IMPULSIVE CONTROL OF DISCRETE-TIME NETWORKED SYSTEMS WITH COMMUNICATION DELAYS

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Abstract: In this paper, an impulsive control scheme is proposed for discrete-time networked control systems (NCSs) with communication delays. This scheme converts the design problem of NCSs to a control design problem of a continuous-time linear time-invariant (LTI) system via output feedback. Necessary and/or sufficient conditions that guarantee global exponential stability of the closed-loop systems are presented. The results suggest a simple design procedure for state/output feedback controllers of the systems. A numerical example is presented to demonstrate the feasibility of the proposed method.

Keywords: Communication networks; Discrete-time systems; Delays analysis; Exponentially stable; Output feedback

1. INTRODUCTION

Recently, networked control systems (NCSs) have gained increasing attention because of the advantages to use real-time networks in control systems, e.g., lower cost and more convenience for installation and maintenance, flexibility and distributed nature in architectures. However, due to network bandwidth restriction, the insertion of communication network in the feedback control loop inevitably leads to communication delays and makes the analysis and design of NCSs complex. Communication delays can deteriorate the performance of NCSs and even can destabilize the systems when they are not considered in the design of NCSs. So far, a variety of efforts have been devoted to analyzing NCSs with communication delays (see, e.g., Nilsson, et al., 1998; Branicky, 2000; Matveev and Savkin, 2001; Zhang, et al., 2001; Kim, et al., 2003; Yu, et al., 2003; Hu and Zhu, 2003; Montestruque and Antsaklis, 2003, 2004 and the references therein). Specifically, Branicky, et al. (2000) and Zhang, et al. (2001) analyzed the stability of NCSs and obtained stability regions using a hybrid systems technique. Kim, et al. (2003) presented linear matrix inequality (LMI) conditions for obtaining maximum allowable delay bounds, which guarantee the stability of NCSs. Based on Lyapunov-Razumikhin function method, Yu, et al. (2003) presented conditions on the admissible bounds of data packet loss and delays for NCSs in terms of LMIs. Based on stochastic control theory, optimal controller design of NCSs with stochastic network delays was investigated in (Nilsson, et al., 1998; Matveev and Savkin, 2001; Hu and Zhu, 2003). For other control schemes, we refer readers to the survey (Tipsuwan and Chow, 2003).

To reduce the network traffic load, Montestruque and Antsaklis (2003, 2004) proposed a model-based control scheme for NCSs without/with delays. Necessary and sufficient conditions for the exponential stability of discrete-time and continuous-time NCSs without/with communica-

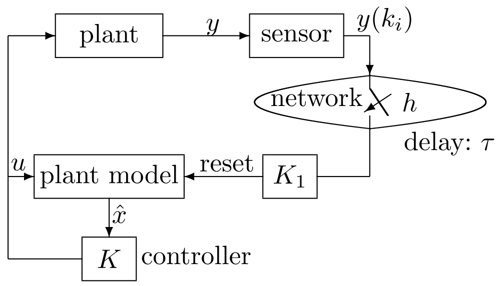

Fig. 1. Proposed configuration of networked control systems.

tion delays were established in both cases of state feedback and output feedback. However, they did not present any method for controller design when communication delays are considered. Moreover, it is in general not an easy task to design the controller based on their conditions. Recently, Mu, et al. (2004a, b, 2005) proposed an improved model based control scheme for NCS without/with delays and presented conditions for exponential stability together with controller design procedures. Particularly, an impulsive model based control scheme for discrete-time NCSs without communication delays was discussed in (Mu, et al., 2004a).

In this paper, we extend the method in (Mu, et al., 2004a) to discrete-time NCSs with communication delays. We consider the case where delay only occurs in the process of samplings passing through the network and is constant and not larger than a sampling period of the sensor. In this case, an impulsive control scheme transfers the controller design problem of NCSs into a design problem for a continuous-time LTI system via output feedback control. The advantages of the scheme for NCSs with network delays are as follow: It introduces additional freedom and hence flexibility in designing the NCSs with network delays. This scheme allows us to design a controller based on an impulsive model for an NCS with a larger sampling period. The NCS configuration considered is shown in Fig. 1.

The paper is organized as follows. Section 2 gives the problem formulation and some preliminaries. Section 3 presents conditions for the global exponential stability of the closed-loop system and a design procedure for the controller. Numerical simulations are presented in Section 4 and some conclusions are drawn in Section 5.

2. PROBLEM FORMULATION AND PRELIMINARIES

The NCS under consideration is described as follows.
Plant: $x(k+1) = Ax(k) + Bu(k), y(k) = Cx(k)$,
Model:

$$\hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}u(k),\ k = k_0, ..., k_0 + \tau - 2,$$
$$\hat{x}(k+1) = \hat{A}\hat{x}(k) + \hat{B}u(k),\ k = k_i + \tau, ..., k_{i+1} + \tau - 2,$$
$$\hat{x}(k_i + \tau) = K_1 y(k_i), \qquad i = 0, 1, 2, ...,$$

Control law: $u(k) = K\hat{x}(k)$,
where k_0 is a nonnegative integer and k_i are positive integers with $k_{i+1} - k_i = h\ (\geq 2)$ constant, h is the sampling period of the sensor, $\tau \in \{2, ..., h\}$ is the constant communication delay, $x(k) \in \mathbb{R}^n,\ u(k) \in \mathbb{R}^m,\ y(k) \in \mathbb{R}^p$, and $\hat{x}(k) \in \mathbb{R}^q$ are the plant state, the plant input, the plant output, and the model state, respectively, K_1 is a $q \times p$ gain matrix, K is an $m \times q$ feedback gain matrix, $A, B, C, \hat{A}$, and $\hat{B}$ are known real constant matrices with appropriate dimensions. Assume that A is invertible and not Schur stable, that $(\hat{A}, \hat{B})$ is controllable, that C is of full row rank, and that $p \geq q$ and $pq \geq n + q$. The last inequalities imply $q \geq 2$. The network is switched off at the initial time k_0. The initial states $x(k_0)$ and $\hat{x}(k_0)$ are arbitrarily selected.

Remark 1. Since τ is constant, the period h also is the period of model state updated.

Define $z(k) = \left[x(k)^T\ \hat{x}(k)^T\right]^T$. The dynamics of overall system for $k = k_i + \tau - 1, ..., k_{i+1} + \tau - 2$ can be described as

$$\begin{aligned} z(k+1) &= \Lambda z(k), \quad k = k_i + \tau, ..., k_{i+1} + \tau - 2, \\ z(k_i + \tau) &= \left[(x(k_i + \tau))^T \ \ (K_1 C x(k_i))^T\right]^T, \\ z(k_0) &= \left[x(k_0)^T \ \ \hat{x}(k_0)^T\right]^T, \end{aligned} \tag{1}$$

where
$$\Lambda = \begin{bmatrix} A & BK \\ 0 & \hat{A} + \hat{B}K \end{bmatrix}.$$

Clearly, the overall system (1) is an impulsive control system with delays.

Our goal is to establish conditions for the trivial solution of system (1) to be globally exponentially stable and based on the conditions, to present design scheme for the controller gain matrix K and the gain matrix K_1.

In the following we establish three lemmas which will be used in the sequel. For convenience, denote

$$S_1 = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0 & 0 \\ K_1 C & 0 \end{bmatrix},$$
$$M = S_1 \Lambda^h + S_2 \Lambda^{h-\tau}, \quad M_0 = S_1 \Lambda^\tau + S_2,$$

where I_n is the $n \times n$ identity matrix. From (1),

$$\begin{aligned} z(k_i + \tau) &= S_1 \Lambda z(k_i + \tau - 1) + S_2 z(k_i) \\ &= \left[S_1 \Lambda^h + S_2 \Lambda^{(h-\tau)}\right] z(k_{i-1} + \tau) \\ &= \cdots \\ &= M^i M_0 z(k_0). \end{aligned}$$

Particularly, for $\tau = h,\ M = M_0$ and
$$z(k_i + \tau) = z(k_{i+1}) = M_0^{i+1} z(k_0).$$

This leads to the following result.

Lemma 1. The response of the system (1) is

$$z(k) = \begin{cases} \Lambda^{(k-k_0)} z(k_0), \quad k = k_0, ..., k_0+\tau-1, \\ \Lambda^{(k-k_i-\tau)} \\ \times M^i M_0 z(k_0), \quad k = k_i+\tau, ..., k_{i+1}+\tau-1, \end{cases}$$

for $0 < \tau < h$ and

$$z(k) = \Lambda^{(k-k_i)} M_0^i z(k_0), \quad k = k_i, ..., k_{i+1}-1,$$

for $\tau = h$, where $i = 0, 1, ...$

From Lemma 1, we immediately obtain the following result.

Lemma 2. The trivial solution of the system (1) is globally exponentially stable if and only if M (M_0 for the case of $\tau = h$) is Schur stable.

We will convert the control problem to a control problem of a continuous-time LTI system via output feedback. For related results we refer to (Alexandridis and Paraskevopoulos, 1996) where it was shown that under certain conditions, the desired pole set of the closed-loop system can be assigned by assigning eigenstructure. In the sequel, we will present some useful results of (Alexandridis and Paraskevopoulos, 1996) for us as below.

Remark 2. Given a controllable and observable LTI system

$$\dot{x} = \bar{A}x + \bar{B}u, \; y = \bar{C}x, \tag{2}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^r$, $\bar{A}, \bar{B}$, and $\bar{C}$ are constant matrices of appropriate dimensions with $\text{rank}(\bar{B}) = m$, $\text{rank}(\bar{C}) = r$, and $mr \geq n$. Under the output feedback law $u = \bar{K}y$, the closed-loop system is

$$\dot{x} = \left(\bar{A} + \bar{B}\bar{K}\bar{C}\right)x. \tag{3}$$

Let $\bar{C}_1$ is any $(n-r) \times n$ constant matrix such that $\left[\bar{C}^T \; \bar{C}_1^T\right]^T$ is nonsingular and $\left[\bar{C}^T \; \bar{C}_1^T\right]^T = [T_1 \; T_2]^{-1}$ with T_2 being an $n \times (n-r)$ matrix. Denote $A_{22} = \bar{C}_1 \bar{A} T_2$. Let $\bar{\Lambda} = \{\Lambda_1, \Lambda_2\}$ be an arbitrarily selected set subject to the following constraints.

a) $\bar{\Lambda}$ contains distinct values,
b) Λ_1 and Λ_2 are self-conjugated sets,
c) Λ_1 contains no eigenvalues of $\bar{A}$ and Λ_2 contains no eigenvalues of A_{22},

where

$$\Lambda_1 = \{\lambda_1, \lambda_2, ..., \lambda_r\}, \Lambda_2 = \{\lambda_{r+1}, \lambda_{r+2}, ..., \lambda_n\}.$$

Necessary and sufficient conditions, which contain two coupled Sylvester matrix equations, for assigning a desired eigenvalue set $\bar{\Lambda}$ to (3) were established in (Alexandridis and Paraskevopoulos, 1996), which reduce the design of output feedback gain matrix $\bar{K}$ to solving the following bilinear algebraic equations

$$a_i^T M_{ij} a_j = 0, \tag{4}$$

where $i = 1, 2, ..., r$ and $j = r+1, r+2, ..., n$,

$$a_i = \left[a_{i1}, a_{i2}, ..., a_{i(m-1)}, 1\right]^T,$$
$$a_j = \left[a_{j1}, a_{j2}, ..., a_{j(r-1)}, 1\right]^T,$$

are mth-order and rth-order parametric vectors, respectively, and

$$M_{ij}^T = \bar{C}\left[I_n + \bar{A}T_2\left(\lambda_j I_{n-r} - A_{22}\right)^{-1}\bar{C}_1\right] \times \left(\lambda_i I_n - \bar{A}\right)^{-1}\bar{B}$$

is an $r \times m$ constant matrix. Particularly, for the case of $m + r \geq n+1$, by preselecting the vectors a_j arbitrarily, (4) is reduced to a set of linear algebraic equations with the vectors a_i's as unknown variables. Denote

$$\Psi_r = [a_1, a_2, ..., a_r], \; U_r = [V_1 a_1, V_2 a_2, ..., V_r a_r], \tag{5}$$

where a_i, $i = 1, 2, ..., r$, verify (4) and

$$V_i = \left(\lambda_i I_n - \bar{A}\right)^{-1}\bar{B} \quad i = 1, 2, ..., r.$$

Lemma 3 (Alexandridis and Paraskevopoulos, 1996). For system (2) and $\bar{\Lambda}$ subject to constraints a)–c) described above, if the output feedback matrix $\bar{K}$ is given by $\bar{K} = \Psi_r(\bar{C}U_r)^{-1}$ with Ψ_r and U_r determined by (5), then the pole set $\bar{\Lambda}$ is assigned to (3).

3. MAIN RESULTS

In this section, we will establish conditions for the trivial solution of the system (1) to be globally exponentially stable and, based on the results, present a design procedure for K and K_1 with h and τ satisfying certain conditions. In the following, we will establish the main results for $0 < \tau < h$. For the case $\tau = h$, similar results can be obtained.

We start from analyzing the structure of M specifically and then, present conditions for M to be Schur stable. For the sake of convenience, denote

$$\Lambda^\delta = \begin{bmatrix} A_1(\delta) & A_2(\delta) \\ 0 & A_3(\delta) \end{bmatrix}, \quad \delta \in \mathbb{Z},$$

where $A_1(\delta) = A^\delta$, $A_3(\delta) = (\hat{A}+\hat{B}K)^\delta$, and $A_2(\delta)$ is certain matrix which depends on the system parameters, K, h, and τ. Particularly, for $\delta = 0$, $\Lambda^\delta = I$ with I the identity matrix. The matrix M is rewritten as

$$\begin{aligned} M &= \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A_1(h) & A_2(h) \\ 0 & A_3(h) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ K_1 C & 0 \end{bmatrix} \\ &\quad \times \begin{bmatrix} A_1(h-\tau) & A_2(h-\tau) \\ 0 & A_3(h-\tau) \end{bmatrix} \\ &= \begin{bmatrix} A_1(h) & A_2(h) \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I_q \end{bmatrix} K_1 \begin{bmatrix} A_1^T(h-\tau)C^T \\ A_2^T(h-\tau)C^T \end{bmatrix}^T \\ &\triangleq A_1 + B_1 K_1 C_1, \end{aligned} \tag{6}$$

where I_q is the identity matrix. Based on Remark 2 and Lemma 3, we have the following result.

Theorem 1. If $K, h,$ and τ are such that the system with a triple (A_1, B_1, C_1) as coefficient matrices is controllable and observable, then K_1 can be designed such that M is Schur stable.

Proof. From (6), we can view M as the closed-loop system matrix of the system (A_1, B_1, C_1) and K_1 as its output feedback matrix. Hence, M is Schur stable if and only if the closed-loop system of (A_1, B_1, C_1) via output feedback is Schur stable. Next we note that K_1 can be designed based on Lemma 3 by assigning eigenstructure such that $A_1 + B_1K_1C_1$ is Schur stable. To this end, we examine the coefficient matrices of (A_1, B_1, C_1) satisfying corresponding conditions of (2).

By assumptions of Theorem 1, (A_1, B_1, C_1) is controllable and observable for certain $K, h,$ and τ. Note that B_1 is of full column rank and that C_1 are of full row rank because C is of full row rank and $A_1(h - \tau)$ is invertible. Also note that $pq \geq n + q$. For the triple (A_1, B_1, C_1) with such $K, h,$ and τ, select the eigenvalues of $A_1 + B_1K_1C_1$ subject to corresponding constraints a)–c) in Remark 2 and to be strictly lying in unit disk. According to the method in Remark 2 and Lemma 3, by solving a set of bilinear algebraic equations, gain matrix K_1 can be found such that $A_1 + B_1K_1C_1$ is Schur stable and so is M. □

Theorem 1 shows that if we have found $K, h,$ and τ such that (A_1, B_1, C_1) is controllable and observable, then we can find further K_1 such that M is Schur stable. Therefore, we need to find $K, h,$ and τ such that (A_1, B_1, C_1) is controllable and observable firstly. However, it is not easy to find such $K, h,$ and τ directly because of the complex expressions of A_1 and C_1. In the sequel, we give a constraint on choosing K.

Theorem 2. For given $K, h, \tau,$ and K_1, if M is Schur stable, then $BK \neq 0$.

Proof. It can be proved by contradiction. Suppose $BK = 0$. Then $A_2(\delta) = 0$ and

$$M = \begin{bmatrix} A_1(h) & 0 \\ CA_1(h-\tau) & 0 \end{bmatrix}.$$

Thus, M is Schur stable if and only if $A_1(h)$ is Schur stable. Since $A_1(h) = A^h$, then M is Schur stable if and only if A is Schur stable. This is in contradiction with A given in the system. □

Theorem 2 imposes a constraint on choosing K. Without loss of generality, we can choose a K with full row rank. Next we give a necessary condition for observability of (A_1, C_1).

Theorem 3. If (A_1, C_1) is observable, then $p \geq q$.

Proof. Since (A_1, C_1) is observable if and only if $\text{rank}(Q_o) = n + q$, where

$$Q_o = \begin{bmatrix} C_1 \\ C_1A_1 \\ \vdots \\ C_1A_1^{n+q-1} \end{bmatrix}.$$

From the definitions of A_1 and C_1 in (6), we can obtain

$$Q_o = \begin{bmatrix} CA_1(h-\tau) & CA_2(h-\tau) \\ P & Q \end{bmatrix},$$

where

$$P = \begin{bmatrix} CA_1(h-\tau)A_1(h) \\ \vdots \\ CA_1(h-\tau)A_1^{n+q-1}(h) \end{bmatrix},$$

$$Q = \begin{bmatrix} CA_1(h-\tau)A_2(h) \\ \vdots \\ CA_1(h-\tau)A_1^{n+q-2}(h)A_2(h) \end{bmatrix}.$$

Note that $A_1(h) = A^h, A_1(h-\tau) = A^{h-\tau}$ and that A is invertible, we get $Q = PA_1(-h)A_2(h)$. After some simple algebraic manipulations, we further obtain

$$Q_o \begin{bmatrix} I & -A_1(-h)A_2(h)] \\ 0 & I \end{bmatrix} = \begin{bmatrix} CA_1(h-\tau) & G \\ P & 0 \end{bmatrix} \triangleq \bar{Q}_o,$$

where $G = C[A_2(h-\tau) - A_1(-\tau)A_2(h)]$. So we have $\text{rank}(Q_o) = \text{rank}(\bar{Q}_o)$. Thus, (A_1, C_1) is observable if and only if $\text{rank}(\bar{Q}_o) = n + q$. In the expression of $\bar{Q}_o$, the orders of $CA_1(h - \tau)$ and G are $p \times n$ and $p \times q$, respectively. Therefore, the following inequalities

$$\text{rank}[\bar{Q}_o(I_n \;\; 0_{n\times q})^T] \leq n, \text{rank}[\bar{Q}_o(0_{q\times n} \;\; I_q)^T] \leq p.$$

hold. So one can get $\text{rank}(\bar{Q}_o) \leq n+p$ and further, $q \leq p$. □

Theorem 3 shows that $q \leq p$ is necessary for the observability of (A_1, C_1). On the other hand, $q \leq p \leq n$ (because C is a $p \times n$ matrix with full row rank) shows that the order of the model used for generating control signal is not larger than that of the plant. This suggests that the controller may be based on a reduced order model, which is of great interest in practical applications.

For a chosen gain matrix K, to determine h and τ such that such that (A_1, B_1, C_1) is controllable and observable, a necessary and sufficient condition for the controllability and observability will be established in following theorem.

Theorem 4. If $q \leq p$ and K satisfies $BK \neq 0$ and $\hat{A} + \hat{B}K$ is invertible, then (A_1, B_1, C_1) is controllable and observable if and only if the following three equalities hold:

$$\text{rank}(Q_1) = n, \quad \text{rank}(Q_2) = n, \tag{7}$$

$$\text{rank}(Q_3) = q, \tag{8}$$

where

$$Q_1 = [A_2(h) \;\; A_1(h)A_2(h) \;\; \cdots \;\; A_1^{n+q-2}(h)A_2(h)],$$

$$Q_2 = \begin{bmatrix} C \\ CA_1(h) \\ \cdots \\ CA_1^{n+q-2}(h) \end{bmatrix}, \quad (9)$$

$$Q_3 = CA_2(-\tau).$$

Proof. First, (A_1, B_1) is controllable if and only if $\text{rank}(Q_c) = n + q$, where

$$Q_c = [B_1 \;\; A_1B_1 \;\; \cdots \;\; A_1^{n+q-1}B_1].$$

Inserting the definitions of A_1 and B_1 in (6) into Q_c, we get

$$Q_c = \begin{bmatrix} 0 & Q_1 \\ I_q & 0 \end{bmatrix},$$

where Q_1 is defined as (9). Clearly, $\text{rank}(Q_c) = n+q$ is equivalent to $\text{rank}(Q_1) = n$.

Next, we will prove that (A_1, C_1) is observable if and only if $\text{rank}(Q_2) = n$ and $\text{rank}(Q_3) = q$. From Theorem 3, (A_1, C_1) is observable if and only if $\text{rank}(\bar{Q}_o) = n+q$. Since $\bar{Q}_o$ is a matrix with $n+q$ columns, then $\text{rank}(\bar{Q}_o) = n + q$ if and only if all column vectors of $\bar{Q}_o$ are linear independent, i.e., $\text{rank}(G) = q$ and $\text{rank}(P) = n$. Since $A_1(h-\tau)A_1(h) = A_1(h)A_1(h-\tau)$ and $A_1(h-\tau)A_1(h)$ is invertible, one has $\text{rank}(P) = \text{rank}(Q_2) = n$ with Q_2 defined in (9). Note that A and $\hat{A} + \hat{B}K$ are invertible. Hence Λ is invertible. Thus, we have $\Lambda^{h-\tau} = \Lambda^{-\tau}\Lambda^h$, i.e.,

$$\begin{bmatrix} A_1(h-\tau) & A_2(h-\tau) \\ 0 & A_3(h-\tau) \end{bmatrix} = \begin{bmatrix} A_1(-\tau) & A_2(-\tau) \\ 0 & A_3(-\tau) \end{bmatrix} \begin{bmatrix} A_1(h) & A_2(h) \\ 0 & A_3(h) \end{bmatrix},$$

from which one can get $A_2(h-\tau) - A_1(-\tau)A_2(h) = A_2(-\tau)A_3(h)$. Also note that $A_3(h) = (\hat{A} + \hat{B}K)^h$ is invertible. So we have $\text{rank}(G) = \text{rank}[CA_2(-\tau)] = \text{rank}(Q_3)$. Therefore, (A_1, C_1) is observable if and only if $\text{rank}(Q_2) = n$ and $\text{rank}(Q_3) = q$. □

Theorem 4 shows that if we can find K, h, and τ satisfying conditions (7) and (8), then (A_1, B_1, C_1) is controllable and observable. Further by Theorem 1, gain matrix K_1 can be designed such that M is Schur stable. From the expressions of Q_1, Q_2, and Q_3, $\text{rank}(Q_1)$ and $\text{rank}(Q_2)$ depend on K and h while $\text{rank}(Q_3)$ relies on K and τ. Hence, once K is obtained, the determination of h is independent of τ and the determination of τ is also independent of h except for $\tau \leq h$. By Theorem 2, we first choose a K with full row rank and such that $\hat{A} + \hat{B}K$ is invertible. Then we can plot the evolutions of $\text{rank}(Q_1)$ and $\text{rank}(Q_2)$ with h and $\text{rank}(Q_3)$ with τ using MATLAB and determine from the plots a positive integer h satisfying (7) and a positive integer τ satisfying (8) and $\tau \leq h$. If no suitable h and τ can be found in $\mathbb{Z}^+$, we need to choose another K and run again. This suggests the following procedure.

Step 1. Choose K with full row rank and such that $\hat{A} + \hat{B}K$ is invertible.

Step 2. Plot the graphs of $\text{rank}(Q_1)$ and $\text{rank}(Q_2)$ vs h and $\text{rank}(Q_3)$ vs τ. Then find positive integers $h, \tau\ (\leq h)$ such that h verifies (7) and τ satisfies (8). If thus h and τ are available, then go to Step 3. Or else return to Step 1.

Step 3. Select the eigenvalues of $A_1 + B_1K_1C_1$ subject to corresponding constraints a)–c) in Remark 2 and strictly lying in unit disk, determine Ψ_r and U_r by solving corresponding algebraic equations (4). Thus, $K_1 = \Psi_r(C_1U_r)^{-1}$.

In Section 4, we will give a numerical example to demonstrate the procedure.

Remark 3. Similar results have been established in continuous-time settings in (Mu, et al., 2005).

4. NUMERICAL EXAMPLE

To illustrate above analysis results, we consider an example. The parameters are as follows.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\hat{A} = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

We choose

$$K = \begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}.$$

The evolutions of $\text{rank}(Q_1)$ and $\text{rank}(Q_2)$ vs h and of $\text{rank}(Q_3)$ vs τ are plotted in Fig. 2, from which we can find that (7) and (8) hold simultaneously for any $h \in \{1, ..., 10\}$ and any $\tau \in \{1, ..., h\}$. So we can choose a larger h if it is desired. Here we select $h = 10, \tau = 9$. Let the poles of $A_1 + B_1K_1C_1$ be $0.1, 0.2, 0.3, -0.1$, and -0.2. By the method in Remark 2 and Lemma 3, we get

$$U_r = \begin{bmatrix} -0.2114 & -0.0001 & -0.0001 \\ 2.9901 & 0.0024 & 0.0015 \\ 0.1601 & 0.0002 & 0.0001 \\ 0 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 \end{bmatrix} \times 10^6,$$

$$\Psi_r = \begin{bmatrix} 0 & 3.7246 & 3.7256 \\ 1.0000 & 1.0000 & 1.0000 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 2.6208 & 0.3397 & 4.9466 \\ 0.7032 & 0.0912 & 1.3273 \end{bmatrix},$$

where $r = 3$. A simulation of the system with $h = 10$, $\tau = 9$, K, and K_1 determined above, and the initial state of the plant $z(0) = [1\ 4\ -2\ -3\ 5]^T$ is shown in Fig. 3.

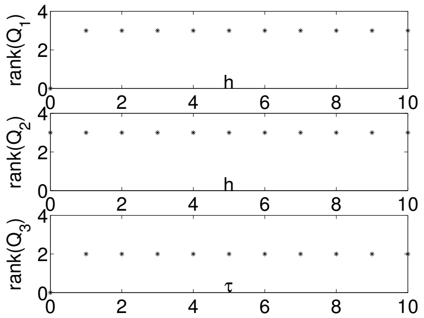

Fig. 2. The evolutions of rank(Q_1), rank(Q_2) vs h and rank(Q_3) vs τ

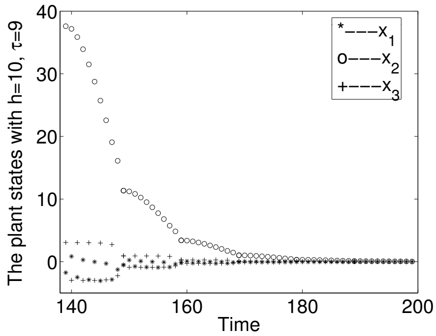

Fig. 3. The evolution of the plant state with time

5. CONCLUSIONS

We have extended the impulsive control scheme proposed in (Mu, et al., 2004a, b, 2005) to discrete-time networked systems with network delays and established necessary and/or sufficient conditions for the global exponential stability of the close-loop systems. Based on the results, a simple design procedure for state/output feedback controllers has been presented for discrete-time NCSs. Numerical results have shown the feasibility and efficiency of the proposed method. Compared with recent work on model-based network systems in (Montestruque and Antsaklis, 2003), the present approach introduces additional freedom and hence flexibility in designing an NCS. Moreover, our method allows of the use of a lower order model to control the plant. This is of practical interest in applications.

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