SLIDING MODE THERMAL CONTROL SYSTEM
FOR FURNACE IN LASER PHYSICS

Anna Khachaturova[1] and Vardan Mkrttchian[2]

[1] Institute for Physical Research of NAS of Armenia,
Ashtarak-2, 378410, Armenia
[2] All Armenian Internet University
P.O.Box 965, Lane Cove, NSW, 2066, Australia

Abstract: The application of the Sliding Mode Thermal Control System to laser spectroscopic system is proposed. An adaptive controller is designed based on results obtained in the sphere of discontinuous system design. The derived Sliding Mode (SM) control algorithm should improve the control performance of the important characteristic of systems such as the thermal field of the oven. The suggested controller program written in LabView environment consists of the following blocks: the sliding mode indicator determining the occurrence of SM; the observer block, identifying the state vector of the system; the anticipatory compensation block. The obtained control algorithm is verified on the real system.

Keywords: Temperature control, Sliding Mode controller, indicator of Sliding Mode, observer; anticipatory compensation, Discontinuous Control and Setting Adjustment.

1. INTRODUCTION

Theoretical and experimental investigations on resonant interaction of laser radiation with atomic vapors are important both for fundamental physics and numerous applications.

Numerous basic studies have been performed on systems involving alkali vapors and their mixtures with buffer gases. There are several reasons why these systems are very suitable subjects for experimental as well as theoretical investigations. Alkali resonance lines are in a spectral range attainable by available lasers and alkali vapors can be easily generated in cells. On the other hand, due to their simple hydrogen-like structure, alkali atoms are convenient subjects for theoretical calculations and for modeling. Many applications such as magnetometry (Scully and Fleischhauer, 1992), electro-magnetically induced transparency, quantum computing, are based on the interaction of these vapors with laser radiation.

In present experiment the thermal regimes of selective excitation of light from boundary of cell's dielectric window and atomic vapor (Chevrollier, *et al*., 1992) are investigated. As is known, selective reflection of light from the boundary of cell's dielectric window and atomic vapor is a convenient spectroscopic tool for this kind of measurement. Relevant spectroscopic investigations imply recording and processing of a large number of selective reflection spectra, along with frequency reference spectra, at various thermal conditions of a vapor cell. The heating unit that provides proper temperature regime of a vapor cell has two independent working areas. Temperature of one of the areas (side arm of a cell) determines alkali metal vapor density, and temperature of the second area determines regime of atom-atom and atom-dielectric surface interaction in the cell (Taylor and Langmuir, 1937; Miles and Harris, 1973; Knappe *et. al.*, 2002). In this connection it is important to precisely set and keep proper values of temperature.

The swift development of electronics, computer and software engineering in recent years had les to broad applications of the adaptive control system in various fields. Thus many complex algorithms can be now applied for real-time control, with PCs with AD/DA cards and modern software such as LabView programming. Within the framework of the present work the Sliding Mode Thermal controller based on LabView software under Windows environment is proposed (Mkrttchian and Khachaturova, 2003; Khachaturova and Mkrttchian, 2003).

2. PROCESS DESCRIPTION AND MODELLING

This section includes the experimental setup description and derives the model of the thermal control system.

2.1 Process description.

The block-scheme of an experimental setup is shown in Fig.1 and operates as follows. The parallel radiation beam of the single –frequency laser diode was directed on a beam splitter. One part of divided radiation beam was directed onto the alkali vapor cell at working temperature 50-300^0 *C***.**

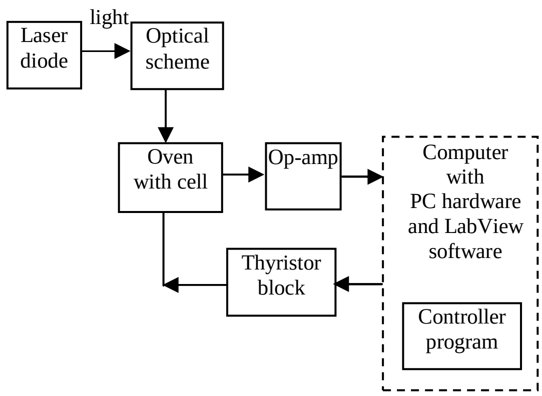

Fig.1. Block-scheme of the experimental setup.

The signal of selective reflection from the “dielectric window-vapor” interface was detected by the photodiode with an operational amplifier. The second radiation beam was directed into a room temperature reference cell with alkali metal vapor, where the saturated absorption scheme has been implemented, allowing to determine frequency positions of atomic hyperfine lines to be determined. Detection of a saturated absorption spectrum was carried out by the second similar photodiode with an operational amplifier. The output signals from the amplifiers of photodiodes were imported to an analog input of the DAQ board, PC-1200AI. And the output (temperature) signals from oven and cell were also imported to analog inputs of the DAQ board (Mkrttchian and Khachaturova, 2004).

2.2 Model of the thermal control system (TCS).

The TCS objective is to control the thermal regime of the oven. The mathematical model of TCS is very useful in determining the sliding surfaces and control functions and is represented by the nonlinear SISO system with partially known parameters. Model equations simulate the dynamics within TCS, which include the response of the oven with a cell inside.

The transfer function of the system is:

$$\frac{\mathrm{Y}(p)}{\mathrm{X}(p)} = \frac{k_T \exp(-p\tau_T)}{(T_T p+1)(T_{TO} p+1)}, \qquad (1)$$

where $k_T = k_{T1}k_{T2}$ - is the gain value, that is a product of a heat source gain (k_{T1}) and plant’s gain (k_{T2}); τ_T is delay; T_T – is time constant of a heat source; $T_{T.O.}$ - time constant of a cell.
From (1) yields:

$$\frac{\mathrm{Y}(p)}{\mathrm{X}(p)} = \frac{k_T \exp(-p\tau_T)}{[T_T T_{TO} p^2 + T_T p + T_{TO} p + 1]} \qquad (2)$$

Applying Laplace transform, yields

$$\frac{d^2Y}{dt^2}T_T T_{TO} + \frac{dY}{dt}(T_T + T_{TO}) + Y = k_T(t-\tau)x \qquad (3)$$

Dividing both parts of the equality (3) on $T_T T_{TO}$, yields

$$\frac{d^2Y}{dt^2} + \left[\frac{T_T + T_{TO}}{T_T T_{TO}}\right]\frac{dY}{dt} + \frac{1}{T_T T_{TO}}Y = \frac{k_T}{T_T T_{TO}}x(t-\tau) \qquad (4)$$

Denote

$$T_1 = \frac{T_T + T_{TO}}{T_T T_{TO}}; \quad T_2 = \frac{1}{T_T T_{TO}}; \quad T_3 = \frac{k_T}{T_T T_{TO}},$$

Then,

$$\frac{d^2Y}{dt^2} + T_1\frac{dY}{dt} + T_2 Y = T_3 x(t-\tau) \qquad (5)$$

The description of the system in normal form is shown as

$$Y = Y_1;$$

$$\dot{Y}_1 = Y_2;$$

$$\dot{Y}_2 = -Y_2 T_1 - Y_1 T_2 + T_3 x(t-\tau) \qquad (6)$$

A set of differential equations can be presented in matrix form:

$$\begin{bmatrix} \dot{Y}_1 \\ \dot{Y}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -T_2 & -T_1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ T_3 \end{bmatrix} x(t-\tau) \qquad (7)$$

3. SLIDING MODE THERMAL CONTROLLER DESIGN

This section synthesizes the sliding mode controller algorithm. Nowadays there are many works (Utkin and Utkin, 2004; Basin *et al.*, 2004) devoted to the development of various types of regulators on Sliding Mode technique. In the suggested algorithm a new block called Sliding Mode Indicator is presented. The block determines the occurrence of SM in the system. If the SM does not occur, the indicator block deliberately introduces the system in SM.

Control within sliding mode enables us to decrease the sensitivity to variations of chip characteristics making them independent upon of the environment. The mentioned problems can be overcome by using asymptotic observers of state and anticipatory devices eliminating delays. Discontinuity of control results in a discontinuity of the right-hand parts of the differential equations describing the system's dynamic properties. Deliberate introduction of objects into the sliding-mode operation will necessitate a continuous monitoring of the sliding-mode occurrence and stability (Utkin, 1982).

Our purpose is to achieve a model suitable for use with all types of solution simulation. Therefore it should be presented as a simple equivalent circuit. The circuit elements are derived from physical concepts, which are more often than not lost whenever a complex model for a solution is extracted from an optimization model. For the Finite Difference Time Domain computation of the object ports are defined on the coaxial lines as it is used in most simulators by observing wave ratios. The ports for solution's equivalent circuit are concentrated or internal ports defined by voltage and current. The resulting S-parameters of the ports are used to extract equivalent circuit elements leading to well defined values with physical senses. The essential components of the solution model are the sliding-mode indicator, observer and the anticipatory device.

3.1 The Sliding Mode Indicator.

As already noted, the approach used is oriented toward a deliberate introduction of sliding modes over the intersection of surfaces on which the control vector components undergo discontinuity. Realization of such an approach implies the knowledge of the conditions of the occurrence of sliding mode. Designed for this purpose was the indicator of sliding modes. From the point of view of mathematics, the problem may be reduced to that of finding the area of attraction to the manifold of the discontinuity surfaces intersection (Dubrovsky and Kortnev, 1968).

On the indicator input two signals x and g from the equivalent circuit are supplied. The indicator compares those two signals recording the moment of changing the signs of function x and g, see (8).

$$g(t) = C_1 x + C_2 \frac{dx}{dt}, \qquad (8)$$

where C_1 and C_2 are constants of object control;
g is function of switching; x is parameter of system; t is time.

According to (Dubrovsky and Kortnev, 1968; Mcrttchian *et al.,* 1999) the existing indicator diagram is shown in Fig.2.

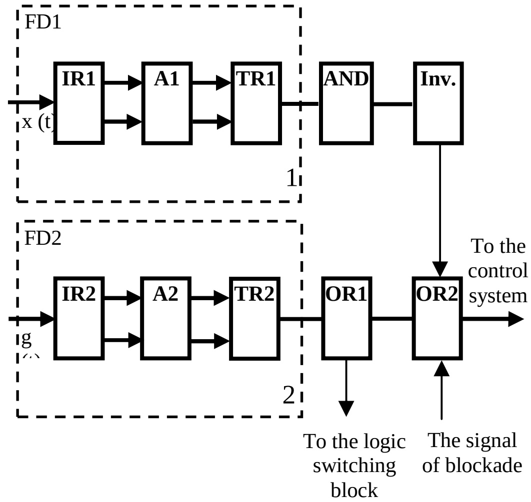

Fig. 2. The diagram of the Sliding Mode Indicator.

In forming devices 1 and 2 (FD1 and FD2) it indicates the moment of changing of the signs of functions *x (t)* and *g (t).* Then in logical cells "AND", INVERTOR- "Inv", and "OR" the operation of sign coincidence is taking place. When the signs of functions *x (t)* and *g (t)* are identical, the indicator logic is "1" (Sliding mode is present), when the signs of function *x (t)* and *g (t)* are not identical, the indicator is "0" (Sliding mode is not present). The device includes the input restructures (IR1 and IR2), amplifiers (A1 and A2), and potential triggers (TR1 and TR2).

3.2 Construction of SM existence region

As it known, the motion of the system operating in the sliding surface is invariant to disturbances, parameter uncertainties, coupling between channels, and nonlinearities seen in the system. The constant coefficients C_i of the sliding surface boundary described by equation (8) are chosen to provide the output tracking motion in sliding mode. This is accomplished through construction of SM existence region and choosing the coefficients C_i from the region.

In this section the graphic method of construction of SM existence region is given, according to (Dubrovsky and Kortnev, 1968). The example of construction of the boundary of sliding mode existence region for the control system is shown in Fig. 3.

Construction of boundaries is carried out as follows. Three systems of coordinates are combined on a plane: $xO\dot{x},\ xOF(M),\ pO_1F(p)$. The points of origin of the two first systems coincide, and the third system is shifted for -1 on axes Ox. The scales on axis O_1p, Ox and on $OF(M)$, $O_1F(p)$ are identical, and the directions of axis $O\dot{x}$, $OF(M)$, $O_1F(p)$ coincide. A plot of characteristic polynomial of the system (9) is constructed in $xO_1F(p)$ system of coordinates given p from 0 to $-\infty$. A plot of function (10) is constructed in $xOF(M)$ system of coordinates.

$$F(p) = p^2 + T_1 p + T_2 \qquad (9)$$

$$F(M) = -T_3 M x^{-1} \qquad (10)$$

where T_3 – is input constant; M- is amplitude of control action. Instead of M the value of M_1 or M_2 is substituted depending on what boundary (upper or lower) B_1 or B_2 is constructed.

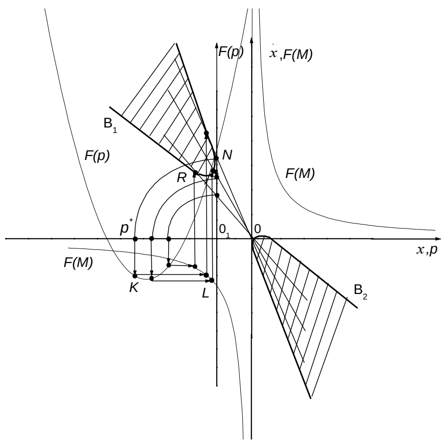

Fig. 3. Construction of the boundary of sliding mode existence region.

Then a point $p^* \in (0, -\infty)$ is taken on axis O_1p. The distance of O_1p^* is marked on an axis $O_1F(p)$ and denoted as a point N. Through the point N the straight line NO is drawn, which coincides with a straight line $g = 0$ (the boundary equation). Then from a point p^* the vertical straight line is conducted up to crossing with polynomial graph (9) in a point K. From point K the horizontal straight line is conducted up to crossing with hyperbolic curve (10) in a point L. The vertical straight line which has been drawn from a point L, crossers the straight line ON in point R, belonging to boundary of area of existence of the sliding mode. Taking various points of p^* and making for them similar constructions, a number of points R is obtained. The curve obtained by connection of points R is the boundary of SM existence region, constructed in system of coordinates $xO\dot{x}$.

The shaded regions on Fig. (3) represent the sliding mode existence area, and B_1, B_2 are the boundaries of these regions.

3.3 The SM observer and anticipatory device.

The problem of identifying the state vector is solved based on data provide by the observer. The state vector of the equivalent circuit is not given. The given data include only certain dimensions of the vector. The problem of identifying the state vector is solved based on data providing by the observer (11).

$$\frac{dy}{dt} = Ay + Bx - L(z - Ky) \qquad (11)$$

In (11) A, B, L, K are the known matrixes; y is exit parameter of system; z is the vector composed in the form of linear combination of the state vector components of the system and in fact, is its estimation. In (11) the state vector of the system can be directly measured, and the last member of (11) characterizes the gap between the intended and actual parameters of the state vector of the equivalent circuit. The value of the gap will be the solution of the homogeneous differential equation with any desired distribution of roots of the characteristic equation. We can receive components of matrix L for each case by solving (11) and transforming the differential integrating links. For construction of the block-diagram of the observer the differential equation of object will be transformed to the convenient form. Let us rewrite the equation (5) in the following form

$$\ddot{y} = -a_{2T1}\dot{y} - a_{1T1}y + k_{OT1}x, \qquad (12)$$

where

$$a_{1T1} = \frac{1}{T_T T_{TO}};\ a_{2T} = \frac{T_T + T_{TO}}{T_T T_{TO}};\ k_{OT1} = \frac{k_{T1}k_{T2}}{T_T T_{TO}}$$

The block-diagram of the observer is built according to the expressions obtained for the observer diagram.

The technique of anticipatory compensation is based upon the preliminary supply of input signal with regard to the duration of delay. Considering the delay, the problem of controlling a multi-level equivalent circuit can be presented as follows, see (13):

$$\frac{dy}{dt} = Ay + Bx(t - \tau)\ , \qquad (13)$$

where τ is delay.

The solution on the equation (13) is:

$$y(t) = \exp(At)y_0(t) + \int_0^t \exp\{A(t-\gamma)\}Bx(\gamma - t)d\gamma \quad (14)$$

The integral in equation (14) is replaced by the sum and γ is replaced by $z+t$ yielding the expression (15):

$$y(t+\tau) = \exp(A\tau)\{y(t) + B\sum_{i=0}^{N} \exp(-Az_i)x(z_i + t - \tau)\frac{\tau}{N}\Delta z\} \quad (15)$$

The equation (15) yields a diagram for delay compensation.

The designed SM controller algorithm for temperature control of the oven with a cell inside is shown in Fig.4.

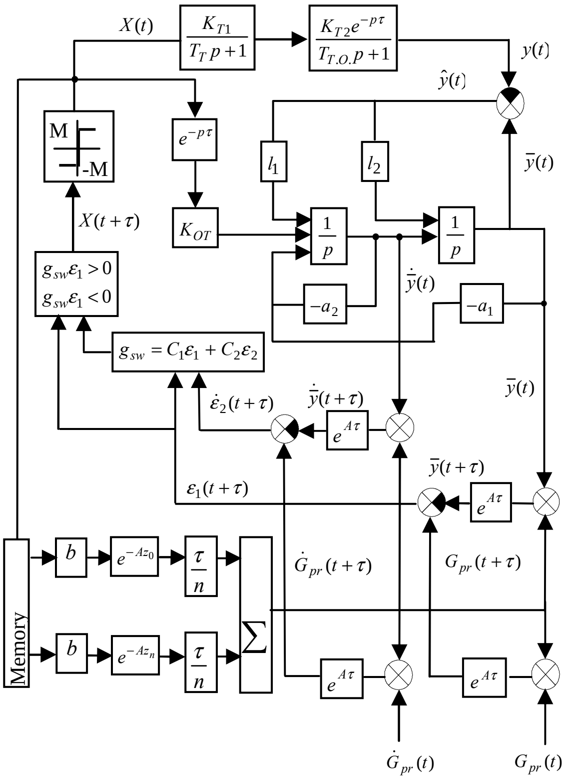

Fig.4. Sliding Mode Controller algorithm for temperature control of oven with a cell inside.

The algorithm includes the following blocks: the block for acquiring data from the thermal point block; the programmer block (set points); the block responsible for knowledge of conditions of occurrence of the SM, the so-called indicator; the observer block, which identifies the state vector of the system; block of anticipatory compensation (Mkrttchian, 1999).

The designed controller operates as follows.
The observer forms function $\bar{y}(t)$ (estimation of the state vector of the system) and its derivatives $\bar{y}(t), \dot{\bar{y}}(t)$, which pass on the block of anticipatory compensation. In that block the functions $\bar{y}(t), \dot{\bar{y}}(t)$ are compensated for value of time delay τ and compared with the task of process control and its derivatives. After comparison the error $\varepsilon(t+\tau)$ and its derivatives are passed on the switching function g_{sw} forming block. The indicator block compares two compensated signals $X(t+\tau)$ and $g_{sw}(t+\tau)$ recording the moment of changing the signs of functions X and g_{sw}. If signs of functions are coincided (Sliding Mode does not occur in the equivalent circuit) then TRUE signal is formed and passed on the digital output of the PC-1200 hardware and then through the op-amplifier to the logic switching block (thyristor). If signs of functions are different, (Sliding Mode occur in the equivalent circuit), then FALSE signal is formed and passed on the digital output of the PC-1200 hardware. Vector $\hat{y}$ characterizes the mismatch between the measured state vector of the system $y\ (t)$ and its estimation, $\bar{y}(t)$. The proper choice of components of matrix L allows the estimation error to be reduced.

4. EXPERIMENTAL RESULTS

The TCS includes: oven with heater and thermocouple; an amplifier that has sufficient gain (k=100) so as to provide the range of voltage needed; the PC hardware; voltage-controlled power supply for the oven heater. Software support is provided by the designed controller program written in the LabView programming language environment. The DAQ board measures output from the operational amplifier. The voltage from the PC hardware is the control signal that will turn ON or OFF the heating element to keep temperature set programmatically and maintain the Sliding Mode. The model arrangement including the oven used in real experiment was allowed to achieve stability of ~0.5 ^{0}C.

The results of applying the designed controller to the thermal control system are shown in Fig.5 and present the graph of the optimally controlled state (temperature).

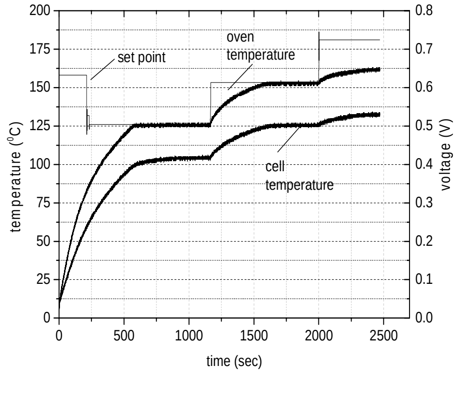

Fig.5. The result of applying the designed controller to the thermal control system.

5. CONCLUSIONS

- This paper formulates the TCS problem in laser using the methods of sliding mode control. The Sliding Mode Controller is designed based on results obtained in the sphere of discontinuous system design. The controller consists of the following blocks: the sliding mode indicator determining the occurrence of SM; the observer block, identifying the state vector of the system, and the anticipatory compensation block.
- The experimental results of applying the designed controller have illustrated the improvement of the achieved performance applied to the spectroscopic system equipped with a thermal system composed of a heating source.

REFERENCES

Basin, M.V., J.G. Rodriguez Gonzalez, L.M. Fridman (2004). Robust integral sliding mode regulator with delayed control input. *Proceedings of the III International Conference on System Identification and Control Problems,* 944-952.

Chevrollier, M., M. Fichet, M. Oria, G. Rahmat, D. Bloch, M. Ducloy (1992). High resolution selective reflection spectroscopy as a probe of long-range surface interaction. *J.Phys.II France* 2, 631-657.

Dubrovsky, E.N. and A.V. Kortnev (1968). Adaptive variable structure systems of automatic control with bounded control action. In: *Variable structure systems and their application in problems of flight automation* (B.N. Petrov and S.V. Emelyanov. (Ed.)), 34-45. Nauka, Moscow.

Knappe, S., J. Kitching, L. Hollberg, R. Wynands (2002). Temperature dependence of coherent population trapping resonances. *Appl. Phys. B* 74, 217-222.

Khachaturova A.A. and Mkrttchian V.S. (2003). Application of control algorithm in a Sliding Mode in spectrum measurement systems and growing of crystals. *Proceedings of the International Conference Physics and Control,* 901-905.

Miles, R.B. and S.E.Harris (1973). Optical Third-Harmonic Generation in Alkali Metal Vapors. *IEEE Journal of Quantum Electronics*, Vol. QE-9, No.4, 470-484.

Mkrttchian, V.S., A. Hovakimyan, A. Hunanyan, T. Kchachaturyan (1999). Special-Purpose Devices Using Techniques of Discontinuous Control and Setting Adjustment (DC & SA) in Control Applications. *Proceedings of the 1999 IEEE International Conference on Control Applications,* 1400-1405

Mkrttchian, V.S. (1999). Extension Theory of Sliding Mode Control for Nonlinear Systems. *Preprints of International Conference Nonlinear Sciences on the Border of Millenniums*. IFMO Press, Saint–Petersburg, 70-73.

Mkrttchian, V.S. and A.A. Khachaturova (2003). Applied LabView Programming for a Temperature Control System Design Modeling. *Proceedings of the International Conference on Computer, Communication and Control Technologies,* 6-12.

Mkrttchian, V.S. and A.A. Khachaturova (2004). Sliding Mode Thermal Control System for laser spectroscopy of alkali metal vapors. *Proceedings of the III International Conference "System Identification and Control Problems*", 975-982.

Scully, M.O. and M. Fleischhauer (1992). High-sensitivity magnetometer based on index-enhanced media. *Phys.Rev.Lett*. 69, 1360-1363.

Taylor, J.B. and I. Langmuir (1937). Vapor pressure of caesium by the positive ion method. *Phys.Rev*. 51, 753.

Utkin, V.A. and A. V. Utkin (2004). Identification linear system via sliding mode technique. *Proceedings of the III International conference on System Identification and Control Problems,* 955-963.

Utkin, V.I. (1982). *Sliding Modes in Problems of Control and Optimization* (in Russian), 1-367. Nauka, Moscow.