# HIERARCHICALLY SUPERVISED OUTPUT REGULATION OF LINEAR PLANTS WITH UNPREDICTABLE PARAMETER STEP CHANGES

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Abstract: A piece-wise time-invariant, linear process is defined allowing it to switch, from time to time, from the actual configuration to another one of a given family of linear, time-invariant plants. When and where switching occurs is not *a priori* known. It is required to find a stabilizing set-point following controller yielding an output step response with some desired specifications. The solution proposed is given by the connection of multiple families of linear controllers with a hierarchically supervised switching scheme. 

Keywords: Supervisory control, switching algorithms, hierarchical control, control system synthesis, stabilizing controllers.

## 1. INTRODUCTION

In this paper we consider the following control problem. A finite family of linear, time-invariant plants is given, the linear process to be controlled is a piece-wise time-invariant plant that, from time to time, suddenly switches among the different possible configurations belonging to the given family. When and where switching occurs is not *a priori* known. It is required to find a stabilizing controller which, for each possible fixed configuration of the process, yield an output step response with some desired time specifications.

Conventional adaptive techniques may not be adequate to cope with this mode-switch control problem because of the too long time interval needed for adaptation (Hilhorst *et al.*, 1994; Narendra *et al.*, 1995). Moreover, even in the case of a fixed linear, time-invariant plant, too strict response specifications can not be adequately satisfied by a single, linear, time invariant controller. For example, a controller producing a step response with a short rise time is like to produce a large overshoot and/or poorly damped oscillations. On the order hand, a smooth behaviour of the step response is often coupled with too long rise and settling times. To this purpose, fuzzy gain scheduling procedures have been recently proposed with reference to a fixed, linear, time-invariant plant (Shamma and Athans, 1990; Shamma and Athans, 1991; Hang and Cao, 1996; Visioli, 1999; Raminez and Lee, 2000; Rugh and Shamma, 2000). The main drawback of this approach is the lack of a tool for a rigorous stability analysis.

The method proposed in this paper situates in the area of supervised switching control (see (Morse, 1995; Narendra *et al.*, 1995; Liberzon, 2003) and references therein). A group of families of linear, time-invariant controllers is defined, each one for each possible configuration of the plant. The switching among the families and inside each family is governed by two supervisors ($S_1$ and $S_2$) operating at two different hierarchical levels. The task of the hierarchically higher supervisor ($S_1$) is to recognize if the currently acting controller family is appropriate for the actual configuration of the plant. This is accomplished by means of a controller falsification procedure detecting closed-loop instability. Moreover $S_1$ drives the control law towards that corresponding to the actual operating conditions.

The hierarchically lower supervisor $S_2$ governs the switching inside each controller family to improve the output step response features obtainable with a single linear, time-invariant controller. The switching logic is based on the real time comparison of some real positive definite cost functionals of the output prediction errors.

The present method is based on the hypothesis that the behaviour of the external reference over a future, bounded time-interval is available and that the time intervals between two consecutive changes of the plant modes and between two consecutive step changes of the set point are long enough.

The paper is organized in the following way. Section 2 states the control problem, the hierarchical organization of the supervised switching control policy is illustrated in section 3, conditions for closed-loop stability are stated in section 4. Numerical results and concluding remarks are given in sections 5 and 6 respectively.

## 2. PROBLEM STATEMENT

Consider the family of linear, time-invariant, discrete-time, reachable and observable s.i.s.o. plants $\mathcal{P} = \{P_1, \cdots, P_N\}$, the linear, piece-wise time-varying plant $\Sigma$ to be controlled switches among the elements of $\mathcal{P}$ at some unpredictable time instants $t_k, k = 1, 2 \cdots$. The initial configuration of $\Sigma$ at $t_0 = 0$ is known, while the new configuration assumed by $\Sigma$ at $t = t_k$, is not known *a priori*. The external reference $r(\cdot)$ is assumed to be a piece-wise constant signal generated as the unforced output response of a s.i.s.o, switched system denoted by $\Sigma_R$. At some time instants $t^{(\ell)}, \ell = 1, 2, \cdots$, $r(\cdot)$ may exhibit jump discontinuities of arbitrary amplitude. Inside each interval $[t^{(\ell-1)}, t^{(\ell)})$, the switched signal generator $\Sigma_R$ behaves like a time-invariant system which is denoted by $\Sigma_R^{(\ell)}$.

It is required to find an output feedback controller $C$, stabilizing $\Sigma$ and yielding an output response $y(\cdot)$ of the closed loop system $\Sigma_C$ forced by $r(\cdot)$ with a little overshoot, well damped oscillations and a fast transient extinguishment.

The time intervals $[t_{k-1}, t_k) \triangleq T_k$ and $[t^{(\ell-1)}, t^{(\ell)}) \triangleq T^{(\ell)}$ between two switching instants of $\Sigma$ and $\Sigma_R$ respectively are assumed to be long enough to allow the definition of a suitable dwell-time, as it will be specified later.

To meet the requirements on the transient output response, $N$ families $\mathcal{C}_i, i = 1, \cdots, N$, of linear, time-invariant, stabilizing regulators $G_i^{(j)}, j = 1, \cdots, N_i$, have been designed, each one for each possible configuration $P_i$ of $\Sigma$. The purpose is to define, for each fixed $P_i$, an overall time-varying control strategy picking up the best features of each single $G_i^{(j)}$.

## 3. THE HIERARCHICAL SWITCHING SCHEME

The time-invariant feedback connection of any fixed element $G_i^{(j)} \in \mathcal{C}_i$ with the corresponding $P_i$ is denoted by $\Sigma_i^{(j)}$ and its output is denoted by $y_i^{(j)}(\cdot)$. The time-varying feedback connection of a fixed $P_i \in \mathcal{P}$ with the time-varying controller generated by the switching inside $C_i$ is denoted by $\Sigma_i$. The time-invariant error system $E_i^{(j)}$ is given by the series connection of the time-invariant external reference generator $\Sigma_R^{(\ell)}$ with $\Sigma_i^{(j)}$, its unforced output response originating from an arbitrary, initial internal state of $E_i^{(j)}$ is denoted by $e_i^{(j)}(\cdot)$ and is given by $e_i^{(j)}(\cdot) = r(\cdot) - y_i^{(j)}(\cdot)$. Without any loss of generality it is assumed that no controller $G_i^{(j)}, j = 1, \cdots, N_i$, stabilizes $P_\ell$, for $i \neq \ell$.

For any fixed configuration $P_i$ of the plant $\Sigma$, the hierarchically lower supervisor $S_2$ governs the switching among the elements of the corresponding stabilizing $\mathcal{C}_i$ according to the following logic. At time $t = 0$, an "*a priori*" chosen $G_i^{(\bar{j})}$ is connected to $P_i$. At the same time, and before the next output $y_i^{(\bar{j})}(t+1)$ of $\Sigma_i^{(\bar{j})}$ is acquired, the output prediction errors $e_i^{(j)}(t+k), k = 0, \cdots, L$, of systems $\Sigma_i^{(j)}, j = i, \cdots, N_i$, are computed. Each $e_i^{(j)}(t+k)$, $j = 1, \cdots, N_i$, is computed setting the corresponding $E_i^{(j)}$ in the same initial conditions of $E_i^{(\bar{j})}$ at $t = 0$. At each time instant $t$, the procedure of prediction computation is repeated setting each $E_i^{(j)}$, in the same conditions of $E_i^{(l)}$, where suffix $l$ corresponds to the controller $G_i^{(l)} \in \mathcal{C}_i$ connected to $P_i$ at the same time instant. The idea is to exploit the output prediction errors to foresee the future performances in the case the actual $G_i^{(l)}$ be changed and in the case it be kept acting. To this purpose the following cost functionals are defined for $j = 1, \cdots, N_i$,

$$J_i^{(j)}(t) = \sum_{k=1}^{L} [{e_i^{(j)}}^2(t+k)(1+p(k))] +$$

$$+ p_1 \sum_{k=1}^{L_1} \max_{(t+k)\in I_m} \left| e_i^{(j)}(t+k) \right| . \qquad (1)$$

Parameter $p(k)$ is a penalty term whose initial value is zero and is increased by one whenever a sign change of $e_i^{(j)}(t+k)$ is observed, the second term of functional (1) is like an infinity norm which penalizes the maximum absolute error, parameter $L_1$ is the number of oscillations observed in the prediction horizon $L$, $I_m$ is the number of error samples belonging to the m-th oscillation, parameter $p_1$ is a free design positive, penalty term.

At each time instant $t$, supervisor $S_2$ switches the actual $G_i^{(j)}$ towards the controller producing the minimum index $J_i^{(j)}(t)$.

For each interval $T_k, k = 1, 2, \cdots$, supervisor $S_2$ starts governing the switching inside the proper family $\mathcal{C}_i$ $L$ time instants before a predicted discontinuity at $t = t^{(\ell)}$ of $r(\cdot)$ and stops after the switching time interval $T_i^{(s)}$ related to $P_i$ has elapsed. The length of $T_i^{(s)}$ is chosen as $L + T_i'^{(s)}$, where $T_i'^{(s)}$ is the estimated maximum time interval for the output $y_i^{(j)}(\cdot)$ of $\Sigma_i^{(j)} j = 1, \cdots, N_i$, to settle around a prescribed percentage of the steady-state after each $t^{(\ell)}$. Each switching interval $T_i^{(s)} \in T_k$ is followed by a dwell-time $D_{k,\ell}$, where suffixes $k$ and $\ell$ mean that $D_{k,\ell}$ is the $\ell - th$ dwell-time occurring inside $T_k$. The minimal length of each $D_{k,\ell}$ is related to the particular plant configuration $P_i$, and is denoted by $\tilde{D}_i$, it will be computed in section 4 on the basis of stability conditions. During each $\tilde{D}_i$ neither abrupt changes of $r(\cdot)$ nor transitions from $P_i$ to $P_j$ $(i \neq j)$ are allowed. The difference $D_{k,\ell} - \tilde{D}_i$ is the (possibly null) time interval where switching is not forbidden but does not take place because neither plant transitions occur nor reference changes are expected within the next $L$ steps.

The two tasks of the hierarchically higher supervisor $S_1$ are to detect the switching instants from $P_i$ to $P_j$ and to drive the control law towards the appropriate family $\mathcal{C}_j$.

By assumption, the closed loop system $\Sigma_C$ behaves like a linear, time-invariant one inside each $D_{k,\ell}$ , $\ell = 1, 2, \cdots$. This allows $S_1$ to accomplish its task of detecting the switching instants, by means of the following controller falsification procedure.

Let $G_i^{(j)}$ and $E_i^{(j)}$ be the controller and the error system respectively, during the dwell-time $\tilde{D}_i \subseteq D_{k,\ell}$, let $\bar{t}_{k,\ell}$ be the first time instant of $\tilde{D}_i$, let $\tilde{t}_{k,\ell}$ be the first time instant after the minimal interval $\tilde{D}_i$ has elapsed, and finally let $\ell_i^{(j)}$ be the state dimension of $E_i^{(j)}$. As $G_i^{(j)}$ stabilizes $P_i$, $E_i^{(j)}$ is state-output stable, hence, $\forall \bar{\sigma} \in (0,1)$ there exists a minimum time interval $\tau_i^{(j)}(\bar{\sigma})$ such that $\forall \tau \geq \max\left(\ell_i^{(j)}, \tau_i^{(j)}(\bar{\sigma})\right)$ the functional

$$V(t, m\tau) = \sum_{h=t-m\tau}^{t-(m-1)\tau} (e_i^{(j)}(h))^2, \quad t \geq \bar{t}_{k,\ell} + m\tau,$$

$m = 1, 2, \cdots$, is monotonically converging to zero as

$$V(t, \tau) < \bar{\sigma} V(t, 2\tau), \quad t \geq \bar{t}_{k,\ell} + 2\tau, t \to \infty, \quad (2)$$

(Corradini and Jetto, 2000). Supervisor $S_1$ starts computing $e_i^{(j)}(h)$ when $t = \bar{t}_{k,\ell}$ and the functional $V(t,\tau)$ when $t \geq \bar{t}_{k,\ell} + 2\tau$, while starts checking inequality (2) as soon as plant transitions are allowed, namely at the first time instant $t = \tilde{t}_{k,\ell}$ after $\tilde{D}_i$. This in turn implies that $\tilde{D}_i$ must satisfy the condition

$$\tilde{D}_i \geq 2 \max_j \left\{ \max\left(\ell_i^{(j)}, \tau_i^{(j)}(\bar{\sigma})\right) \right\} \triangleq \eta_i(\bar{\sigma}) . \quad (3)$$

If for some $\hat{t}_k \geq \tilde{t}_{k,\ell}$ inequality (2) is not satisfied, this means that $G_i^{(j)}$ is not anymore a stabilizing controller for the plant, so that a switching from $P_i$ to $P_\ell$ , $\ell \neq i$, has surely occurred for some $t_k \leq \hat{t}_k$.

At $t = \hat{t}_k$ supervisor $S_1$ stops checking inequality (2) and starts the identification of the new configuration $P_\ell$ computing the following functionals

$$F_l(\hat{t}_k) = \sum_{k=\hat{t}_k}^{\hat{t}_k + L_2} |y_l(k) - y(k)|, l = 1, \cdots, N, \, l \neq i,$$

where $y_l(\cdot)$ and $y(\cdot)$ are the outputs produced by each model $P_l, l = 1, \cdots, N, \, l \neq i$ and by the new unknown configuration of $\Sigma$ respectively, forced by the same control input produced by the currently acting controller $G_i^{(j)} \in \mathcal{C}_i$. Switching is performed towards the controller family corresponding to the model producing the minimum $F_l$. The duration $L_2$ of the identification interval is the maximum number of coefficients of the transfer function of each $P_i, i = 1, \cdots, N$.

The supervisory action of $S_1$ stops at time $\hat{t}_k + L_2$, otherwise, if $S_1$ does not detect any plant transition, it stops its supervisory action $L$ time instants before the next step-change of $r(\cdot)$. In both cases, the last task of $S_1$ is to drive the switched controller $C$ towards an *a priori* chosen element $G_i^{(j)}$ of the appropriate family $C_i$.

## 4. STABILITY ANALYSIS

Sufficient conditions for the internal stability of the switched closed loop system $\Sigma_C$ are derived imposing, for each $P_i \in \mathcal{P}$, a minimum length $\tilde{D}_i$ to the corresponding dwell-times intervals $D_{k,\ell}$.

It is convenient to subdivide the time axis into intervals $[t'_{k-1}, t'_k) \triangleq T'_k$ with

$$t'_k - t'_{k-1} - 1 = T_k^{(0)} + \sum_{\ell} T''_{k,\ell} + L'_{2,k} \; , \qquad (4)$$

where: $t'_{k-1}$ is the time instant in which supervisor $S_1$ switches controller $C$ towards family $C_i$ corresponding to the plant configuration $P_i$ identified at the end of the previous interval $T'_{k-1}$, $T_k^{(0)}$ is the possibly null initial time interval of $T'_k$ in which switching inside $C_i$ is stopped because no abrupt change of the external reference $r(\cdot)$ occurs (clearly $T_0^{(0)} = 0$ because the first step change of $r(\cdot)$ occurs in $t'_0 = 0$), $T''_{k,\ell}$ is given by $T''_{k,\ell} = T_i^{(s)} + D_{k,\ell}$ , $L'_{2,k}$ is given by $L'_{2,k} = L_2 + \hat{t}_k - t_k$. During interval $L'_{2,k}$ the closed-loop system is frozen on an unstable configuration.

Denote by $A_{i,j}$ the dynamic matrix of $\Sigma_i^{(j)}$ and by $\Phi_i(\cdot,\cdot)$ the state transition matrix of $\Sigma_i$. The minimal length $\tilde{D}_i$ of $D_{k,\ell}$, $i = 1, \cdots, N$, is derived imposing the condition

$$\left\| \Phi_i \left( t + T_i^{(s)} + \tilde{D}_i, t \right) \right\| < 1, i = 1, \cdots, N. \qquad (5)$$

Let $\left( \max_j \|A_{i,j}\| \right)^{T_i^{(s)}} \triangleq M_i$ and let $A_{i,n}$ denote the (stable) *a priori* unknown dynamic matrix of the configuration $\Sigma_i^{(n)}$ where $\Sigma_i$ remains frozen at the end of a generic switching interval, one has

$$\begin{aligned} &\left\| \Phi_i \left( t + T_i^{(s)} + \tilde{D}_i, t \right) \right\| \leq \\ &\leq \left\| A_{i,n}^{\tilde{D}_i} \right\| \cdot \left\| \Phi \left( t + T_i^{(s)}, t \right) \right\| \leq \left\| A_{i,n}^{\tilde{D}_i} \right\| \cdot M_i. \end{aligned}$$

The norm $\left\| A_{i,n}^t \right\|$ is converging to zero for $t$ going to infinite, hence there exists a time instant $t_i^{(n)}$ such that $\left\| A_{i,n}^t \right\| \cdot M_i < 1, \; \forall t \geq t_i^{(n)}$. As $A_{i,n}$ is not known *a priori*, the minimal length $\tilde{D}_i$ of the dwell-time must be computed on the basis of $\max_n \left\{ t_i^{(n)} \right\} \triangleq \theta_i$. Recalling (3), one has:

$$\tilde{D}_i = \max \left\{ \eta_i(\bar{\sigma}), \theta_i \right\}. \qquad (6)$$

Condition (6) states an off-line computable lower bound for the dwell-times $D_{k,\ell}$ related to $\Sigma = P_i, i = 1, \cdots, N$. Nevertheless, the above condition only holds for the dwell-time $D_{k,\ell}$ with $\ell = 2, 3, \cdots$ The minimum length $\tilde{D}_i$ of $D_{k,1}$ has to be computed differently because the time interval $T''_{k,1} = T_i^{(s)} + D_{k,1}$ is preceded by $T_k^{(0)}$ and by the interval $L'_{2,k-1} \in T'_{k-1}$ over which the closed-loop system is frozen on an unstable configuration. As the value of $T_k^{(0)}$ is not *"a priori"* known, the minimal length $\tilde{D}_i$ of $D_{k,1}$ can be computed only after interval $T_k^{(0)}$ has elapsed. Assume that, at the switching instant $t_{k-1} \in T'_{k-1}$, plant $\Sigma$ switches from $P_\ell$ to $P_i$, while the controller $C$ remains frozen on an element $G_\ell^{(m)} \in C_\ell$ until $P_i$ has been identified.

During $L'_{2,k-1} = L_2 + \hat{t}_{k-1} - t_{k-1}$, the closed-loop system $\Sigma_C$ is frozen on the unstable configuration, given by the feedback connection of $G_\ell^{(m)}$ with $P_i$. Let $A_{i,\ell,m}$ denote the dynamic matrix of $\Sigma_C$ during $L'_{2,k-1}$, let $A_{i,j}$ be the stable dynamic matrix of $\Sigma_C \equiv \Sigma_i^{(j)}$ during $T_k^{(0)} \in T_k$ and let $\tilde{t}_{k-1,\bar{\ell}} \in T'_{k-1}$, the first time instant after the minimum length $\tilde{D}_i$ of the last $D_{k-1,\bar{\ell}} \subset T'_{k-1}$ has elapsed.

As $t_{k-1}$ is unknown, the minimal length $\tilde{D}_i$ of $D_{k,1}$ must be computed allowing $t_{k-1}$ vary all over the permitted interval $[\tilde{t}_{k-1,\ell}, \hat{t}_{k-1}]$.

Let $\max_{0 \leq t \leq \hat{t}_{k-1} - \tilde{t}_{k-1,\ell}} \left\| A_{i,\ell,m}^{(L_2+t)} \right\| \triangleq K_{i,\ell,m}$ and let $t_i'^{(n)}$ be such that $K_{i,\ell,m} \cdot \left\| A_{i,j}^{T_k^{(0)}} \right\| \cdot M_i \cdot \left\| A_{i,n}^t \right\| < 1, \quad \forall t \geq t_i'^{(n)}$. By arguing as in equation (6) and defining $\max_n \left\{ t_i'^{(n)} \right\} \triangleq \theta'_i$, one has that the minimal length $\tilde{D}_i$ of $D_{k,1}$ must be computed as

$$\tilde{D}_i = \max \left\{ \eta_i(\bar{\sigma}), \theta'_i \right\}. \qquad (7)$$

## 5. NUMERICAL RESULTS

A family $\mathcal{P} = (P_1, P_2, P_3)$ has been obtained discretizing three continuous time plants with a sampling period $T_s = 0.1s$. The transfer function of each $P_i$ is of the kind $P(d) = (b_1 d + b_0 d^2)/(1 + a_1 d + a_0 d^2)$. The values of the coefficients of $P(d)$ are reported in Table 1. For each $P_i$, a family $\mathcal{C}_i$ of $N_i = 3$ controllers $G_i^{(j)}$ has been designed by the pole placement technique. The transfer function of each $G_i^{(j)}$ is of the kind $G(d) = (q_2 + q_1 d + q_0 d^2)/(1 + p_1 d + p_0 d^2)$. To each $G_i^{(j)}$ a different set point response speed of the respective $\Sigma_i^{(j)}$ corresponds: $G_i^{(1)}$ produces a slow response, $G_i^{(3)}$ produces a fast response, $G_i^{(2)}$ produces an intermediate closed-loop dynamics. The values of the coefficients of each $G(d)$ are reported in Table 2. Plant $\Sigma$ is in the configuration $P_1$ in the time interval $T_1 = [0, 70s)$, and in the configuration $P_2$ in $T_2 = [70s, \infty)$. The external reference $r(\cdot)$ is the following piece-wise constant signal: $r(t) = 1, \forall t \in T^{(1)} = [0, 30)$, $r(t) = 1.5, \forall t \in$

$T^{(2)} = [30, 80)$, $r(t) = 3, \forall t \in T^{(3)} = [80, \infty)$. This experimental situation is consistent with the dwell-time $\tilde{D}_i$ computed as explained in section 4. Functional (1) has been implemented choosing $L = 3s$ and $p_1 = 10$. The switching intervals are $T_1^{(s)} = 5s$, $T_2^{(s)} = 6s$, $T_3^{(s)} = 10s$. The switching controller started from $G_1^{(2)}$. The results of the numerical simulation, reported in Figure 1, show the effectiveness of the proposed controller. The spike observed around $t_1 = 70s$ is due to the plant transition. Details of two parts of the simulations are reported in Figure 3 and 4. Both diagrams and parameters reported in Table 3 evidence the improvement introduced by switching. The switching sequence is shown in Figure 2.

Table 1 Parameters of the process family

| | $b_1$ | $b_0$ | $a_1$ | $a_0$ |
|---|---|---|---|---|
| $P_1$ | 0.0057 | 0.0056 | −1.9312 | 0.9418 |
| $P_2$ | 0.0001 | 0.0001 | −1.9800 | 0.9802 |
| $P_3$ | 0.0206 | 0.0169 | −1.5114 | 0.5488 |

Table 2 Parameters of the controller families

| | $q_2$ | $q_1$ | $q_0$ | $p_1$ | $p_0$ |
|---|---|---|---|---|---|
| $G_1^{(1)}$ | 71.79 | −135.64 | 64.00 | −0.64 | −0.361 |
| $G_1^{(2)}$ | 73.03 | −137.18 | 64.47 | −0.63 | −0.364 |
| $G_1^{(3)}$ | 84.82 | −153.82 | 69.64 | −0.60 | −0.397 |
| $G_2^{(1)}$ | 11.98 | −23.68 | 11.70 | −1.91 | 0.911 |
| $G_2^{(2)}$ | 102.9 | −201.64 | 98.74 | −1.75 | 0.750 |
| $G_2^{(3)}$ | 3102 | −5458 | 2416 | −0.9 | −0.1 |
| $G_3^{(1)}$ | 1.670 | −2.52 | 0.91 | −1.52 | 0.523 |
| $G_3^{(2)}$ | 4.140 | −6.11 | 2.18 | −1.32 | 0.324 |
| $G_3^{(3)}$ | 66.15 | −71.53 | 20.83 | −0.36 | −0.639 |

Table 3 Transient response parameters relative to Figure 3

| | **Rise Time** | **Settling Time** | **Overshoot** |
|---|---|---|---|
| **3(*a*)** | 0.2*s* | 1.5*s* | 13.2% |
| **3(*b*)** | 0.1*s* | 4.4*s* | 16.2% |
| **4(*a*)** | 0.3*s* | 0.5*s* | 0.0% |
| **4(*b*)** | 0.5*s* | 2.4*s* | 6.8% |

## 6. CONCLUSIONS

A class of mode-switch processes has been considered and a stabilization control problem including strict specifications on the transient response has been formulated. This problem calls for control algorithms which behave in a less conservative manner than robust or conventional adaptive techniques. A switching control policy driven by a hierarchically organized supervisory policy has been proposed. The resulting time-varying controller is able to quickly adjust the parameters of the control law exploiting the information carried by suitably defined performance indicators. The reported simulation results confirm the validity of the newly developed method.

The free design parameters of the control scheme are the prediction horizon $L$ and the number $N_i$ of controllers $G_i^{(j)} \in C_i, i = 1, \cdots, N$. When choosing them, numerical considerations should be also taken into account. The number $N_o$ of operations for the on-line computations of functionals (1) is $N_o \propto L \cdot N_i$. Hence the choice of $L$ and $N_i$ should also depend on the length of the sampling period and on the available computer. A too large value of $L$ could make the cost functional (1) insensitive to short term differences, while, in theory, the greater $N_i$ is chosen, the better it is. In practice, the value $N_i = 3, i = 1, \cdots, N$, is a reasonable choice. It corresponds to the possibility of assigning a slow, a fast and an intermediate closed-loop dynamics. The switching logic is organized in such a way that the two supervisors never act the same time. Supervisor $S_2$ only acts during the switching periods $T_i^{(s)}$, where $S_1$ is not acting, therefore the hierarchical structure of the control scheme does not represent an additional burden from the

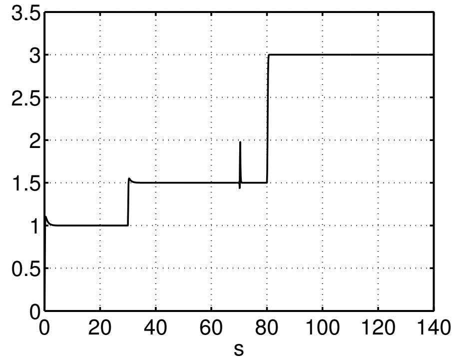



Fig. 1. Output step response of $\Sigma_C$.

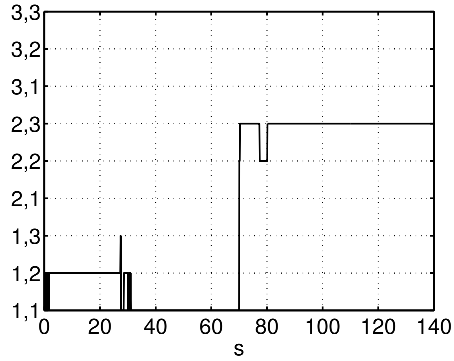
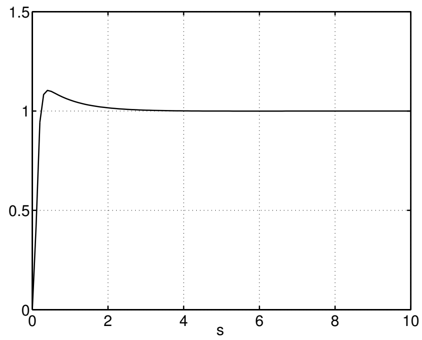
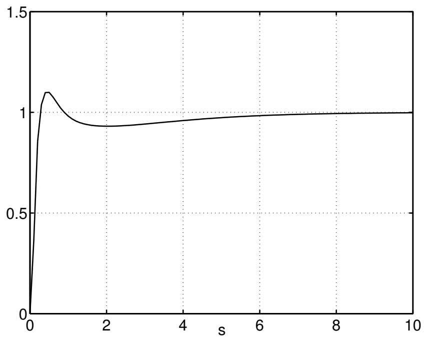



Fig. 2. Switching sequence starting from $G_1^{(2)}$. The pair of integers i,j on the ordinate axis identifies the controller $G_i^{(j)}$.



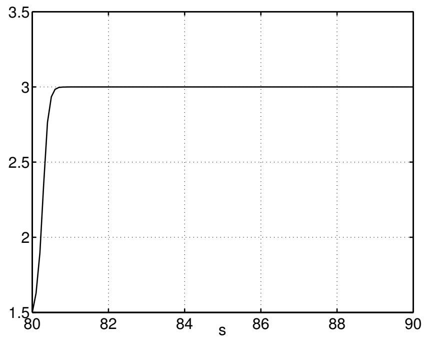

(a)



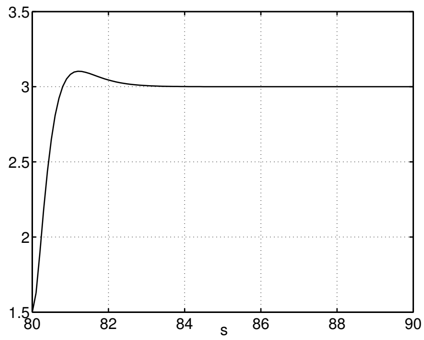

(b)

Fig. 3. Diagram (a): Detail of the output step response of $\Sigma_C \equiv \Sigma_1$ over the first $10s$ of simulation. Diagram (b): Detail of the output step response of the time invariant $\Sigma_1^{(2)}$over the same time interval of Figure 3(a).

computational point of view. Numerical results confirmed the validity of the proposed approach.




(a)




(b)

Fig. 4. Diagram (a): Detail of the output step response of $\Sigma_C \equiv \Sigma_2$. Diagram (b): Detail of the output step response of the time invariant $\Sigma_2^{(2)}$ over the same time interval of Figure 4(a).

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