DESIGN OF PID CONTROLLERS FOR DECOUPLED MULTI-VARIABLE SYSTEMS

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Abstract: A PID design method is presented. The work done is part of an attempt to find an automatic tuning algorithm for two times two systems in the process industry. The method tries to minimize the impact of load disturbances. The method was developed mainly because previous design methods with the same objectives could not handle the processes obtained in decoupled systems.

Keywords: PID control, MIMO, Design, Decoupling, Nyquist diagrams

1. INTRODUCTION

The PID controller is the most common controller (Åström and Hägglund, 2001). There are several good PID design methods for SISO systems (Kristiansson and Lennartson, 2002), (Panagopoulos and Åström, 1999), (Panagopoulos *et al.*, 2002), (Hägglund and Åström, 2002), (Hägglund and Åström, 2004). The work presented in this paper is part of an effort to find a good tuning algorithm for two times two systems. If two PID controllers are to be used to control a two times two system, the system would in most cases have to be decoupled. The diagonal elements of the decoupled system would then consist of two parallel coupled processes with, possibly, different time delays and different signs. Most PID tuning algorithms are not appropriate for these systems. The algorithm proposed here can handle that kind of element as well as more easily tuned processes.

2. THE PROBLEM

Many PID design methods exist. These methods are normally based on the idea to first approximate the process dynamics with a simple model, and then base the design on this model. This approach works well on SISO processes in the process industry, since these processes often are well described by the simple models. An example of that is methods that use step responses for tuning (Ziegler and Nichols, 1942), (Hägglund and Åström, 2002), (Hägglund and Åström, 2004). These methods have in common that they require the process to have quite simple dynamics.

When PID controllers are to be used for multi-variable control of processes with strong cross couplings the situation is different. In many cases the system has to be decoupled. Even if the elements of the system have simple dynamics, decoupling may result in complicated diagonal elements that consist of parallel coupled processes that might have different signs and different time delays. An example of such a diagonal element could be:

$$G = \frac{1.7}{(s+1)^4}e^{-12s} - \frac{1}{(s+1)^4}e^{-5s} \qquad (1)$$

The step response of this process is shown in Figure 1.

If PID controllers are used to control a system with diagonal elements like this, methods that rely on simple process dynamics, like step response methods, are not appropriate. A PID design method that do not rely

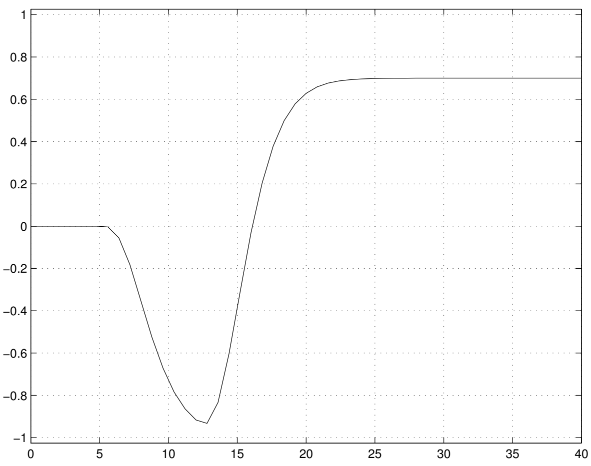

Fig. 1. Step response of the process (1).

on simple process dynamics will be presented in this paper.

3. THE DESIGN PROCEDURE

3.1 The Controller

The PID controller is described by

$$C = K\left(1 + \frac{1}{T_i s} + T_d s\right) \tag{2}$$

where K is the proportional gain, T_i is the integral time and T_d is the derivative time. A pure PID controller would have infinite high frequency gain. It is both undesirable and impossible to realize such a controller. Therefore a low pass filter would be required. A second order low pass filter is used here.

3.2 The design objective

The design objective is to minimize the integrated absolute error, IAE, at step load disturbances subject to a bound on the sensitivity function. Previously the integrated error, IE, together with the bound on the sensitivity function and other constraints has been used to approximate the IAE (see Introduction). Further it has previously been shown that the IE of a step load disturbance is directly proportional to the inverse of the integral gain of the controller -a fact that makes minimization of the IE easier than direct minimization of the IAE. However, if the step load disturbance response shifts signs it is not good enough to calculate the IE.

The bound on the sensitivity function can be interpreted as a circle centered in -1 in the complex plane that the Nyquist curve of the open loop system has to stay outside. The bound is called M_s and the circle is called the M_s circle. The radius of the M_s circle is $R = 1/M_s$.

3.3 The design method

An upper bound on the sensitivity function is specified. The space of possible controllers is discretized in the parameters T_i, T_d, and K. For each combination of T_i and T_d, a K, that puts the Nyquist curve of the open loop system on the edge of the M_s circle in a way such that the Nyquist curve does not encircle the point -1, is found, if possible. For each controller a step load disturbance is simulated and the integrated absolute error, IAE, is calculated. The controller that gives the smallest IAE is chosen.

3.4 The sign

Since the algorithm should be able to handle processes with different signs, a sign is added to the PID controller. The output of the process after a step change of the control signal is simulated. If the output goes to a positive value or towards plus infinity the sign is chosen positive. If the output goes to a negative value or towards minus infinity the sign is chosen negative. The controller is in either case connected to the process using negative feedback.

3.5 Td and Ti

The controller (2) has one pole in the origin, two filter poles and two zeros. The zeros are located in:

$$z = -\frac{1}{2T_d} \pm \sqrt{\frac{1}{4T_d^2} - \frac{1}{T_i T_d}} \tag{3}$$

If T_i is less than $4T_d$ the zeros are complex conjugated with a real part $a = -1/2T_d$. The imaginary part will increase with decreasing T_i. If T_i is greater than $4T_d$, the zeros will be real and centered around $a = -1/2T_d$.

$1/2T_d$ is swept over the frequency region of interest. This region could for example be 0.001Hz to 1000Hz with the grid points spread in a logarithmic fashion. In this way many processes can be covered.

For each value of T_d, T_i is swept over a reasonable region. In most cases it is not interesting to get a controller with zeros that have very large imaginary parts or a controller with zeros at frequencies far below or above the non-integrator poles and the zeros of the process, so this region is limited.

3.6 K

For every pair of T_d and T_i, a K that gives the system the prespecified maximum value of the sensitivity function without making the system unstable has to be found. For stable processes this corresponds to finding a K that puts the Nyquist curve on the edge of the M_s circle without making it encircle the point -1. An algorithm that checks if the point -1 is encircled has to be used.

A large K is chosen as a starting value. K is decreased until the point -1 is not encircled and the peak of the sensitivity function is less than the prespecified M_s value. If K is lowered under a certain bound, without making the system satisfy these specifications, the conclusion that no stable closed loop system exists for the present combination of T_i and T_d, is drawn. Subsequently K is gently increased until the Nyquist plot is close to the edge of the M_s circle. Off course, it would be easy to put a bound on K when the proposed design method is used.

3.7 IAE

The integrated absolute error is defined as:

$$IAE = \int_0^\infty |e(t)|dt \tag{4}$$

where $e(t)$ is the control error at the time t. It is calculated by integration of a simulation of a step load disturbance response. The controller with the smallest IAE is then chosen. To improve the accuracy the algorithm can be repeated with the intervals of T_d and T_i centered around the T_d and T_i values of the first controller and with a narrower grid.

4. EXAMPLE 1

Another algorithm that tries to minimize the load disturbance step response was presented in (Panagopoulos *et al.*, 2002), (Hägglund and Åström, 2004). That algorithm (called MIGO tuning) works well on a large class of processes but fails when it comes to more complicated processes like two parallel coupled processes with different time delays and different signs.

The proposed algorithm was compared in an example with the MIGO tuning algorithm. A simple process that both algorithms could handle was used:

$$G = \frac{1}{(s+1)^4} \tag{5}$$

The grid used in the proposed design method was the following: $1/2T_d$ was first divided into 10 grid points between 0.001 and 10000 Hz in a logarithmic fashion. The best controller for these values was calculated as described above. Subsequently $1/2T_d$ was again divided into 10 grid points with the best value of the first round in the middle. The calculations were done in Matlab (R13) and Simulink on a 2.66 GHz Pentium 4. It took $88s$ to find the controller.

The algorithm that is proposed in this paper is most easily explained in the Nyquist plot of the open loop system. Figure 2 shows a Nyquist plot of the process (5). The controller tries to find a way to bend the Nyquist plot to the edge of the M_s circle and at the same time to minimize the impact of load disturbances on the closed loop system.

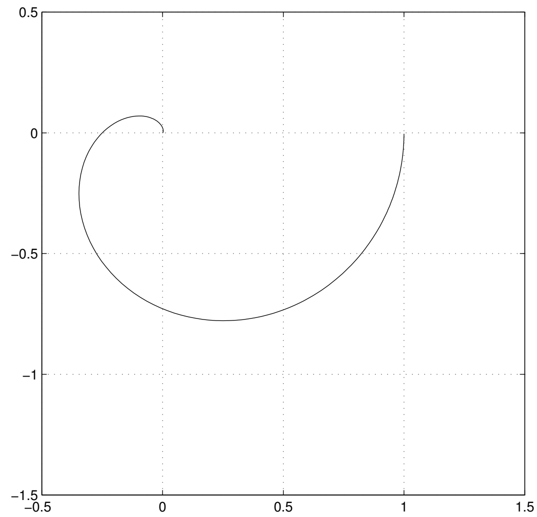

Fig. 2. Nyquist diagram of the process (5).

4.1 The controllers

The MIGO tuning algorithm tries to minimize the integrated area error by minimization of the integrated error IE subject to a constraint on the sensitivity function and some additional constraints. The MIGO tuning with an M_s value of 1.4 gave the parameters $K = 1.19$, $T_i = 2.22$ and $T_d = 1.20$. The controller was filtered by a low pass filter. The poles of the low pass filter were chosen sufficiently high not to compromise the controller properties claimed in (Hägglund and Åström, 2004).

The design method proposed in this paper with the same M_s value gave the parameters $K = 1.18$, $T_i = 2.27$ and $T_d = 1.28$. The low pass filter used had two poles in $p = -19.6$ Figure 3 shows the Nyquist curves of the loop transfer functions.

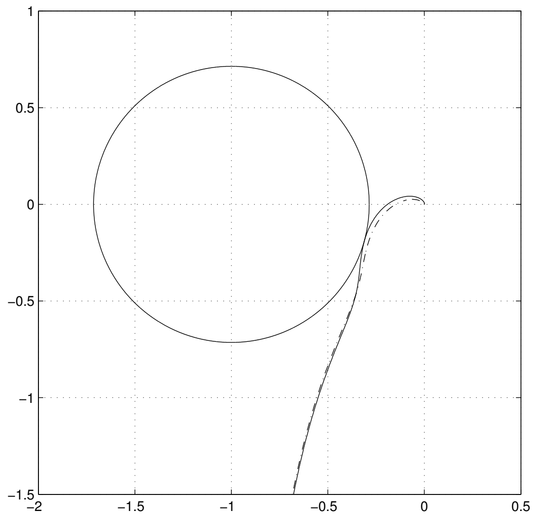

Fig. 3. Nyquist diagram of the loop transfer functions in Example 1. The controllers were designed by MIGO (dot-dashed line) and the design method proposed in this paper (full line).

The step load disturbance responses (y) and the control signals (u) are shown in Figure 4.

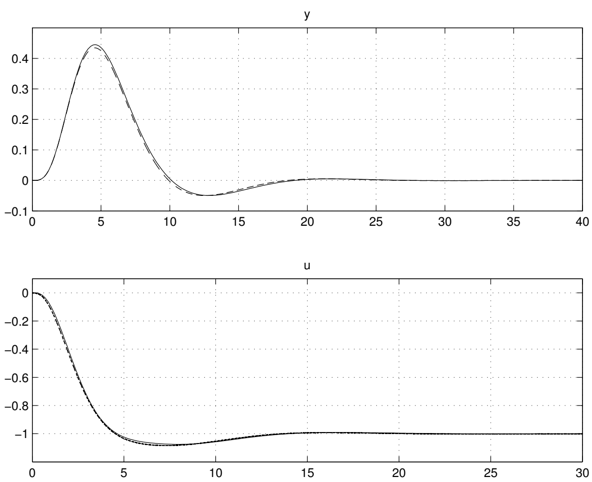

Fig. 4. Step load disturbance response of the closed loop systems in Example 1. The controllers were designed by MIGO (dot-dashed line) and the design method proposed in this paper (full line).

The integrated area error of the MIGO controlled system was calculated as:

$$IAE = 2.35 \tag{6}$$

The integrated area error of the system controlled by the controller that the proposed design method resulted in was calculated to:

$$IAE = 2.43 \tag{7}$$

4.2 Conclusions from the first example

There were some differences in the prerequisites for the two methods. In the MIGO design an M circle was used instead of an M_s circle. The M circle is slightly bigger than the M_s circle (Hägglund and Åström, 2004). Further the filter was not a part of the design in the MIGO case and the minimization was done on the IE instead of the IAE. However, the point was not to make an exact comparison of the methods, but rather to show that the proposed design method works as well as the MIGO method on an simple process.

5. EXAMPLE 2

The proposed algorithm was used to determine a controller for an example of two parallel coupled processes with different dead time and different signs, see (1). This is a kind of process that is expected to appear at diagonal elements of a decoupled two times two system in the process industry. A Nyquist plot of the process is shown in Figure 5.

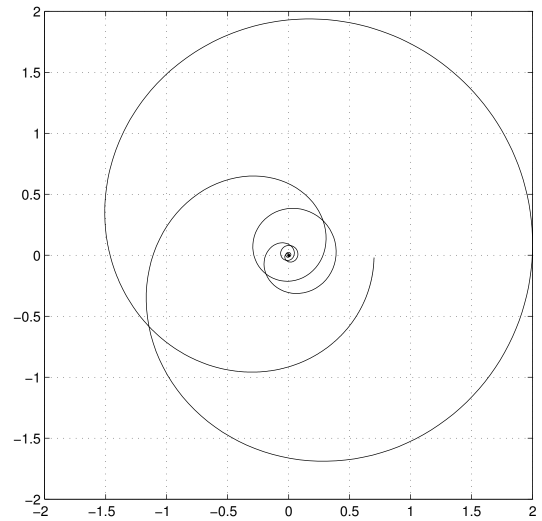

Fig. 5. Nyquist diagram of the process (1).

The same grid as in Example 1 was used. The calculations were done in Matlab (R13) and Simulink on a 2.66 GHz Pentium 4. It took $151s$ to find the controller. The proposed design method gave the controller parameters $K = 0.152$, $T_i = 6.50$, $T_d = 2.83$. The controller low pass filter had its poles in $p = -8.82$. Figure 6 shows a Nyquist plot of the loop transfer function. It is easy to see that the specification that the Nyquist plot should touch the edge of the M_s circle holds. Further, we know that the algorithm has compared a lot of different PID controllers that fulfill this specification and chosen the one that gives the smallest IAE of a step load disturbance response.

Figure 7 shows a step load disturbance response of the process (y) controlled by the controller and the control signal (u). The sign of the step response changes, indicating that a minimum of the IE would not be a good approximation of the IAE in this case.

The integrated area error of the controlled system was calculated as:

$$IAE = 54.6 \tag{8}$$

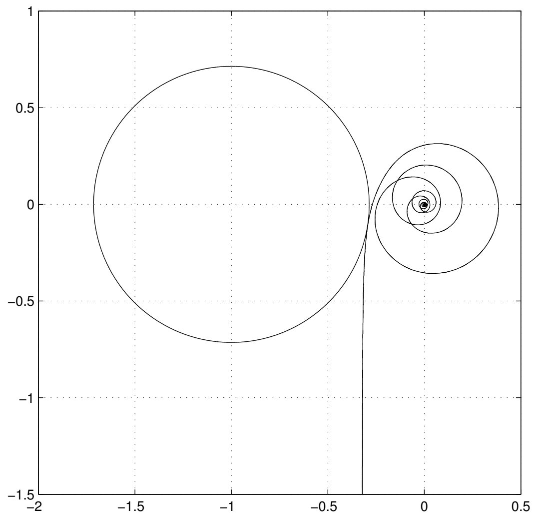

Fig. 6. Nyquist diagram of the loop transfer function in Example 2.

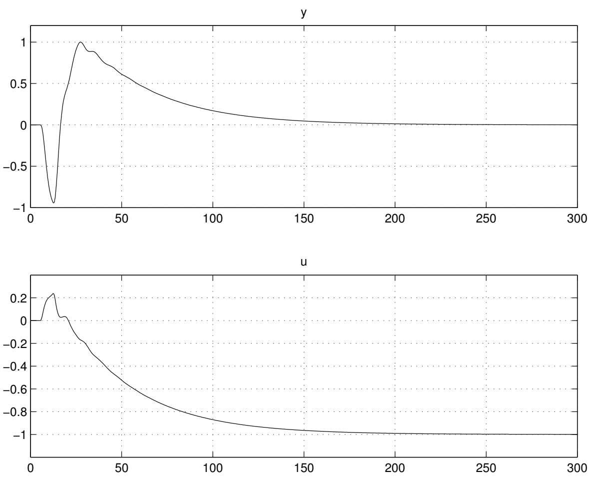

Fig. 7. Step load disturbance response of the closed loop system in Example 2.

This example shows that the proposed design method is able to handle processes that are quite complicated, in this case a process that is expected to appear among the cases that the method has to cope with.

6. EXAMPLE 3

In Example 5 of (Skogestad, 2001) several controllers for the process (9) were tuned with different methods. The best one was a SIMC-PID controller. This controller was given on cascade form (Skogestad, 2001) but was converted to the form in (2).

$$G = \frac{1}{(s+1)(0.2s+1)(0.04s+1)(0.008s+1)} \quad (9)$$

A controller determined with the design method proposed in this paper was compared with the SIMC-PID controller. The tuning was done under the same prerequisites as in Example 1 and Example 2. The tuning time was $91s$.

Figure 8 shows a Nyquist plot of the process (9).

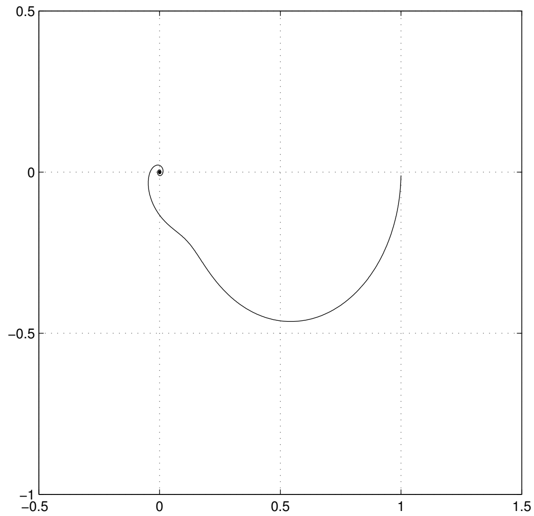

Fig. 8. Nyquist plot of the process (9).

6.1 *The controllers*

The SIMC-PID controller had the recommended parameters $K = 21.8$, $T_i = 1.22$ and $T_d = 0.180$. It was filtered by a low pass filter. The poles of the low pass filter were chosen sufficiently high not to compromise the controller properties claimed in (Skogestad, 2001). The design method proposed in this paper with $M_s = 1.58$ gave the parameters $K = 18.1$, $T_i = 0.632$ and $T_d = 0.117$. The low pass filter used had two poles in $p = -321$. Figure 9 shows the Nyquist curve of the loop transfer functions.

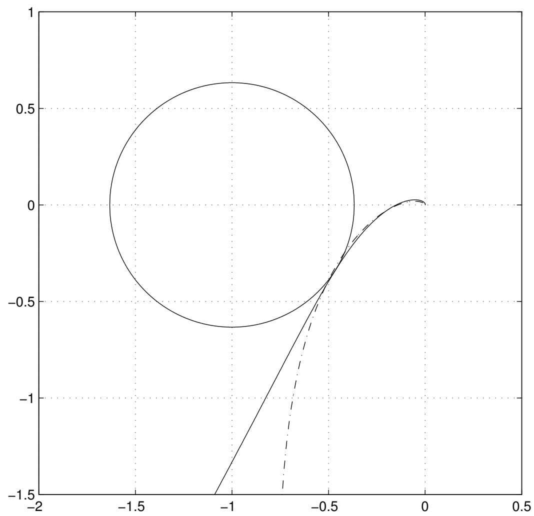

Fig. 9. Nyquist diagram of the loop transfer functions in Example 3. The controllers were SIMC-PID (dot-dashed line) and a controller produced by the design method proposed in this paper (full line).

The step load responses (y) and the control signals (u) are shown in Figure 10.

The integrated absolute error of the SIMC-PID controlled system was calculated as:

$$IAE = 0.0559 \quad (10)$$

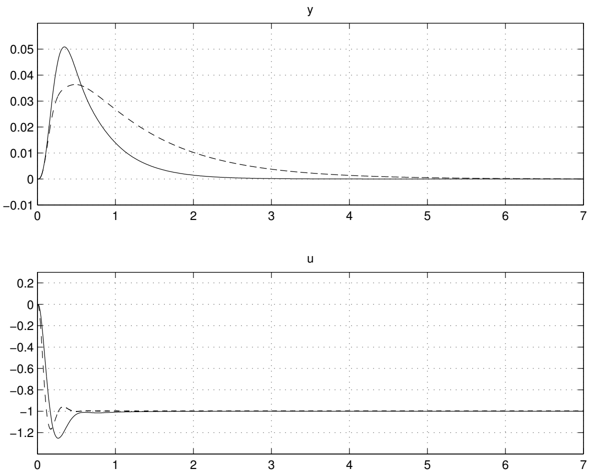

Fig. 10. Step load disturbance response of the closed loop systems in Example 3. The controllers were SIMC-PID (dot-dashed line) and a controller determined by the design method proposed in this paper (full line).

The integrated area error of the system controlled by the controller that the proposed design method resulted in was calculated to:

$$IAE = 0.0349 \quad (11)$$

6.2 Conclusions from the third example

The tuning method proposed in this paper resulted in a controller that has considerably better load disturbance attenuation properties than the controller proposed in (Skogestad, 2001), even though it had the same M_s value.

7. CONCLUSIONS

A design method for PID controllers has been proposed. The aim is to find controllers that minimize the impact of load disturbances under a bound on the sensitivity function. The method has in an example been shown to work as well as another method with the same tuning objectives. Further, the method has been shown to work in an example in the difficult case of two parallel coupled processes with different time delays and signs. It has also been compared in an example with a controller proposed in (Skogestad, 2001). The implementation of the design method was not time optimized. The time to find a controller was approximately 1 to 3 minutes in the examples. The time could probably be significantly shortened if effort was put into time optimization.

8. FURTHER WORK

A design method for PID controllers that is effective in a wide class of processes has been presented. The motivation for this work was to find an algorithm that could be suitable for decoupled process industry two times two systems, without degradation in performance on simpler systems. The next step in this work will be to find criteria that can be used to automatically decouple two times two systems.

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