AN OBSERVER-BASED FAULT-ACCOMMODATING CONTROLLER FOR NONLINEAR SYSTEMS IN THE PRESENCE OF SENSOR FAILURES

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Abstract: This paper addresses the issue of designing a fault accommodating control algorithm which, after a sensor fault occurrence, takes information about the degraded sensor into account and drives the system such that performance recovery is achieved. A general class of single input nonlinear systems is considered, containing also a disturbance term. An observer is used to reconstruct the degraded state variable, and is activated as soon as the Fault Detection and Identification unit signals the location and occurrence of a sensor fault. Asymptotical convergence of the observer within an arbitrary finite time is proved, under some assumptions about the plant structure

Keywords: Fault Tolerant Systems, Robust Estimation, Sliding Mode Control.

1. INTRODUCTION

Ensuring high reliability of industrial processes is currently becoming a key concern of industrial automation, due to the increasing awareness about the risks associated with system malfunction in terms of technical parts of the plants, personnel and environment. Correspondingly, more and more attention is currently being given to the problem of designing Fault Tolerant Control (FTC) algorithms, able to recover, after fault occurrence, performances close to the nominal desired performances. As discussed in (Blanke *et al.*, 2001), fault tolerance can be defined as "the ability of a controlled system to maintain control objectives despite the occurrence of a fault" and "can be obtained through Fault Accommodation" (FA), i.e. "through a change in controller parameters or structure to avoid the consequences of a fault" (Blanke *et al.*, 2001). According to (Noura *et al.*, 2000), fault tolerant controllers can be divided into active or passive approaches, and belong to different categories, the more widely known of which is perhaps that of adaptive control (Bodson and Groszkiewicz, 1997) . The so called integrated approaches (Noura *et al.*, 2000) constitute another trend, where the fault monitoring algorithm determines also the control law (Nett *et al.*, 1988). Alternatively, the fault tolerant control problem has been formulated as an optimization problem, assuming that the considered plant is linear, and heuristic techniques have been used as well (Babuska, 2001) (Zidani *et al.*, 2003) (Napolitano *et al.*, 1999) (Hoblos *et al.*, 2000). Finally, the last category relies on supervised control schemes, where a Fault Detection and Isolation (FDI) unit provides information about the location and time occurrence of any fault.

In this approach, faults are compensated via an appropriate control law triggered according to the diagnosis of the system. Within the framework of the so called active approaches, and with reference to this latter fault tolerant control method, the present paper addresses the issue of designing a fault accommodating control algorithm which, after the occurrence of a sensor fault, takes information about the degraded sensor into account and drives the system such that performance recovery is achieved. To the authors' knowledge, Variable Structure Control (VSC) techniques have been rarely used in this framework. Indeed, the well known robustness features of sliding mode control (Utkin, 1992) appear particularly well suited for handling nonlinear and uncertain single input plants subject to unknown but bounded sensor failures. Sensor fault tolerant controllers have been studied mostly for linear and/or parametrizable systems (Yang *et al.*, 1999) (Campos-Delgado and Zhou, 2003) (Bennett *et al.*, 1999), while few results are available about nonlinear systems (Qu *et al.*, 2003) (Qu *et al.*, 2001). In this paper, a general class of single input nonlinear systems is considered, containing also a disturbance term. An observer is used to reconstruct the degraded state variable, and is activated as soon as the FDI unit signals the location and occurrence of a sensor fault. Asymptotical convergence of the observer within an arbitrary finite time is proved, under some mild assumptions about the plant structure. Note that the condition of input-to-state stability (needed f.i. by (Qu *et al.*, 2003)) is not required, as shown also by the simulation example.

2. PROBLEM FORMULATION

An uncertain nonlinear system of the following form is given:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{B}(u + d(\mathbf{x}, t)) \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector and $u \in \mathbb{R}$ is the plant input. The nonlinear function $\mathbf{f}(\mathbf{x}) : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and the state-input map $\mathbf{B} \in \mathbb{R}^n$ represent known parts of system dynamics, while $d(\mathbf{x}, t) : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}$ describes possible matched disturbances and/or uncertainties.

As in many realistic situations, the system state vector is supposed available through sensors subject to potential failure. Denoting the measurement of $\mathbf{x}$ by $\bar{\mathbf{x}}$, one has, without loss of generality:

$$\mathbf{x} = \bar{\mathbf{x}} + \Delta h(\mathbf{x}) \tag{2}$$

where $\Delta h(\mathbf{x})$ is an unknown function representing both the occurrence and the magnitude of possible sensor faults. As in (Qu *et al.*, 2003), in fact, the normal operation mode corresponds to $\Delta h(\mathbf{x}) = 0$, while in faulty conditions, the magnitude of the fault itself can range from a small offset to an unknown scaling factor

$$\Delta h(\mathbf{x}) = \delta \mathbf{x} \tag{3}$$

$||\Delta h(\mathbf{x})||$ being possibly greater than $||\mathbf{x}||$ in severe cases. Only one fault is assumed to occur at the same time.

The following assumptions are made:

Assumption 2.1. The uncertain function $d(\mathbf{x}, t)$ is bounded by a known function:

$$|d(\mathbf{x}, t)| \leq \rho(\mathbf{x}) \tag{4}$$

and a bound on the sensor failure model (3) is available as well:

$$|\delta| \leq \delta_{max} \tag{5}$$

Correspondingly, there exist a bound $\rho_b(\mathbf{x})$ such that

$$|[\mathbf{B}d(\mathbf{x}, t)]_i| \leq \rho_{bi}(\mathbf{x}), \quad i = 1, \ldots, n \tag{6}$$

where the subscript i denotes the i-th component of the vector.

Assumption 2.2. There exists at least a component $f_j(\mathbf{x})$ of $\mathbf{f}(\mathbf{x})$ such that the following function

$$F_j(\mathbf{x}, \tilde{x}_k) \stackrel{\text{def}}{=} f_j(x_1, \ldots, x_k, x_{k+1}, \ldots x_n) + \\ - f_j(x_1, \ldots, \tilde{x}_k, x_{k+1}, \ldots, x_n), \tag{7}$$

satisfies:

$$(x_k - \tilde{x}_k) F_j(\mathbf{x}, \tilde{x}_k) \leq 0 \quad \forall\, \tilde{x}_k : \ (x_k - \tilde{x}_k) \neq 0 \tag{8}$$

denoting by the subscript k the component subject to fault.

Assumption 2.3. There exists a linear sliding surface

$$s(\mathbf{x}) = \mathbf{C}\mathbf{x} = \sum_{i=1}^{n} c_i x_i, \quad c_i > 0, \quad i = 1, \ldots, n \tag{9}$$

such that the achievement of a sliding motion on it ensures the asymptotic stabilization of the plant (1).Note that a suitable selection of (9) ensures also the achievement of desired performances in the fault-free system, according to standard results (see (Utkin, 1992)).

Remark 2.1. While Assumptions 2.1, 2.3 correspond to standard hypotheses fulfilled by most physical systems, Assumption 2.2 is apparently restrictive. It should be noticed, however, that for linear systems it simply corresponds to the presence of at least a negative value in the k-th column (corresponding to the sensor subject to fault) of the dynamical matrix. It follows that any stable linear plant satisfy Assumption 2.2, and unstable linear plants stabilizable by static output feedback (in the fault-free condition) fulfill Assumption 2.2 as well.

The problem addressed in this paper can be formalized as follows:

Problem 1. The problem here considered consists in finding a fault-tolerant controller guaranteeing the robust asymptotical stabilization of the plant (1), under the Assumptions 2.1-2.3, in the presence of failures of the form (2) of the sensors measuring the state variables.

Note that only the problem of designing a fault-tolerant robust controller is being addressed here, leaving to further studies how to detect and identify the fault itself.

3. A PRELIMINARY RESULT

Once the sliding surface (9) has been determined, a stabilizing controller can be found straightforwardly in the fault-free case. Indeed, it is enough to compute an input $u = u(\mathbf{x})$ such that

$$s(\mathbf{x})\dot{s}(\mathbf{x}) < 0 \quad \forall \mathbf{x} \tag{10}$$

which ensures the fulfillment of the sliding condition in the worst case. A possible controller of this type is (see, for instance, (Utkin, 1992)):

$$u(\mathbf{x}) = -[\mathbf{CB}]^{-1}[\mathbf{Cf}(\mathbf{x})] - \rho(\mathbf{x})sgn(s(\mathbf{x})) \tag{11}$$

Whenever a sensor fault has occurred, the above quantities are no more available, since $\bar{\mathbf{x}} \neq \mathbf{x}$. Introduce the following state observer:

$$\dot{\hat{\mathbf{x}}} = \mathbf{f}(\hat{\mathbf{x}}) + \mathbf{B}\mathbf{u} + \nu \tag{12}$$

where ν will be designed later on. The observation error is $\Delta\mathbf{x} = \mathbf{x} - \hat{\mathbf{x}}$, whose dynamics are:

$$\dot{\hat{\mathbf{x}}} = \dot{\mathbf{x}} - \dot{\mathbf{\Delta x}} = \mathbf{f}(\mathbf{x}) + \mathbf{B}(\mathbf{u} + \mathbf{d}(\mathbf{x},\mathbf{t})) - \dot{\mathbf{\Delta x}} \tag{13}$$

It follows that, in faulty conditions, what is available to the designer is the observed sliding surface:

$$s(\hat{\mathbf{x}}) = \mathbf{C}\hat{\mathbf{x}} = \sum_{i=1}^{n} c_i \hat{x}_i \tag{14}$$

instead of (9), and the observed controller

$$u(\hat{\mathbf{x}}) = -[\mathbf{CB}]^{-1}[\mathbf{Cf}(\hat{\mathbf{x}})] - \rho(\hat{\mathbf{x}})sign(s(\hat{\mathbf{x}})) \tag{15}$$

instead of (11). Since the condition (10) cannot be checked experimentally, consider the corresponding observed function instead:

$$W(\hat{\mathbf{x}}) = s(\hat{\mathbf{x}})\dot{s}(\hat{\mathbf{x}}) \tag{16}$$

whose sign can be detected simply checking wether the observed sliding surface (14) is increasing or decreasing.

Hypothesizing that a fault on the k-th state component has occurred and has been detected, the following Lemma can be proved. It provides a set of allowed values for the observer input ν_k ensuring the knowledge of the sign of the unknown observer error $x_k - \hat{x}_k$. In this vein, it is likely to introduce the following Assumption.

Assumption 3.1. Consider the case when a sensor failure on the k-th component of the state vector of the plant (1) has occurred at a time $\bar{t}$. It is assumed that a known $M(\bar{t})$ exists such that

$$|x_k(\bar{t}) - \hat{x}_k(\bar{t})| < M(\bar{t}) \tag{17}$$

Remark 3.1. Assumption 3.1 simply requires that, when a fault occurs, the observer output immediately after the fault time $\bar{t}$ differs from the measured variable immediately before the same time $\bar{t}$ for an arbitrarily large but bounded quantity. Basically, what is hypothesized is that both the faulty variable and the observer output, at the very time the failure occurs, remain finite, which is likely to occur in real plants.

Under Assumption 3.1, the following Lemma can now be proved. Define:

$$\bar{F}_k(\hat{x}_k, \bar{t}) \stackrel{\text{def}}{=} \sup_{\xi \in \mathbb{R}^n : |\xi - \hat{x}_k| < M(\bar{t})} |f_k(\xi, \bar{t}) - f_k(\hat{x}_k, \bar{t})| \tag{18}$$

and

$$\Delta_{j,k}(\hat{x}_k, \bar{t}) = \sum_{\substack{i=1 \\ i \neq j}}^{n} c_i \bar{F}_i(\hat{x}_k, \bar{t}) + c_k \bar{F}_k(\hat{x}_k, \bar{t}) \tag{19}$$

Lemma 2. It is given the plant (1) under the Assumptions 2.1, 2.2, 3.1. Assume that a sensor fault on the k-th component of the state vector has occurred. The following input for the observer (12) $\nu_k =$

$$\begin{array}{c}
\begin{cases}
\nu_k < -\dfrac{1}{c_k}\left(\mathbf{CB}\rho(\hat{\mathbf{x}}) + \sum\limits_{\substack{i=1 \\ i \neq k}}^{n} c_i \rho_{bi}(\hat{\mathbf{x}}) + \Delta_{j,k}(\hat{x}_k, \bar{t})\right) \\
\quad if \quad s(\hat{\mathbf{x}}) > 0 \\
\nu_k > \dfrac{1}{c_k}\left(\mathbf{CB}\rho(\hat{\mathbf{x}}) + \sum\limits_{\substack{i=1 \\ i \neq k}}^{n} c_i \rho_{bi}(\hat{\mathbf{x}}) + \Delta_{j,k}(\hat{x}_k, \bar{t})\right) \\
\quad if \quad s(\hat{\mathbf{x}}) \leq 0
\end{cases} \\
if \quad W(\hat{\mathbf{x}}) > 0 \\
\begin{cases}
\nu_k > \dfrac{1}{c_k}\left(\mathbf{CB}\rho(\hat{\mathbf{x}}) + \sum\limits_{\substack{i=1 \\ i \neq k}}^{n} c_i \rho_{bi}(\hat{\mathbf{x}}) + \Delta_{j,k}(\hat{x}_k, \bar{t})\right) \\
\quad if \quad s(\hat{\mathbf{x}}) > 0 \\
\nu_k < -\dfrac{1}{c_k}\left(\mathbf{CB}\rho(\hat{\mathbf{x}}) + \sum\limits_{\substack{i=1 \\ i \neq k}}^{n} c_i \rho_{bi}(\hat{\mathbf{x}}) + \Delta_{j,k}(\hat{x}_k, \bar{t})\right) \\
\quad if \quad s(\hat{\mathbf{x}}) \leq 0
\end{cases} \\
if \quad W(\hat{\mathbf{x}}) \leq 0
\end{array} \tag{20}$$

and

$$\nu_i = 0, \quad i = 1, \dots, n, \quad i \neq k \tag{21}$$

ensures that

$$sign(x_k - \hat{x}_k) = -sign(s(\hat{\mathbf{x}})) \cdot sign(W(\hat{\mathbf{x}})) \tag{22}$$

Proof. Assume first that $W(\hat{\mathbf{x}}) > 0$. One has:

$$s(\hat{\mathbf{x}}) \left[\mathbf{Cf}(\mathbf{x}) - \mathbf{Cf}(\hat{\mathbf{x}}) - \mathbf{CB}\rho(\hat{\mathbf{x}}) sign(s(\hat{\mathbf{x}})) + \right. \\ \left. + \mathbf{CB}d(\mathbf{x}, t) - \mathbf{C}\dot{\Delta\mathbf{x}}\right] > 0 \tag{23}$$

Consider the case $s(\hat{\mathbf{x}}) > 0$. Since $\Delta\mathbf{x} = [0\ 0\ \dots\ x_k - \hat{x}_k\ 0 \dots 0]^T$, it follows that: $\mathbf{C}\dot{\Delta\mathbf{x}} = c_k(\dot{x}_k - \dot{\hat{x}}_k) = c_k \left[f_k(\mathbf{x}) - f_k(\hat{\mathbf{x}}) + [\mathbf{B}d(\mathbf{x}, t)]_k - \nu_k\right]$ and inequality (23) gives the following expression:

$$\sum_{\substack{i=1 \\ i \neq k}}^{n} c_i \left[f_i(\mathbf{x}) - f_i(\hat{\mathbf{x}})\right] - \mathbf{CB}\rho(\hat{\mathbf{x}}) + \mathbf{CB}d(\mathbf{x}, t) \\ -c_k[\mathbf{B}d(\mathbf{x}, t)]_k + c_k\nu_k > 0$$

Adding and subtracting the quantity $c_k(f_k(\mathbf{x}) - f_k(\hat{\mathbf{x}}))$ one has $\sum_{i=1}^{n} c_i \left[f_i(\mathbf{x}) - f_i(\hat{\mathbf{x}})\right] - c_k(f_k(\mathbf{x}) - f_k(\hat{\mathbf{x}})) - \mathbf{CB}\rho(\hat{\mathbf{x}}) + \sum_{\substack{i=1 \\ i \neq k}}^{n} c_i[\mathbf{B}d(\mathbf{x}, t)]_i + c_k\nu_k > 0$ or, equivalently $\sum_{\substack{i=1 \\ i \neq j}}^{n} c_i \left[f_i(\mathbf{x}) - f_i(\hat{\mathbf{x}})\right] - c_k(f_k(\mathbf{x}) - f_k(\hat{\mathbf{x}})) + c_j(f_j(\mathbf{x}) - f_j(\hat{\mathbf{x}})) - \mathbf{CB}\rho(\hat{\mathbf{x}}) - \sum_{\substack{i=1 \\ i \neq k}}^{n} c_i[\mathbf{B}d(\mathbf{x}, t)]_i$ $+c_k\nu_k > 0$, where the component $f_j(\mathbf{x})$ of $\mathbf{f}(\mathbf{x})$ satisfies Assumption 2.2. Taking the worst case at time $t = \bar{t}$ one gets:

$$-\sum_{\substack{i=1 \\ i \neq j}}^{n} c_i \bar{F}_i(\hat{x}_k, \bar{t}) - c_k \bar{F}_k(\hat{x}_k, \bar{t}) + c_j(f_j(\mathbf{x}) - f_j(\hat{\mathbf{x}})) \\ - \mathbf{CB}\rho(\hat{\mathbf{x}}) - \sum_{\substack{i=1 \\ i \neq k}}^{n} c_i \rho_{bi}(\hat{\mathbf{x}}) + c_k\nu_k > 0 \tag{24}$$

It is easy to verify from (24) that, if ν_k is chosen such that

$$-\Delta_{j,k}(\hat{x}_k, \bar{t}) - \mathbf{CB}\rho(\hat{\mathbf{x}}) - \sum_{\substack{i=1 \\ i \neq k}}^{n} c_i \rho_{bi}(\hat{\mathbf{x}}) - c_k\nu_k > 0 \tag{25}$$

then $(f_j(\mathbf{x}) - f_j(\hat{\mathbf{x}}))$ is positive since it is greater than a positive quantity ($c_j > 0$ by assumption). Since the expression (20) for ν_k verifies (25) in the worst case, it follows that $f_j(\mathbf{x}) - f_j(\hat{\mathbf{x}}) = F_j(x_1, \dots, x_k, \hat{x}_k, x_{k+1}, \dots, x_n) > 0$, since $x_i = \hat{x}_i$ for $i = 1, n$, $i \neq k$. As a consequence $(x_k - \hat{x}_k) < 0$ in view of (8).

Assume now that $s(\hat{\mathbf{x}}) < 0$. The inequality analogous to (23) gives: $\sum_{\substack{i=1 \\ i \neq k}}^{n} c_i \left[f_i(\mathbf{x}) - f_i(\hat{\mathbf{x}})\right] + \mathbf{CB}\rho(\hat{\mathbf{x}}) + \mathbf{CB}d(\mathbf{x}, t) - c_k[\mathbf{B}d(\mathbf{x}, t)]_k + c_k\nu_k < 0$. By following the same logic as in the case $s(\hat{\mathbf{x}}) > 0$, and taking the worst case one has:

$$\sum_{\substack{i=1 \\ i \neq j}}^{n} c_i \bar{F}_i(\hat{x}_k, \bar{t}) + c_k \bar{F}_k(\hat{x}_k, \bar{t}) + c_j(f_j(\mathbf{x}) - f_j(\hat{\mathbf{x}})) \\ + \mathbf{CB}\rho(\hat{\mathbf{x}}) + \sum_{\substack{i=1 \\ i \neq k}}^{n} c_i \rho_{bi}(\hat{\mathbf{x}}) + c_k\nu_k < 0 \tag{26}$$

so, if ν_k is chosen such that

$$\Delta_{j,k}(\hat{x}_k, \bar{t}) + \mathbf{CB}\rho(\hat{\mathbf{x}}) + \sum_{\substack{i=1 \\ i \neq k}}^{n} c_i \rho_{bi}(\hat{\mathbf{x}}) - c_k\nu_k < 0 \tag{27}$$

then $f_j(\mathbf{x}) - f_j(\hat{\mathbf{x}})$ is negative since it is less than a negative quantity. Since the expression (20) for ν_k verifies (27) in the worst case, il follows that

$$f_j(\mathbf{x}) - f_j(\hat{\mathbf{x}}) = F_j(x_1, \dots, x_k, \hat{x}_k, \dots, x_n) < 0$$

since $x_i = \hat{x}_i$ for $i = 1, n$, $i \neq k$. As a consequence $(x_k - \hat{x}_k) > 0$ in view of (8). Iterating the same argument for $W(\hat{\mathbf{x}}) < 0$ in both cases $s(\hat{\mathbf{x}}) < 0$ and $s(\hat{\mathbf{x}}) > 0$, the expression (22) is obtained. △

4. FAULT TOLERANT ROBUST CONTROL

It is now possible to show that a subset of (20) exists guaranteeing also the convergence of the faulty state observer within a finite time, as stated in the following Theorem.

Theorem 1. It is given the plant (1) under the Assumptions 2.1, 2.2, 3.1. Assume that a sensor failure on the k-th component of the state vector has occurred at the time $\bar{t}$. Define:

$$G_k(\hat{\mathbf{x}}, \bar{t}) \stackrel{\text{def}}{=} \left(\bar{F}_k(\hat{x}_k, \bar{t}) + \rho_{bk}(\hat{\mathbf{x}}) + \eta\right) \tag{28}$$

with $\eta > 0$. The following input for the observer (12): $\nu_k =$

$$\begin{cases} \nu_k < \min\left\{-\frac{1}{c_k}\left(\mathbf{CB}\rho(\hat{\mathbf{x}}) + \sum_{\substack{i=1 \\ i \neq k}}^{n} c_i \rho_{bi}(\hat{\mathbf{x}}) + \right.\right. \\ \left.\left. + \Delta_{j,k}(\hat{x}_k, \bar{t})\right), -G_k(\hat{\mathbf{x}}, \bar{t})\right\} \\ \qquad if \quad s > 0 \\ \nu_k > \max\left\{\frac{1}{c_k}\left(\mathbf{CB}\rho(\hat{\mathbf{x}}) + \sum_{\substack{i=1 \\ i \neq k}}^{n} c_i \rho_{bi}(\hat{\mathbf{x}}) + \right.\right. \\ \left.\left. \Delta_{j,k}(\hat{x}_k, \bar{t})\right), G_k(\hat{\mathbf{x}}, \bar{t})\right\} \\ \qquad if \quad s \leq 0 \end{cases} \\ \qquad\qquad if \quad W(\hat{\mathbf{x}}) > 0 \tag{29}$$

$$
\begin{cases}
\nu_k > \max\left\{\frac{1}{c_k}\left(\mathbf{CB}\rho(\hat{\mathbf{x}}) + \sum_{\substack{i=1\\ i\neq k}}^{n} c_i\rho_{bi}(\hat{\mathbf{x}}) + \right.\right. \\
\left.\left. + \Delta_{j,k}(\hat{x}_k,\bar{t})\right), G_k(\hat{\mathbf{x}},\bar{t})\right\} \\
\qquad if \quad s > 0 \\
\nu_k < \min\left\{-\frac{1}{c_k}\left(\mathbf{CB}\rho(\hat{\mathbf{x}}) + \sum_{\substack{i=1\\ i\neq k}}^{n} c_i\rho_{bi}(\hat{\mathbf{x}}) + \right.\right. \\
\left.\left. \Delta_{j,k}(\hat{x}_k,\bar{t})\right), -G_k(\hat{\mathbf{x}},\bar{t})\right\} \\
\qquad if \quad s \leq 0
\end{cases}
\qquad if \quad W(\hat{\mathbf{x}}) \leq 0 \tag{30}
$$

and

$$\nu_i = 0, \quad i = 1,\ldots,n, \quad i \neq k \tag{31}$$

ensures the vanishing of the observation error $(x_k - \hat{x}_k)$ within an arbitrary finite time $\Delta t < \frac{M(\bar{t})}{\eta}$.

Proof. With reference to the following sliding surface:

$$\sigma = x_k - \hat{x}_k \tag{32}$$

the achievement of a sliding motion on (32) in a finite time, corresponding to the condition

$$\sigma\dot{\sigma} < -\eta|\sigma| \tag{33}$$

guarantees the vanishing of the observation error within an arbitrary finite time (depending on η). The inequality (33) yields:

$$\sigma\left(f_k(\mathbf{x}) - f_k(\hat{\mathbf{x}}) + [\mathbf{B}d(\mathbf{x},t)]_k - \nu_k + \eta sign(\sigma)\right) < 0 \tag{34}$$

Assume first that $W(\hat{\mathbf{x}}) > 0$ and $s(\hat{\mathbf{x}}) > 0$. From Lemma 2 it follows that $\sigma < 0$, hence inequality (34) provides, taking the worst case:

$$-\bar{F}_k(\hat{x}_k,\bar{t}) - \rho_{bk}(\hat{\mathbf{x}}) - \nu_k - \eta > 0 \tag{35}$$

i.e.

$$\nu_k < -\left(\bar{F}_k(\hat{x}_k,\bar{t}) + \rho_{bk}(\hat{\mathbf{x}}) + \eta\right) \tag{36}$$

For Lemma 2 to hold, the previous inequality has to be coupled with the first condition of (20), and the first case of (29) immediately follows. Iterating the same argument for the case $s(\hat{\mathbf{x}}) < 0$, and for $W(\hat{\mathbf{x}}) < 0$ in both cases $s(\hat{\mathbf{x}}) < 0$ and $s(\hat{\mathbf{x}}) > 0$, the expression (29) is obtained. $\triangle$

Lemma 2 and Theorem 4 can be easily combined to design a control law ensuring robustness against sensor failures. The following Corollary, in fact, directly follows from previous results.

Corollary 3. It is given the plant (1) under the Assumptions 2.1-3.1. Assume that a sensor failure on the k-th component of the state vector can occur. The following fault-tolerant control law:

(1) u is set equal to (15);
(2) as soon as the fault occurs, the observer (12) is invoked. Its convergence is achieved within the arbitrary finite time Δt (depending on the choice of η);

ensures the achievement of a sliding motion on (9). Hence system global stabilization and performance recovery are achieved regardless of the fault affecting state measurement.

Remark 4.1. Note that Corollary 3 implicitly assumes that no extra failures are allowed to occur within the convergence time Δt. Anyway, this requirement is not restrictive since Δt can be chosen arbitrarily small.

Remark 4.2. Both Lemma 2 and Theorem 4 have been derived using Sliding mode control, and could produce chattering in the neighborhood of $s(\mathbf{x}) = 0$. This drawback can be easily overcome using the well known Boundary Layer Method (Slotine and Sastry, 1983).

5. SIMULATION RESULTS

Theoretical results described in Section 4 have been validated by simulation using the following plant taken from (Khalil, 2002):

$$\dot{x} = -\frac{x}{1+x^2} + u + d \tag{37}$$

with $d = x\sin(x)$ and initial condition $\mathbf{x}(0) = 0.5$. Though the system considered for simulations may appear scarcely realistic from a practical viewpoint, it has been chosen in view of the fact that the nominal plant is *not* input-to-state stable (Khalil, 2002), hence the fault-tolerant controller recently proposed by (Qu *et al.*, 2003) cannot be applied.

The failure model (3) has been used, with $\delta_{max} = 3$. A severe sensor failure has been considered to occur on x at time $\bar{t} = 1.45\ s$, when the sensor jumps from its current value to the maximum value of its range. A fault detection module is supposed to activate the observer (12) at that time $\bar{t}$. Using the fault tolerant controller (15), plant asymptotic stabilization has been achieved within $\Delta t \simeq 0.8\ s$ from the occurrence of the failure. The (straightforward) surface $s = x$ has been chosen, and a boundary layer of width $\epsilon = 0.001$ has been used to avoid chattering. The following controller settings has been adopted: $c_1 = 1$, $\eta = 1$, $\rho(\hat{\mathbf{x}}) = |\hat{\mathbf{x}}|$.

Simulations show that the proposed control maintains robust stability during the fault, and is capable to restore the normal operation a short time after the failure. Results are shown in Fig.1 (Measured variable $\bar{x}_2$ and true variable x_2), Fig.2 (Observation error), Fig.3 (Control input).

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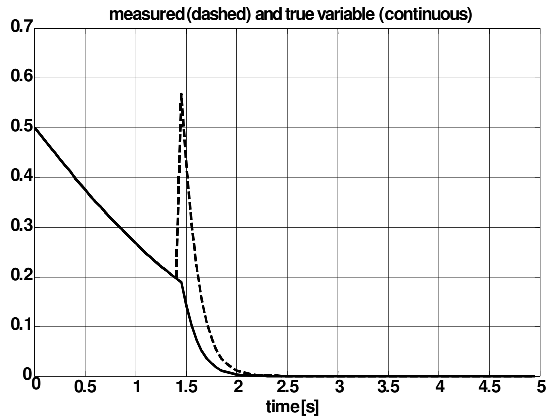

Fig.1 - Measured (dashed line) and true (continuous line) state variable x.

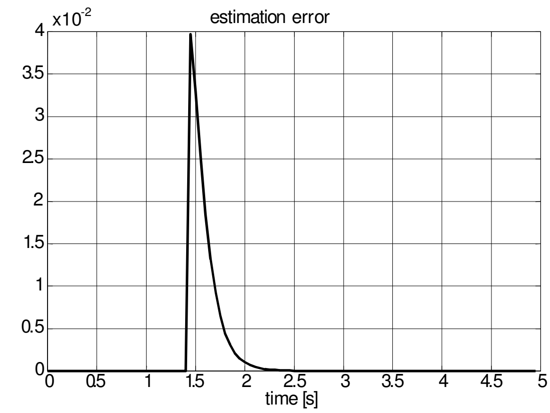

Fig.2 - Observation error $x - \hat{\mathbf{x}}$.

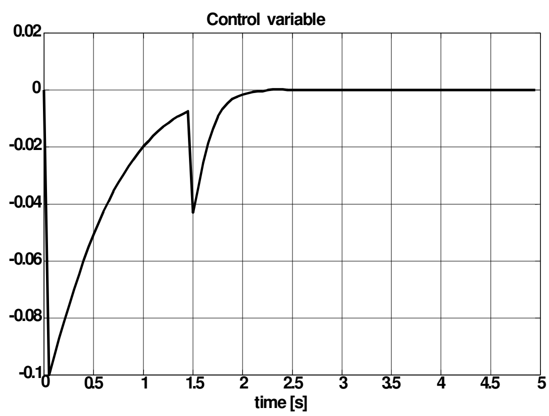

Fig.3 - Control input u.