CONTROL-ORIENTED NARX MODELING OF MAGNETO-RHEOLOGICAL DAMPERS

Alberto Leva [1], **Luigi Piroddi**

Dipartimento di Elettronica e Informazione, Politecnico di Milano
Via Ponzio, 34/5 - 20133 Milano (Italy)

Abstract: This paper presents a methodology for identifying variable-structure models of magneto-rheological dampers (MRDs) that are structurally simple, easy to estimate and well suited for model-based control. Linear-in-the-parameters NARX models are adopted, and an identification method is developed based on the minimisation of the simulation error. Both the model structure and the parameters are selected by the identification method, thus no *a priori* structural information is needed. Some validation tests are reported. *Copyright© 2005 IFAC*

Keywords: Magneto-rheological dampers; NARX models; identification.

1. INTRODUCTION

Semi-active control devices have proven effective in a number of applications, see e.g. (Dupont and Stokes, 1997): they do not transfer energy to the controlled system, so they are as reliable as passive devices, but can approach the performances of active ones. In recent years, interesting technological solutions have been proposed based on the use of magneto-rheological (MR) fluids, see e.g. (Carlson *et al.*, 1995; Jolly *et al.*, 1998). These fluids contain micron-sized, magnetically polarisable particles, which form chain-like structures when an external field is applied, thus changing the damper's viscous behaviour. However, the use of such devices for control is hindered by the fact that their dynamics are inherently hysteretical and highly nonlinear, so that modeling them in a way suitable for control is far from trivial. This work presents a technique, based on NARX (Nonlinear AutoRegressive with eXogenous inputs) models (Leontaritis and Billings, 1985*a*; Leontaritis and Billings, 1985*b*), to describe such devices in a way suited for model-based control. Models to be used in that context must be not only accurate, but also robust. In fact (contrary e.g. to the predictive framework) they have to be reliable in simulation, as most of the measurements required for prediction (e.g., forces) are unavailable. They must be simple, to improve robustness and simplify control computations. Also invertibility with respect to the control variables is a desirable feature. Notice that explaining complex dynamics with a small parameter set not only requires structure selection in the model identification, but makes that capability the actual core of the identification itself. The motivation of this research comes from the fact that several modeling paradigms (briefly reviewed later on) for MRDs do not appear to enjoy these properties up to the necessary amount, so that the models obtained are difficult to use for control. It will be shown that with the proposed technique, that has several advantages with reference to control-orientation, a comparable level of accuracy can be achieved with respect to standard, widely accepted modeling approaches.

[1] Corresponding author. Voice (39) 02 2399 3410, Fax (39) 02 2399 3412, e-mail leva@elet.polimi.it

2. BRIEF REVIEW OF MRD MODELLING

The major modeling techniques proposed so far for MRDs can be broadly divided in three groups. **First-principle models** are written on the basis of dynamic balance equations: examples are in (Carlson *et al.*, 1995; Kamath *et al.*, 1996). Due to the complexity of devices' geometry and physics, these models must contain some heuristics, typically in modeling the fluid flow. Thy can be accurate and have parameters that can be interpreted physically, but they are complex, especially if control-orientation is an issue, generally difficult to invert and/or linearise, and also to identify, as parameters appear in a highly nonlinear way. **Phenomenological mechanical models** are models in which mechanical blocks such as masses, springs and dampers are assembled, based on the *qualitative* interpretation of experimental data. Several examples, like the Bingham and the Bouc-Wen models, are in (Dyke *et al.*, 1996; Spencer Jr. *et al.*, 1997). Such models do have a physical interpretation but cannot represent nonlinear and hysteretical phenomena satisfactorily: for example, the Bingham model does not reproduce the preyield/postyield transition well enough. To overcome these limitations, they are often completed by nonlinear blocks implemented as fixed-structure dynamic systems, or the parameters of the mechanical elements are made dependent on the inputs, sometimes dynamically (Spencer Jr. *et al.*, 1997), but this hinders any model interpretation seriously. **Local physical models** are designed to catch one of the relevant operating conditions. The overall model is then obtained by combining the outputs of the single ones by means of "shaping functions" chosen heuristically, see e.g. (Kamath and Wereley, 1997). While parameters of local models can be interpreted physically, however, this is not true for the shaping functions. In addition, it is not easy to state in which sense an operating condition is the "combination" of two or more others, because the transition from an operating condition to another is normally due to the internal dynamics of the device, which are exactly what is neglected when employing a local modeling approach. The quality of these models is strongly dependent on the shaping functions, that defy a physical interpretation. In all these approaches, a model structure is typically assumed *a priori* and, after a parameter fitting process, the resulting model is tested and validated. If it is found to be inaccurate, the structure is modified, based on more or less heuristic reasoning, and the whole procedure is repeated. Besides not being optimal (in any sense), the model structures adopted do not naturally fit in an identification framework, and complex tuning techniques must be employed. Also, models are not developed with a control-oriented implementation in mind, and are not easy to incorporate in known control frameworks.

3. THE PROPOSED APPROACH

The class of polynomial NARX models (Leontaritis and Billings, 1985*a*; Leontaritis and Billings, 1985*b*) has many appealing features for nonlinear black-box identification: they are a discrete time straightforward counterpart of nonlinear differential equations; they can model purely linear systems as well as a large class of nonlinearities; they are linear in the parameters, and the computer implementation of them and of their identification is straightforward. In the discrete time domain, the general form of a polynomial NARX model (Peyton-Jones and Billings, 1989) is

$$y(k) = \sum_{m=0}^{l} \sum_{p=0}^{m} \sum_{n_1=1}^{n_y} \cdots \sum_{n_m=1}^{n_u} c_{p,m-p}(n_1, \ldots, n_m) \prod_{i=1}^{p} y(k-n_i) \prod_{i=p+1}^{m} u(k-n_i) \tag{1}$$

where $0 \leq m \leq l$ is the degree of nonlinearity, l is the maximum degree of nonlinearity, each m-th order term is the product of a p-th order factor in y and a $(m-p)$-th order factor in u and of a coefficient $c_{p,m-p}(n_1, \ldots, n_m)$ depending on the delays of the y and u terms considered, n_y and n_u being the maximum delays for the y and u terms, respectively. In practice, identified models tend to contain a limited subset of all the possible regressors. In addition to (1), additional terms may be included for coping with specific modeling needs, as long as linearity in the parameters is preserved. This idea is exploited in this work, where we consider a simple extension to the notation (1) to account for multiple elementary regressors: if multiple inputs $u_1, \ldots, u_{N_u}$ appear in the model, a suitable number of subscripts and arguments is added to the c coefficients in analogy to the notation defined in (1). For example if 2 inputs are present, terms such as $c_{2,1,1}(1,2,1,3)y(k-1)y(k-2)u_1(k-1)u_2(k-3)$ could appear. By this modification, more complicated functions of the input signals can be easily introduced, such as non integer powers and other non polynomial functions, while the resulting NARX model - in the native input(s) and output - preserves linearity in the parameters.

4. IDENTIFICATION AND VALIDATION

The identification of NARX models of the type considered can be done with standard Least

Squares (LS), but once the model structure has been selected. To overcome this limitation, various algorithms have been proposed to estimate structure and parameters *together*, typically based on orthogonalisation techniques and simple heuristic procedures that provide a stop condition when no further regressor improves the cost function: the reader can refer to (Billings *et al.*, 1989; Korenberg *et al.*, 1987; Leontaritis and Billings, 1987; Mao and Billings, 1997). To apply an LS based technique, it is essential that the linear regression paradigm is preserved; this requires that the 1-step prediction error variance is minimised. Then, the order in which parameters are included is relevant for the model selection process (Korenberg *et al.*, 1987): to this end, a "forward-regression orthogonal" variant has been developed (Billings *et al.*, 1989). What is actually maximised by the most important algorithms, however, is not the output variance explained by the model, but rather the *increment* of explained variance provided by a new parameter; therefore, there is no guarantee that the model obtained be optimal in any sense. Recall that in this context should be *simulation* models as accurate as possible are desired, as in practice it is impossible to employ a prediction model for control unless *accurate* measurements of all the past input/output signals involved exist. For example, a prediction model of an MRD would be inappropriate since past force values are not commonly available. For all these reasons, we employ an identification algorithm based on the minimisation of the *simulation* error which extends that proposed in (Piroddi and Spinelli, 2003). A search procedure is devised for the exploration of incremental model structures, aiming at the minimum *simulation* error variance is selected. The algorithm is stopped when the introduction of a further regressor does not significantly improve this figure of merit. Note that this approach prevents unstable models from being selected, as is often the case with methods based exclusively on the prediction error variance. The identification algorithm can be summarised as follows. To initialise the procedure, it is necessary to provide a set of I/O data and the set of possible regressors. Optionally, also a first guess of the model structure can be provided; otherwise, the model will start out as an empty structure. If an initial model structure is given, the corresponding parameters are estimated with LS, the model is simulated and the cost function (mean square simulation error, possibly weighted if this is convenient) is computed. At each step, all the candidate regressors are examined for inclusion in the model. For each of them, the current model structure is augmented with it, and the resulting model is estimated. The regressor is accepted if it makes the cost function decrease at least by a specified percentage. If the candidate regressor set is exhausted, the best regressor is accepted anyway. When a regressor is added, an iterative pruning subprocedure is also performed. For each regressor, the submodel obtained by eliminating it from the current model is estimated. If the best reduced model still makes the cost function decrease with respect to the previous major iteration, it becomes the current model. The subprocedure is repeated until no regressors can be eliminated without a performance loss. Finally, the overall procedure is stopped when a prespecified accuracy is obtained or after a given maximum number of iterations. With respect to the algorithm presented in (Piroddi and Spinelli, 2003), three main extensions are introduced. First, the model has more than one input. Then, the elementary regressors are non-polynomial functions (e.g., sign and absolute value): this helps a lot at representing highly nonlinear and hysteretical phenomena. Finally, the cost function is a weighted sum of squared simulation errors, namely

$$J = \frac{1}{N}\sum_{i=1}^{N}\frac{(y(i) - y_{sim}(i))^2}{max(|y(i)|, 0.1)}, \qquad (2)$$

where N is the total number of samples and y_{sim} the (normalised) output of the model being evaluated. This reduces the systematic errors at low output values (typical of this application).

5. APPLICATION TO MRD DEVICES

To assess the validity of the proposed methodology, we present a comparison with a "reference" modeling approach representative of the relevant literature, namely that of (Spencer Jr. *et al.*, 1997). The goal is to show that, with no *a priori* structural assumptions, a NARX model can be found which displays a comparable accuracy with respect to a phenomenological model, albeit coming from a procedure which estimates the model structure and parameters *together*. A thorough experimental comparison would require availability of the same data used in (Spencer Jr. *et al.*, 1997) for the model tuning, plus several alternative data sets of the same MRD to ensure robustness. For the purpose of the work, which is methodological rather than experimental, it is then more appropriate to generate identification and validation data from the reference model and show that these can be accurately replicated with the proposed approach. The model considered includes an algebraic function (a Bouc-Wen model), has the structure of figure 1, and is made of the equations

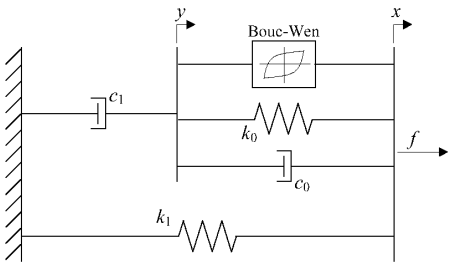

Fig. 1. The reference model by Spencer *et al.*

$$\begin{aligned} f &= \alpha z + c_0(\dot{x}-\dot{y}) + k_0(x-y) + k_1(x-x_0) \\ \dot{y} &= \alpha z + c_0\dot{x} + k_0(x-y) \\ \dot{z} &= -\gamma|\dot{x}-\dot{y}||z|z|^{n-1} - \beta(\dot{x}-\dot{y})|z|^n + A(\dot{x}-\dot{y}) \end{aligned} \quad (3)$$

where x and y have the meaning indicated by figure 1, f is the force and z is an auxiliary variable required by the Bouc-Wen model. Parameters α, c_0 and c_1 depend dynamically on the applied voltage E as follows: $\alpha = \alpha_a + \alpha_b\eta E/(s+\eta)$, $c_0 = c_{0a} + c_{0b}\eta E/(s+\eta)$, $c_1 = c_{1a} + c_{1b}\eta E/(s+\eta)$. Note that the reference model is not easily invertible with respect to the control variable, as the relationship $E = E(x, \dot{x}, f)$ is extremely complex to obtain analytically. The reference model has been implemented in Simulink and simulated. The parameters used in the simulations are $c_{0a} = 20.2$, $c_{0b} = 2.68$, $k_0 = 15$, $c_{1a} = 350$, $c_{1b} = 70.7$, $k_1 = 5.37$, $x_0 = 0$, $\alpha_a = 44.9$, $\alpha_b = 638$, $\gamma = 39.3$, $\beta = 39.3$, $A = 47.2$, $n = 2$, $\eta = 251$—values taken from (Dyke *et al.*, 1996; Spencer Jr. *et al.*, 1997). The reference model has been fed with sinusoidal displacement inputs of different amplitudes and frequencies, and stepwise field inputs covering the entire range. These inputs and the force signal resulting from this simulation have been employed for the NARX identification (40 seconds of data have been recorded with a 1 ms sampling rate). Note, incidentally, that these data would be easily repeatable in an experimental environment. Prior to identification all signals have been normalised to the [-1,1] range for numerical reasons. The choice of the initial candidate regressor set is particularly important for the identification success. This implies choosing the elementary regressors (e.g. the velocity), as well as the powers and delays each one of them is allowed to appear with in the model. After a few attempts, some facts have been noticed. Purely algebraic, and in particular not autoregressive, models are not adequate. This means that the estimated force must depend on its past values (recall that a *simulation* model is required). The force does not depend on the displacement significantly, but rather on the velocity and field terms. The dependence of the force on the velocity is too complex to be represented by integer powers precisely enough (switching behaviours are observed). Finally, systematic estimation errors were observed at low force values. Hence, the following elementary regressors have been selected: $y(k) = f(k)$ (with power 1 to 2, delay 1), $u_1(k) = E(k)$ (powers 1 to 4, delay 0), $u_2(k) = |v(k)|^{0.2}$ (power 1 to 10, delay 0 to 1), $u_3(k) = sign(v(k))$ (power 1, delay 0 to 1), $v(k)$ being the sampled velocity. The identification algorithm has been initialised with a linear model containing only past estimated forces, field and velocity terms. Cost function weighting has been used to reduce the errors at low force. The final model (at iteration 32) is

$$\begin{aligned} &y(k) = \\ &0.84506u_1(k)u_2(k)^2u_3(k) + 1.00465y(k-1) \\ &-0.9934y(k-1)u_2(k-1)^3 + 0.22914u_2(k)^{10}u_3(k) \\ &-0.41012u_1(k)u_2(k)u_3(k) + 0.04469u_1(k)u_3(k) \\ &+0.60627y(k-1)^2u_1(k)u_3(k) + 0.23977u_2(k-1)^7u_3(k-1) \\ &-0.209y(k-1)u_1(k)^2 + 0.02209u_2(k-1)^3u_3(k-1) \\ &+0.10544y(k-1)u_1(k)^4u_2(k)^3 + 0.12693y(k-1)u_1(k)^4 \\ &-0.49617y(k-1)^2u_1(k)^2u_3(k) \end{aligned} \quad (4)$$

and the corresponding value of the cost function is $J_{32} = 0.001596$. Note that the parameterisation is numerically "well-conditioned", in that the values of all parameters range in two orders of magnitude only. The remarkable performance of this model can be appreciated by examining figures 2 and 3 (in all the comparison figures the reference model output is drawn in dash-dot line, and the estimated model output in solid line). The model performance on different data sets is also reported. Figure 4 (top) shows its behaviour in response to a composite displacement signal containing filtered random series, saturated and non-saturated ramps and a chirp signal, while the applied voltage is a chirp signal in the first 2 seconds, and piecewise constant at random values thereafter. Figure 4 (middle) describes a ramp displacement simulation, with constant voltage at the minimum (0 V) in the first half of the simulation, and at the maximum (2.25 V) in the second part. Finally, an earthquake-like displacement signal is applied in the simulation of figure 4 (bottom), with a constant voltage of 2.25 V. For robustness issues and in view of control purposes, however, this model may be considered too complex and - which is worse - it may be affected by overfitting: if the unweighted cost functions on the validation data sets (figure 5) are monitored, it appears that after iteration 20 approximately they do not decrease with the same rate and regularity as before. The model at iteration 20 can then be considered a good compromise between complexity and accuracy. This compact model is

$$\begin{aligned} &y(k) = \\ &0.04081u_2(k-1)^2u_3(k-1) + 0.53543u_1(k)u_2(k)^2u_3(k) \\ &+0.90878y(k-1) - 0.47837y(k-1)u_2(k-1)^3 \\ &+0.24809u_2(k)^{10}u_3(k) - 0.16867u_1(k)u_2(k)u_3(k) \end{aligned} \quad (5)$$

and corresponds to the cost function value $J_{20} = 0.004768$ (about three times J_{32}). In spite of this, accuracy on the identification data set is not severely affected, while performance on the vali-

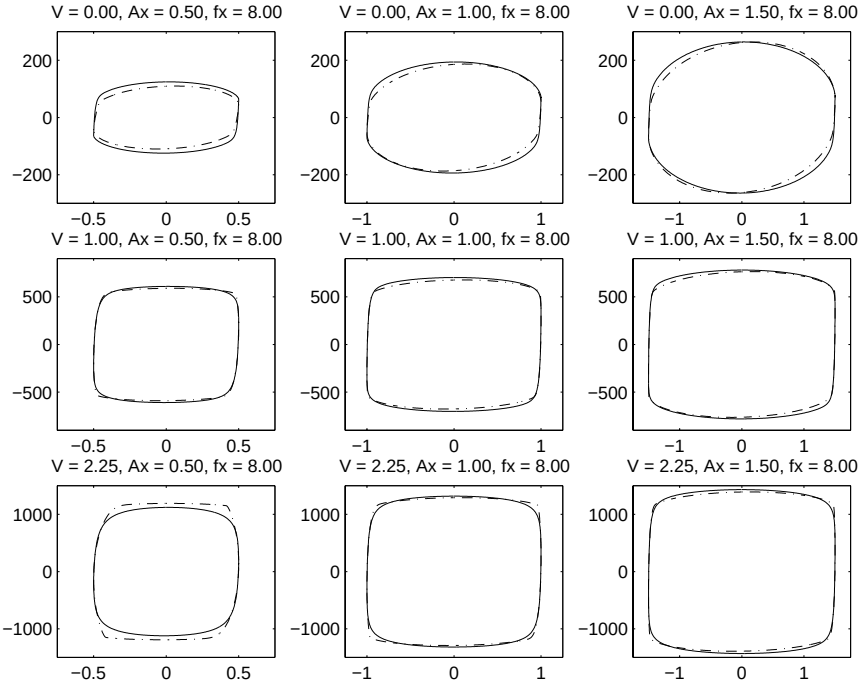

Fig. 2. Force vs. displacement with the 13-parameters model (V is the applied voltage, A_x and f_x the displacement amplitude and frequency).

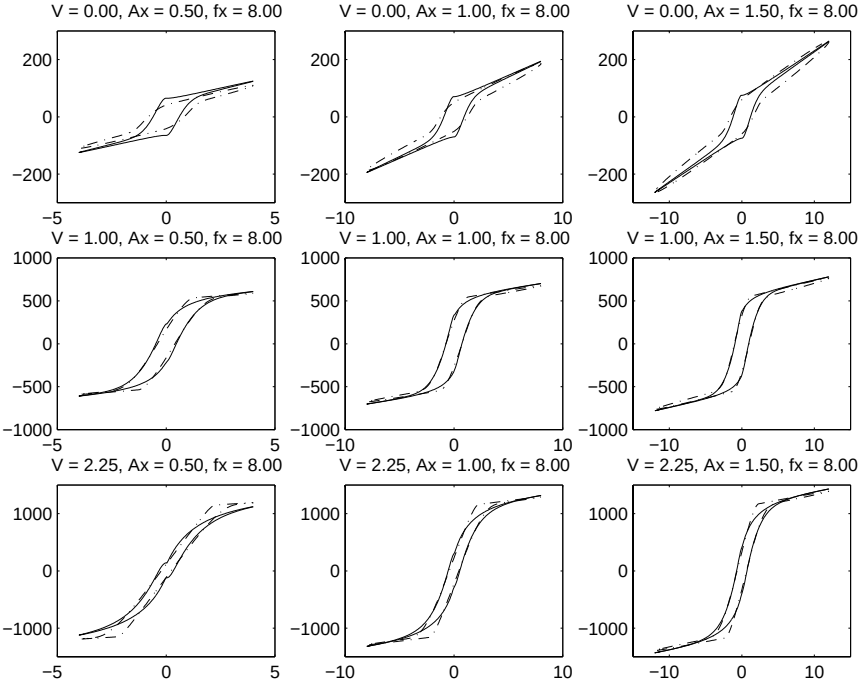

Fig. 3. Force vs. velocity with the 13-parameters model (V is the applied voltage, A_x and f_x the displacement amplitude and frequency).

dation data sets is even improved. The robustness of this model can be appreciated by evaluating its performance on the same data sets of figure 4, as shown in figure 6. As a final (and important) remark, observe that both models (4) and (5) are invertible with respect to the control variable E in a straightforward way.

6. CONCLUDING REMARKS

A technique has been presented for the modeling of MRDs, based on NARX models and a suitable identification algorithm. The described work can be seamlessly extended to the modeling of similar damping devices, such as shock absorbers,

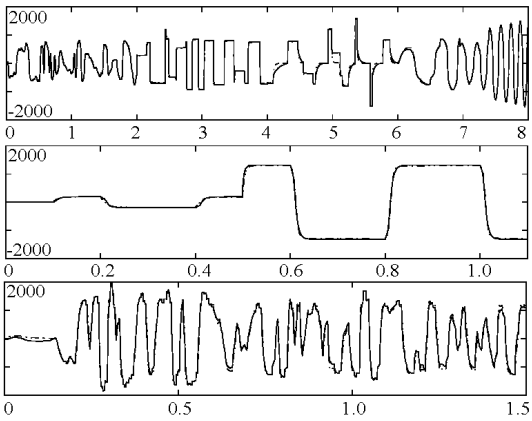

Fig. 4. Validation of the 13-parameters model.

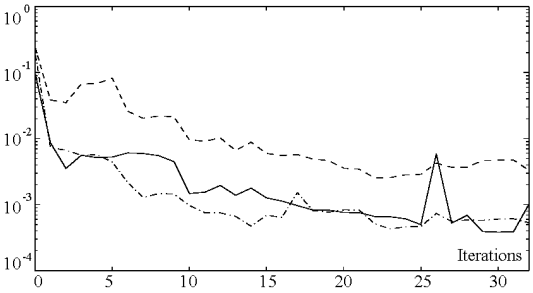

Fig. 5. Cost function (log scale) on the three validation data sets.

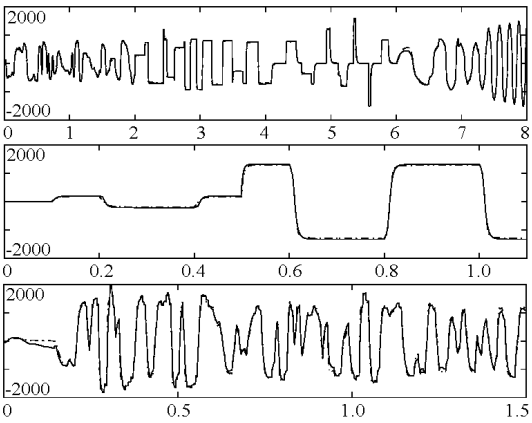

Fig. 6. Validation of the 6-parameters model.

suspension systems and so forth. The technique has been validated on a literature reference model and has been shown to yield simple, accurate and robust models, which can be exploited in the development of model-based control systems. Future research will be focused on the development of control strategies based on the obtained models.

7. REFERENCES

Billings, S.A., S. Chen and M.J. Korenberg (1989). Identification of mimo non-linear systems using a forward-regression orthogonal estimator. *Int. J. of Control* **49**(6), 2157–2189.

Carlson, J.D., D.M. Catanzarite and K.A. St. Clair (1995). Commercial mr fluid devices. In: *Proc. 5th Int. Conf. on Electro-Rheological, Magneto-Rheological Suspensions and Associated Technology*.

Dupont, P. and A. Stokes (1997). Semi-active control of friction dampers. *J. Sound and Vibration* **202**(2), 203–218.

Dyke, S.J., B.F. Spencer Jr., M.K. Sain and J.D. Carlson (1996). Modeling and control of magnetorheological dampers for seismic response reduction. *Smart Mat. Struct.* **5**, 565–575.

Jolly, M.R., J.W. Bender and J.D. Carlson (1998). Properties and applications of commercial magnetorheological fluids. In: *Proc. 5th SPIE Int. Symp. on Smart Structures and Materials*.

Kamath, G.M. and N.M. Wereley (1997). A nonlinear viscoelastic-plastic model for electrorheological fluids. *Smart Mat. Struct.* **6**, 351–359.

Kamath, G.M., M.K. Hurt and N.M. Wereley (1996). Analysis and testing of bingham plastic behavior in semi-active electrorheological fluid dampers. *Smart Mat. Struct.* **5**, 576–590.

Korenberg, M., S.A. Billings, Y.P. Liu and P.J. McIlroy (1987). Orthogonal parameter estimation algorithm for non-linear stochastic systems. *Int. J. of Control* **48**, 193–210.

Leontaritis, I.J. and S.A. Billings (1985*a*). Input-output parametric models for non-linear systems - part i: deterministic non-linear systems. *Int. J. of Control* **41**, 303–328.

Leontaritis, I.J. and S.A. Billings (1985*b*). Input-output parametric models for non-linear systems - part ii: stochastic non-linear systems. *Int. J. of Control* **41**, 329–344.

Leontaritis, I.J. and S.A. Billings (1987). Model selection and validation methods for non-linear systems. *Int. J. of Control* **45**, 311–341.

Mao, K.Z. and S.A. Billings (1997). Algorithms for minimal model structure detection in non-linear dynamic system identification. *Int. J. of Control* **68**, 311–330.

Peyton-Jones, J.C. and S.A. Billings (1989). Recursive algorithm for computing the frequency response of a class of non-linear difference equation models. *Int. J. of Control* **50**, 1925–1940.

Piroddi, L. and W. Spinelli (2003). An identification algorithm for polynomial narx models based on simulation error minimization. *Int. J. of Control* **76**(17), 1767–1781.

Spencer Jr., B.F., S.J. Dyke, M.K Sain and J.D. Carlson (1997). Phenomenological model of a magnetorheological damper. *ASCE J. Eng. Mech.* **123**(3), 230–238.