ROBUST ADAPTIVE TRACKING CONTROL FOR A CLASS OF PERTURBED UNCERTAIN NONLINEAR SYSTEMS

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Abstract: A novel robust adaptive controller is presented for a wide class of strict feedback uncertain nonlinear system with unknown virtual control coefficients under bounded exogenous disturbances. Combined Nussbaum gain with the backstepping technique, the proposed design algorithm, which does not require a priori knowledge of the signs of the unknown virtual control coefficients, is proved to be able to guarantee the resulting closed-loop system globally uniformly ultimately bounded (GUUB). Moreover, the output of the system is proven to converge to a small neighborhood of the origin. Simulation results are presented to validate the effectiveness of the proposed controller.

Keywords: Uncertain nonlinear systems, robust adaptive control, backstepping design, virtual control gain function, Nussbaum type gain.

1. INTRODUCTION

During the past few years, a lot of researches and significant progress have been made in the adaptive control of nonlinear systems via feedback linearization (Sastry and Isidori, 1989), (Kanellakopoulos *et al.*, 1991), which has evolved as a powerful methodology. For a class of nonlinear systems transformable to a parametric strict-feedback canonical ones, a recursive design procedure, adaptive backstepping approach, has been presented (Isidori, 1995), (Krstic *et al.*, 1995), (Marino and Toper, 1993). The overparametrization problem was soon eliminated via introducing the concept of tuning function (Krstic *et al.*, 1992). Recently, nonlinear damping was also introduced in the controller to improve transient performance (Kanellakopoulos, 1993). However, all these adaptive controllers deal with the case of parametric uncertainties only, seldom with the uncertainties on modeling and external disturbances, nor with the case including unknown virtual control gain functions.

When there is no a priori knowledge about the signs of virtual control coefficients, adaptive control of such systems becomes much more difficult. The first solution was given in (Nussbaum, 1983) for a class of first-order linear systems, where the Nussbaum type gain was originally proposed. Without the requirement for monotone increasing arguments for the Nussbaum functions, the same technique has been extended to higher order systems for constant virtual control coefficients in (Ye and Jiang, 1998), (Ge and Wang, 2002), using decoupled backstepping. Recently, with respect to both unknown virtual control gain function and unknown time delay systems, two adaptive neural controllers are presented for a class of strict-

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feedback nonlinear systems in (Ge *et al.*, 2003) using the same technique and integral Lyapunov functions.

Motivated by the works (Ye and Jiang, 1998; Ge and Wang, 2002), and based on the Nussbaum-type gain and the decoupled backstepping techniques, a novel robust adaptive tracking controller is presented for a wide class of perturbed uncertain nonlinear systems with unknown virtual control gain functions, which are complex than the systems in (Ye and Jiang, 1998), (Ge and Wang, 2002), and without the help of integral Lyapunov functions, the design procedure is simple than that in (Ge *et al.*, 2003), such that the closed-loop system is globally uniformly ultimately bounded (GUUB), and additionally, the tracking error is proven to be able to converge to a small neighborhood of the origin as small as desired by an appropriate choice of the design parameters in the controller. In a word, the main contribution is that, this paper enlarges the class of perturbed strict-feedback uncertain nonlinear systems for which global robust adaptive tracking control can be designed.

2. PROBLEM FORMULATION AND PRELIMINARIES

In this paper, we treat the control problem of nonlinear systems transformable to the following strict-feedback form

$$\begin{cases} \dot{x}_i = g_i\left(\bar{x}_i\right)x_{i+1} + \theta_i^T\psi_i\left(\bar{x}_i\right) + \Delta_i\left(t,x\right), \\ \qquad\qquad 1 \le i \le n-1 \\ \dot{x}_n = g_n\left(x\right)u + \theta_n^T\psi_n\left(x\right) + \Delta_n\left(t,x\right) \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = \left[x_1,...,x_i\right]^T$, $x = (x_1,...,x_n) \in R^n$, $u \in R$, $y \in R$ are the state variables, system input and output respectively, $\theta_i \in R^p$ denotes the unknown constant parameter vector, $\psi_i\left(x_1,...,x_i\right)$ is known smooth function, g_i is unknown virtual control gain function, and its sign is also unknown, $\Delta_i\left(t,x\right)$ is the bounded uncertainties, which can include modeling uncertainties, external disturbances, etc. The control goal is to design a robust adaptive tracking controller such that the output $y(t)$ can follow a desired reference signal $y_d(t)$, while all the signals and states involved are GUUB.

Assumption 1 **:** There exist constants g_{i0} and known smooth functions $\bar{g}_i\left(\bar{x}_i\right)$ such that $0 < g_{i0} \le \left|g_i\left(\bar{x}_i\right)\right| \le \bar{g}_i\left(\bar{x}_i\right)$, $\forall \bar{x}_i \in R^i$. In addition, $\bar{g}_i\left(\bar{x}_i\right)$ take value in the unknown closed intervals $I_i := \left[l_i^-, l_i^+\right] \subset [g_{i0}, +\infty)$.

Assumption 2 **:** There exists an completely unknown constant b_i and known smooth function $\psi_{1,i}$, such that

$$g_i(\bar{x}_i) = b_i\psi_{1,i}(\bar{x}_i) \tag{2}$$

Remark 1**:** In the works of (Ye and Jiang, 1998),(Ge and Wang, 2002), g_i 's are constants, and (Ye and Jiang, 1998) did not consider the perturbed uncertainties. So those systems are one special case of the systems in this paper.

Assumption 3**:** There exists an unknown positive constant λ_i, for all $(t,x) \in R^+ \times R^n$,

$$\left|\Delta_i\left(t,x\right)\right| \le \lambda_i\phi_i\left(\bar{x}_i\right) \tag{3}$$

where ϕ_i is a known nonnegative smooth function.

Assumption 4**:** The reference signal $y_d(t)$ has up to its nth time derivative, and $Y_d = \left[y_d, \dot{y}_d, ..., y_d^{(n)}\right]^T$ is bounded.

Now we introduce a useful Lemma on Nussbaum-type function gain (Nussbaum, 1983) which will be used throughout this paper (For clarity, an even Nussbaum function $N\left(\kappa\right) = \exp(\kappa^2)\cos\left(\frac{\pi\kappa}{2}\right)$ is used throughout this paper.).

Lemma 1**:**(Ge *et al.*, 2003) Let $V(\cdot)$ and $\kappa(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(t) \ge 0$,$\forall t \in [0, t_f)$, $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds:

$$\begin{aligned} V\left(t\right) \le C_0 + e^{-c_1 t}\int_0^t g\left(x\left(\tau\right)\right)N\left(\kappa\right)\dot{\kappa}e^{c_1 t}d\tau \\ + e^{-c_1 t}\int_0^t \dot{\kappa}e^{c_1 t}d\tau, \forall t \in [0, t_f) \end{aligned} \tag{4}$$

where $c_1 > 0$, $g\left(x\left(\tau\right)\right)$ is a time-varying function which takes values in the unknown closed interval $I := [l^-, l^+]$ with $0 \notin I$, and C_0 represents some suitable constant, then $V(t)$, $\kappa(t)$ and $\int_0^t \left(g\left(x\left(\tau\right)\right)N(\kappa(\tau))+1\right)\dot{\kappa}(\tau)d\tau$ must be bounded in $[0, t_f)$.

Remark 2**:** According to (Ryan, 1991), if the solutions to the closed-loop system exist, then $t_f = \infty$. So the boundedness result in Lemma 1 is able to be extended to globally uniformly ultimately bounded(GUUB).

3. DESIGN PROCEDURE AND STABILITY ANALYSIS

For simplification and conciseness, we define Lyapunov candidate function V_i , for $i = 1, ..., n$, as follows

$$V_i = \frac{1}{2}z_i^2 + \frac{1}{2}\tilde{\Theta}_i^T\Gamma_i^{-1}\tilde{\Theta}_i + \frac{1}{2\gamma_i}\tilde{\lambda}_i^{*2} \tag{5}$$

where $\gamma_i > 0$, Γ_i is positive definite matrix to be design later. define estimation errors $(\tilde{\cdot}) = (\cdot) - (\hat{\cdot})$, $(\hat{\cdot})$is the estimation of $(\cdot)$.

Theorem 1**:** Consider the closed-loop adaptive system consisting of the system (1) under Assumptions 1-4, the robust adaptive tracking control law (6) and the intermediate stabilizer (7) and adaptation laws (10) and (11), for $i = i, ..., n$,

$$u = \alpha_n \tag{6}$$

$$\alpha_i = N(\kappa_i)\xi_i \tag{7}$$

$$\dot{\kappa}_i = \xi_i z_i \tag{8}$$

$$\xi_i = c_i z_i + \hat{\Theta}_i^T \Phi_i + \beta_i + \hat{\lambda}_i^* \bar{\phi}_i \tanh\left(\frac{z_i \bar{\phi}_i}{\varepsilon_i}\right) \tag{9}$$

$$\dot{\hat{\Theta}}_i = \Gamma_i\left[\Phi_i z_i - \sigma_{\theta i}\left(\hat{\Theta}_i - \dot{\hat{\Theta}}_i^0\right)\right] \tag{10}$$

$$\dot{\hat{\lambda}}_i^* = \gamma_i z_i \bar{\phi}_i \tanh\left(\frac{z_i \bar{\phi}_i}{\varepsilon_i}\right) - \gamma_i \sigma_{\lambda i}\left(\hat{\lambda}_i^* - \lambda_i^0\right) \tag{11}$$

can guarantee the following properties: i) all the signals and solutions in the closed-loop system remain GUUB, ii) for any given $\varepsilon^* > \sqrt{\sum_{i=1}^n 2\rho_i}$, there exists a $T > 0$, such that $|z(t)| \le \varepsilon^*$ for all $t \ge T$. Furthermore, ε^* can be made as small as desired by an appropriate choice of the design parameters such that the tracking error $z_1 = y(t) - y_d(t)$ satisfies the property of $\lim_{t\to\infty}|z_1(t)| \le \varepsilon^*$. Correspondingly, the system output $y(t)$ satisfies

$$|y(t)| \le \sqrt{2(\rho_1 + C_1)} + |y_d(t)|. \tag{12}$$

where c_i, ε_i, $\sigma_{\theta i}$,$\sigma_{\lambda i}$, θ_i^0 ,λ_i^0 are positive design parameters. And

$$C_1 = \sup\int_0^t \left((g_1 N(\kappa_1) + 1)\dot{\kappa}_1 + g_1 z_2^2\right) e^{-c_{11}(t-\tau)} d\tau.$$

***Proof*:** The proof will be obtained via the following backstepping design procedure.

First, we define $\rho_i = \delta_i / c_{ii}$,

$$\delta_i = \lambda_i^* \varepsilon_i + \frac{1}{2}\sigma_{\theta i}\left|\Theta_i - \Theta_i^0\right|^2 + \frac{1}{2}\sigma_{\lambda i}\left|\lambda_i^* - \lambda_i^0\right|^2, \tag{13}$$

$$c_{ii} := \min\left\{2\left(c_i - \frac{1}{4}\right), \frac{\sigma_{\theta i}}{\lambda_{\max}\left(\Gamma_i^{-1}\right)}, \sigma_{\lambda i}\gamma_i\right\}. \tag{14}$$

***Step 1*:** Let $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$, then

$$z_1 \dot{z}_1 = z_1\left[g_1(z_2 + \alpha_1) + \theta_1^T \psi_1 + \Delta_1 - \dot{y}_d\right] \tag{15}$$

Note that (3) and the Schwarz inequality, and let $i = 1$ in (5), then the derivative of V_1 can be obtained

$$\begin{aligned}\dot{V}_1 \le& \left[g_1(x_1) z_1 z_2 + z_1 g_1(x_1)\alpha_1 + \Theta_1^T \Phi_1(x_1) z_1 \right.\\ &\left. - z_1 \dot{y}_d\right] + |z_1|\lambda_1 \bar{\phi}_1(x_1) - \tilde{\Theta}_1^T \Gamma^{-1}\dot{\hat{\Theta}}_1 - \tilde{\lambda}_1^T \gamma_1^{-1}\dot{\hat{\lambda}}_1\end{aligned} \tag{16}$$

where $\Theta_1 = \theta_1$, $\bar{\phi}_1 = \phi_1$, $\Phi_1 = \psi_1$.

By substituting (7)-(11), for $i = 1$, into (16), then

$$\begin{aligned}\dot{V}_1 \le& g_1(\bar{x}_1) z_1 z_2 + g_1(\bar{x}_1) N(\kappa_1)(\dot{\kappa}_1) + \dot{\kappa}_1\\ &+ \hat{\lambda}_1 |z_1| \bar{\phi}_1 - \hat{\lambda}_1 z_1 \bar{\phi}_1(x_1)\tanh\left(\frac{z_1 \bar{\phi}_1(x_1)}{\varepsilon_1}\right)\\ &- c_1 z_1^2 + \sigma_{\theta 1}\tilde{\Theta}_1^T\left(\hat{\Theta}_1 - \Theta_1^0\right) + \sigma_{\lambda 1}\tilde{\lambda}_1^T\left(\hat{\lambda}_1 - \lambda_1^0\right)\\ \le& -\left(c_1 - \frac{1}{4}\right) z_1^2 - \frac{1}{2}\sigma_{\theta 1}\left|\tilde{\Theta}_1\right|^2 - \frac{1}{2}\sigma_{\lambda 1}\left|\tilde{\lambda}_1\right|^2\\ &+ (g_1 N(\kappa_1) + 1)\dot{\kappa}_1 + \delta_1 + g_1^2 z_2^2\\ \le& -c_{11} V_1 + (g_1 N(\kappa_1) + 1)\dot{\kappa}_1 + \delta_1 + g_1^2 z_2^2\end{aligned} \tag{17}$$

In the above analysis, the Young's inequality $g_1(\bar{x}_1) z_1 z_2 \le \frac{1}{4} z_1^2 + g_1^2(\bar{x}_1) z_2^2$ and the facts $0 \le |x| - x\tanh\left(\frac{x}{\varepsilon}\right) \le 0.2785\varepsilon, \varepsilon > 0, x \in R$, $\tilde{\theta}^T\left(\hat{\theta} - \theta_0\right) = \frac{1}{2}\left|\tilde{\theta}\right|^2 + \frac{1}{2}\left|\hat{\theta} - \theta_0\right|^2 - \frac{1}{2}|\theta - \theta_0|^2$ are used.

Similar to (Ge and Wang, 2002)and (Ge *et al.*, 2003), then

$$\begin{aligned}0 \le& V_1(t)\\ \le& \rho_1 + V_1(0) + e^{-c_{11}t}\int_0^t g_1 N(\kappa_1)\dot{\kappa}_1 e^{c_{11}\tau} d\tau\\ &+ e^{-c_{11}t}\int_0^t \dot{\kappa}_1 e^{c_{11}\tau} d\tau + e^{-c_{11}t}\int_0^t g_1^2 z_2^2 e^{c_{11}\tau} d\tau\end{aligned} \tag{18}$$

From (18), if there is no extra term $e^{-c_{11}t}\int_0^t g_1^2 z_2^2 e^{c_{11}\tau} d\tau$ within the inequality, it can be concluded that $V_1(t)$, κ_1, as well as α_1, $\hat{\theta}_1$ are all GUUB by Lemma 1 and Remark 2. Due to the presence of the extra term in (18), Lemma 1 cannot be applied directly. However, if z_2 can be regulated as bounded, then the regulation of z_1 is achieved.

From Assumption 1, the following inequality can be obtained (Ge *et al.*, 2003),

$$e^{-c_{11}t}\int_0^t g_1^2 z_2^2 e^{c_{11}\tau} d\tau \le \frac{1}{c_{11}} l_1^{+2} \sup_{\tau\in[0,t]} z_2^2(\tau) \tag{19}$$

Thus, if z_2 is regulated as bounded, then the boundedness of the extra term can be readily concluded from (19). The effect of the extra term will be tackled in the following steps.

Step i ($2 \le i \le n-1$) Similar procedures are taken recursively for each step of $i = 2, \cdots, n-1$.

Define $z_i = x_i - \alpha_{i-1}$, then we can obtain

$$\begin{aligned}z_i \dot{z}_i = z_i&\left[g_i(z_{i+1} + \alpha_i) + \theta_i^T \psi + \Delta_i + \beta_i\right.\\ &\left. - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\left(b_j \psi_{1j}(\bar{x}_i) x_{j+1} + \theta_j^T \psi_j + \Delta_j\right)\right]\end{aligned} \tag{20}$$

where

$$\begin{aligned}\beta_i =& -\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\lambda}_j}\dot{\hat{\lambda}}_j\\ &- \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}} y_d^{(j+1)} - \frac{\partial\alpha_{i-1}}{\partial\kappa_{i-1}}\dot{\kappa}_{i-1}\end{aligned} \tag{21}$$

From Assumption 3, the following inequality holds

$$z_i\left(\Delta_i-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Delta_j\right)\le\lambda_i^*|z_i|\bar{\phi}_i(\bar{x}_i) \quad (22)$$

where $\lambda_i^*=\max\{\lambda_1,...,\lambda_i\}$, and

$$\bar{\phi}_i(\bar{x}_i)=\phi_i+\sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|\phi_j \quad (23)$$

From Assumption 2, define

$$\Theta_i=\left[b_1,...,b_{i-1},\theta_i^T,\theta_1^T,...,\theta_{i-1}^T\right]^T \quad (24)$$

$$\Phi_i=\left[-\frac{\partial\alpha_{i-1}}{\partial x_1}\psi_{1,1}x_2,...,-\frac{\partial\alpha_{i-1}}{\partial x_{i-1}}\psi_{1,i-1}x_i,\right.$$
$$\left.\psi_i^T,-\frac{\partial\alpha_{i-1}}{\partial x_1}\psi_1^T,...,-\frac{\partial\alpha_{i-1}}{\partial x_{i-1}}\psi_{i-1}^T\right]^T \quad (25)$$

Then (20) becomes

$$z_i\dot{z}_i\le\left[g_i(\bar{x}_i)z_iz_{i+1}+z_ig_i(\bar{x}_i)\alpha_i+\Theta_i^T\Phi_i(\bar{x}_i)z_i+\beta_i\right]+\lambda_i^*|z_i|\bar{\phi}_i(\bar{x}_i) \quad (26)$$

Note that (5) and (7)~(11), similar to Step 1, we can get

$$\dot{V}_i\le-\left(c_i-\frac{1}{4}\right)z_i^2-\frac{1}{2}\sigma_{\theta i}\left|\tilde{\Theta}_i\right|^2-\frac{1}{2}\sigma_{\lambda i}\left|\tilde{\lambda}_i^*\right|^2+\left(g_iN(\kappa_i)+1\right)\dot{\kappa}_i+g_iz_{i+1}^2+\delta_i$$
$$\le-c_{ii}V_i+\left(g_iN(\kappa_i)+1\right)\dot{\kappa}_i+\delta_i+g_i^2z_{i+1}^2 \quad (27)$$

Similar to Step 1, then

$$V_i(t)\le\rho_i+V_i(0)+e^{-c_{11}t}\int_0^t g_iN(\kappa_i)\dot{\kappa}_ie^{c_{ii}\tau}d\tau+e^{-c_{11}t}\int_0^t\dot{\kappa}_ie^{c_{11}\tau}d\tau+e^{-c_{11}t}\int_0^t g_i^2z_{i+1}^2e^{c_{11}\tau}d\tau \quad (28)$$

Similar to the analysis in Step 1, if z_{i+1} is regulated as bounded in the next step, then the boundedness of z_i can be readily concluded.

***Step n*:** Define $z_n=x_n-\alpha_{n-1}$. By setting $i=n$ in (5) and (7)~(11), and $z_{n+1}=0$, then

$$\dot{V}_n\le-c_nz_n^2-\frac{1}{2}\sigma_{\theta n}\left|\tilde{\Theta}_n\right|^2-\frac{1}{2}\sigma_{\lambda n}\left|\tilde{\lambda}_n^*\right|^2+\left(g_nN(\kappa_n)+1\right)\dot{\kappa}_n+\delta_n$$
$$\le-c_{nn}V_n+\left(g_nN(\kappa_n)+1\right)\dot{\kappa}_n+\delta_n \quad (29)$$

Let $\rho_n=\delta_n/c_{nn}$, similarly,

$$V_n(t)\le\rho_n+V_n(0)+e^{-c_{11}t}\int_0^t g_nN(\kappa_n)\dot{\kappa}_ne^{c_{nn}\tau}d\tau+e^{-c_{11}t}\int_0^t\dot{\kappa}_ne^{c_{11}\tau}d\tau \quad (30)$$

By choosing $c_0=\rho_n+V_n(0)$, According to Lemma 1 and Remark 2, it can be concluded that $V_n(t)$, $\kappa_n(t)$, hence α_n, $z_n(t)$, $\hat{\Theta}_n$ and $\hat{\lambda}_n^*$ are GUUB. From the boundedness of $z_n(t)$, the boundedness of the extra term at Step $(n-1)$ is obtained. Applying Lemma 1 for $(n-1)$ times backward, it can be seen from the above iterative design procedures that V_i, α_i, z_i, $\hat{\Theta}_i$, $\hat{\lambda}_i^*$ and hence $x_i(t)$ are GUUB, $i=1,\cdots,n$. Since $y(t)=x_1(t)=z_1(t)+y_d(t)$, from the definition of V_1 and (17), the property (12) can be easily obtained. In addition, by appropriately choosing the design constants the regulation of the tracking error z_1 to any prescribed accuracy can be achieved while keeping the boundedness of all signals and states involved.

The proof is complete.

4. ILLUSTRATIVE EXAMPLE

4.1 A Mathematical Example

Consider the following uncertain system,

$$\begin{cases}\dot{x}_1(t)=\left(1+2x_1^2\right)x_2+2x_1^2+0.6x_2\sin(x_2)\\ \dot{x}_2(t)=\left(3+\cos(x_1x_2)\right)u+0.5x_2\sin(x_2)\\ \qquad +0.5\left(x_1^2+x_2^2\right)\sin(t)\\ y(t)=x_1(t)\end{cases} \quad (31)$$

where $g_1(x_1)=1+2x_1^2$, $g_2(x)=3+\cos(x_1x_2)$, $\psi_1(x_1)=x_1^2$, $\psi_2(x)=x_2\sin(x_2)$, $\Delta_2=0.5\left(x_1^2+x_2^2\right)\sin(t)$, $\Delta_1=0.6\sin(x_2)$, $\theta_1=2$, $\theta_2=0.5$. If choose $\lambda_1=0.6$, $\lambda_2=0.5$, $\phi_1=1$, $\phi_2=\left(x_1^2+x_2^2\right)$, then Assumption 3 holds. The reference signal is chosen as in (Ge, *et al.*, 2003): $y_d=0.5(\sin(t)+\sin(0.5t))$. Consequently, the robust adaptive tracking controller can be achieved as in (9)-(11) and (6), $i=1,2$, $n=2$. Where $\lambda_1^*=\lambda_1$, $\lambda_2^*=\max\{\lambda_1,\lambda_2\}$, $\beta_1=-\dot{y}_d$, $\beta_2=-\frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1-\frac{\partial\alpha_1}{\partial\hat{\lambda}_1}\dot{\hat{\lambda}}_1-\sum_{j=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(j)}}y_d^{(j+1)}-\frac{\partial\alpha_{i-1}}{\partial\kappa_1}\dot{\kappa}_1$, $\bar{\phi}_1=\phi_1$, $\bar{\phi}_2(x)=\phi_2+\left|\frac{\partial\alpha_1}{\partial x_1}\right|\phi_1$, $\Theta_1=\theta_1$, $\Theta_2=\left[b_1,\theta_2^T,\theta_1^T\right]^T$, $\psi_{1,1}(x_1)=0.5+x_1^2$, $b_1=2$, $\Phi_1=\psi_1$, $\Phi_2=\left[-\frac{\partial\alpha_1}{\partial x_1}\psi_{1,1}x_2,\psi_2^T,-\frac{\partial\alpha_1}{\partial x_1}\psi_1^T\right]^T$.

The design constants are chosen as $c_1=1$, $c_2=20$, $\sigma_{\theta 1}=0.5$, $\sigma_{\theta 2}=2$, $\sigma_{\lambda 1}=\sigma_{\lambda 2}=2$, $\gamma_1=\gamma_2=0.05$, $\Gamma_1=\Gamma_2=0.5$, $\varepsilon_1=\varepsilon_2=0.01$, $\sigma_{b1}=0.5$. The following initial conditions are adopted as $x(0)=[-0.5,0]^T$, $\theta_1^0=0.5$, $\theta_2^0=0.1$, $\hat{\theta}_1(0)=\hat{\theta}_2(0)=\hat{\lambda}_1(0)=\hat{\lambda}_2(0)=\hat{b}_1(0)=0$.

Simulation results in Figs.1-3 show the effectiveness of the presented controller for system (31). Fig. 1 shows that the output tracks the reference signal y_d perfectly after 10 seconds just like (Ge *et al.*, 2003). Figs. 2-3 illustrate the boundedness of some signals and variables respectively.

4.2 Practical Example- Linear Track-keeping Control of Ships

To illustrate the practicability of the interesting systems in this paper, a practical example on ship

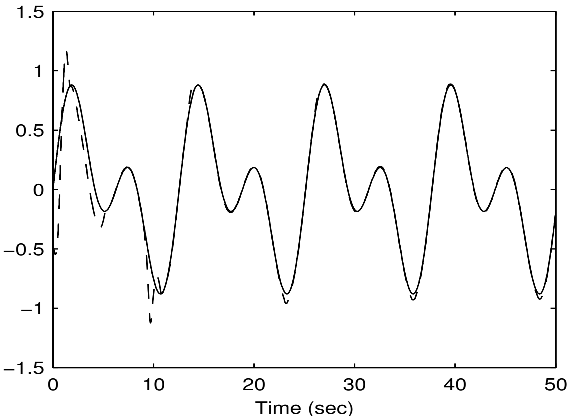

Fig. 1. Output $y(t)$ ("- -") and reference y_d ("–").

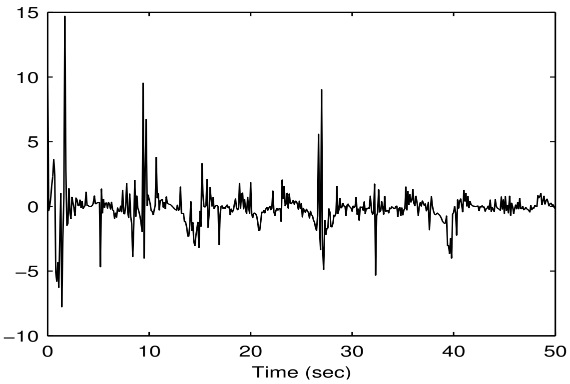

Fig. 2. Control input $u(t)$.

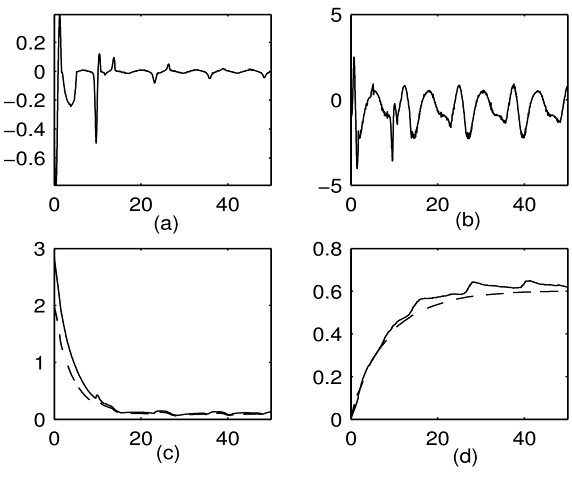

Fig. 3. (a) tracking error z_1. (b) state x_2. (c) Norms $\|\hat{\Theta}_1\|$("- -") and $\|\hat{\Theta}_2\|$("–"). (d) Estimates $\hat{\lambda}_1$("- -") and $\hat{\lambda}_2$("–").

linear track-keeping control design is presented. The following nonlinear ship straight-line motion equations (Li and Yang, 2004) is

$$\begin{cases} \dot{y} = U\sin(\psi) \\ \dot{\psi} = r \\ \dot{r} = f_2(r) + b(t)u + w \end{cases} \tag{32}$$

where y, ψ, r and U denote the sway displacement (cross-track error), heading angle, yaw rate and cruise speed respectively. f_2 depicts the uncertain dynamics, $b = K/T$. In this paper, we assume that U, T, K and α are unknown but constant. u denotes rudder angle input; w denotes equivalent external perturbations induced by current, wave and wind.

Remark 3: In practice, the high gain $b = K/T$ is slowly time-varying, i.e., $b = g(t)$, and the sign of b is determined via the ship test or the trial-and-error way. So in this sense, our proposed algorithm can ease the controller design in practice.

With the coordinate transformation

$$\begin{cases} x_1 = \psi + \arcsin\left(\dfrac{ky}{\sqrt{1+(ky)^2}}\right) \\ x_2 = r \end{cases} \tag{33}$$

We can get the following systems of the form (1)

$$\begin{cases} \dot{x}_1 = f_1(\cdot) + x_2 + \Delta_1 \\ \dot{x}_2 = f_2(x) + g(t)u + \Delta_2 \\ \eta = x_1 \end{cases} \tag{34}$$

where $f_1 = kU\sin(\psi) \Big/ \left(1+(ky)^2\right)$,$\Delta_1 = 0$,$\Delta_2 = w$, $\eta = x_1$ is the output.

Now, the regulation of (32) becomes that of (34)(refer to (Li and Yang, 2004) for details).

With our proposed algorithm (6)∼(11), the simulation results are demonstrated by Fig.4 and Fig.5 as follows.

Remark 4: The ship roll stabilization system, which is a typical 2nd order dynamic system

$$\begin{aligned} &(I_{xx} + J_{xx})\ddot{\varphi} + N\dot{\varphi} + W\dot{\varphi}|\dot{\varphi}| \\ &\quad + Dh\varphi\left(1 - (\varphi/\varphi_v)^2\right) = F_C + F_W, \end{aligned} \tag{35}$$

controlled by fin with actuator, can be transformed into the form of (1) as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_2(\bar{x}_2) + g_2(\bar{x}_2)x_3 + \Delta_2 \\ \dot{x}_3 = f_3(x) + g_3(x)u + \Delta_3 \end{cases} \tag{36}$$

refer to (Yang *et al.*, 2004) (Yang *et al.*, 2003) and (Yang and Zhou, 2005) for details.

5. CONCLUSION

In this paper, a novel robust adaptive tracking controller has been developed for a large class of strict feedback uncertain nonlinear perturbed systems without a priori knowledge of the signs of the unknown virtual control coefficients. It proved that the proposed control algorithm is able to guarantee GUUB of all the signals. In addition, the output of the system has been proven to converge to a small neighborhood of the origin. Numerical simulation results are presented to validate the effectiveness of the proposed approach.

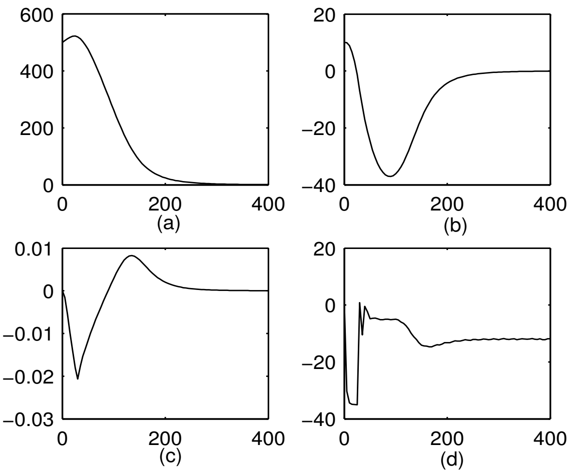

Fig. 4. (a) Cross-track error y(m). (b) Heading error ψ(deg). (c) Yaw rate r(rad/s). (d) Rudder input u(deg).

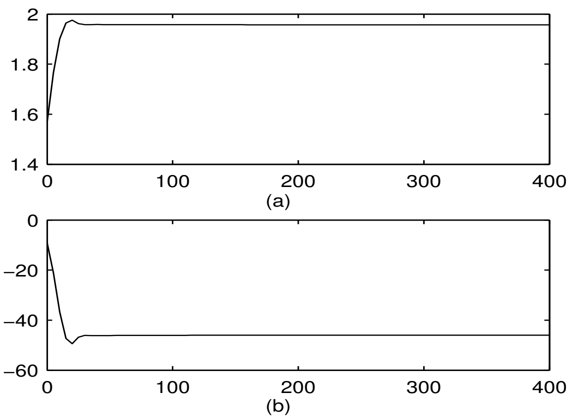

Fig. 5. (a) Adaptation parameter κ. (b) Nussbaum function $N(\kappa)$.

In a word, this paper enlarges the class of strict-feedback uncertain nonlinear systems with unknown virtual control gain functions under perturbation for which global robust adaptive tracking control can be designed.

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