menga G., (2000), First-order hybrid Petri nets: a model for optimization and control, *IEEE-TRA*, vol. 16, no. 4, pp. 382 –399.

Cohen G., Gaubert S. Quadrat J.P, (1999), Max-plus algebra and system theory: where we are and where to go now, *Annual Reviews in Control*, vol. 23 pp. 207-219.

David R., Alla H, (1992), *Petri nets and grafcet – tools for modelling discrete events systems*, Prentice Hall, London.

Diaz M. (editor), (2001), *Les réseaux de Petri: Modè les fondamentaux*, pp. 119 – 158, Hermes, Paris.

Egilmez K., Sharifnia A., (1994), Optimal control of a manufacturing system based on a novel continuous flow model with minimal WIP requirement, *Proc. Int. Conf. On CIM*, pp. 113-118, Albany, USA.

Giua, A.; DiCesare, F., (1994), Petri net structural analysis for supervisory control., *IEEE -TRA*, vol. 10, no. 2, pages 185-195.

Hagan M.T., Demuth H., Beale M., (1995), *Neural network design*, PWS pub. C.., Boston, USA.

Hanzalek Z., (2003), Continuous Petri nets and polytopes, *Proc. IEEE-SMC03*, pp. 1513-1520, Washington, USA.

Krogh B.H., Kowalewski S., (1996), State feedback control of condition / event systems, *Math. Comput. Mod.*, vol. 23, no. 11/12, pp. 161-173.

Lefebvre D., (1999), Feedback control designs of manufacturing systems modelled by continuous Petri nets, *International Journal of Systems Science*., vol. 30, no. 6, pp. 591-600.

Lefebvre D, Leclercq E., Druaux F., Thomas P., (2003), Source and sink transitions controllers for continuous Petri nets : a gradient – based approach, *Proc. IFAC - ADHS03*, pp. 229 – 234, Saint Malo, France.

Lefebvre D, Leclercq E., Druaux F., Thomas P., (2004), Commande des flux dans les réseaux de Petri continues par propagation du gradient, *Proc. CIFA04*, Douz, Tunisie.

Murata T., (1989), Petri nets: properties, analysis and applications, *Proc. IEEE*, vol.77, no. 4, pp 541-580.

Ramadge P.J., Wonham W.M., (1987), Supervisory control of a class of discrete event processes, *SIAM J. Cont. Opt*., vol. 25, pp. 206-230.

Sharifnia A., (1994), Stability and performance of distributed production control methods based on continuous flow models, *IEEE TAC*, vol. 34, no. 4, pp. 725 – 737.

Silva M., Recalde L., (2003) On fluidification of Petri nets : from discrete to hybrid and continuous models, *Proc. IFAC - ADHS*, pp. 9 – 20, Saint Malo, France.

Thomas P., (1997), *Contribution à l’identification de systèmes non linéaires par réseaux de neurones*, Thèse de Doctorat, Université de Nancy.

Uzam, M.; Jones, A.H.; Yucel, I., (1999), A rule-based methodology for supervisory control of discrete event systems modeled as automation Petri nets, *Int. Jour. of Intelligent Control Systems*, vol. 3, no. 3, pp. 297-325.

Widrow B., Lehr M.A., (1990), 30 years of adaptative neural networks: Perceptron, Madaline, and backpropagation, *Proc. IEEE*, vol. 78, no. 9, pp. 1415-1442.

Zaytoon J., (editor), (1998), Hybrid dynamical systems, *APII - JESA*, vol 32, n° 9-10.