DESIGN AND PERFORMANCE ANALYSIS OF TRACKING CONTROLLER OF NONLINEAR COMPOSITE SYSTEMS USING NEURAL NETWORKS

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Abstract: A new tracking controller scheme is presented for a class of nonlinear composite system using dynamic neural networks. Lyapunov stability theory is used to guarantee a uniform ultimate boundedness property for the tracking error and all other signals in the closed loop. The controller derived is smooth. In addition, the performance criteria of the mean-square performance are provided to quantify the control performance of proposed method. No a priori knowledge of an upper bound on the "optimal" weights and modelling errors is required. Numerical simulation examples are used to illustrate and clarify the theoretical results.

Keywords: Nonlinear composite systems; Tracking control; Dynamic neural networks; Adaptive control; Performance

1. INTRODUCTION

Tracking is the key issue in control systems performance, in which the state of a given plant is forced to follow a prespecified bounded reference trajectory (Rovithakis, 1999). In the linear systems case, the problem has found a satisfactory solution, even if the system contains parametric and dynamic uncertainties, or even if external disturbances affect its dynamics.

However, analogous results have not been reported when the controlled system is nonlinear composite systems. Due to their massive parallelism, very fast adaptability and inherent approximation capabilities, neural networks have extensively been used mostly as approximation models of unknown nonlinearities. Therefore, the complex systems that includes uncertain and possibly unknown nonlinearities have been dealt with mainly through a neuro-control approach in many literatures (Hunt et al., 1992;Narendra et al., 1990).

The key relationship between neural and adaptive control arises from the fact that neural networks can approximate arbitrarily well the static and the dynamic nonlinear systems. Thus one can substitute an unknown system by a neural network model, which is of known structure but contains a number of unknown parameters (synaptic weights), plus a modelling error term. Thus transforming the original problem into a nonlinear robust adaptive control problem. The bridge to connect the theory with applications in neurocontrol literature was provided by Lyapunov stability theory. A number of interesting works have already appeared in this direction (Chen et al., 1995;Rovithakis et al., 1997;Rovithakis et al., 1994;Lewis et al., 1995).

[1] This work was supported by the National Natural Science Foundation of China under Grant 60274009

Based on dynamic neural networks, tracking control of affine nonlinear systems is discussed in literature (Rovithakis, 1999), however there isn't analogous research results in the nonlinear composite systems. Composite systems consist of some subsystems by inner connections, it has practical application background, for example, composite systems exist in electric power systems, robot systems, computer networks, long-distance communications etc. Thus research on the composite systems has attracted extensive consideration (Yan et al., 1998;Zhang, 2000;Zhang, 2002). Composite systems with matching condition is discussed by static neural networks in literature (Zhang, 2002). On the basis of literature (Rovithakis, 1999), we investigate the problem of tracking control of nonlinear composite systems and performance analysis by dynamic neural networks in this paper. The following definitions of symbols will extensively be used through the paper.

Supposing $x_i = (x_{i1}, x_{i2}, \ldots, x_{in_i})^T \in R^{n_i}$, $|x_i|$ denotes the usual Euclidean norm of a vector; if x_i is a scalar, then $|x_i|$ denotes its absolute value. Suppose $|x_i|_1 = \sum_{j=1}^{n_i} |x_{ij}|$, if A_i is a matrix, then $||A_i||$ denotes the Frobenius norm, defined as $||A_i||^2 = \sum_{i,j} |a_{ij}|^2 = tr\{A_i^T A_i\}$ where $tr\{\cdot\}$ denotes the trace of a matrix.

2. PROBLEM FORMULATION

We consider nonlinear composite systems

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i + h_i(x), \ \ i = 1, 2, \ldots, N \quad (1)$$

where $x_i \in R^{n_i}$ is the states, $u_i \in R^{m_i}$ is the control input, $f_i(x_i)$ is an unknown smooth vector function, $g_i(x_i)$ is an unknown matrix function, $h_i(x)$ is an interconnection term. $f_i(x_i)$, $g_i(x_i)$, $h_i(x)$ are continuous, locally Lipschitz. The control objective is to force the state to follow a given bounded reference trajectory $x_{im} \in R^{n_i}, i = 1, 2, \ldots, N$, x_{im} is generated from an exosystem of the form

$$\dot{x}_i = B_{im}(x_{im}) + h_{im}(x_m) \quad (2)$$

where $x_{im} \in R^{n_i}$ is the states, $h_{im}(x_m)$ is an interconnection term, whose dynamics are assumed to be unknown, but satisfy a locally Lipschiz and a continuity property.

Nonlinear system is described using the dynamic neural networks in literature (Rovithakis, 1999;Rovithakis et al., 1994). According to this, the system (1) is described by the dynamic neural networks as follows.

$$\begin{aligned} \dot{x}_i = & -A_i x_i + W_{i1} S_{i1}(x_i) + W_{i2} S_{i2}(x_i) u_i \\ & + W_{iN} S_{iN}(x) + \omega_i(x_i, u_i, x) \end{aligned} \quad (3)$$

where A_i is a $n_i \times n_i$ matrix with positive eigenvalues, which for simplicity can be taken diagonal. W_{i1}, W_{i2} and W_{iN} are $n_i \times L_{i1}$, $n_i \times L_{i2}$ and $n_i \times L_{iN}$ matrices of adjustable synaptic weights respectively. $S_{i1}(x_i)$ is a $L_{i1}-$dimensional vector, $S_{i2}(x_i)$ is a $L_{i2} \times m_i$ matrix, $S_{iN}(x)$ is a $L_{iN}-$dimensional vector, they are smooth monotone functions which select Sigmoid function; $\omega_i(x_i, u_i, x)$ is the modelling error term, it suffices following assumption.

Assumption 1: There exist appropriately small positive constants ϖ_i such that $|\omega_i(x_i, u_i, x)| \leq \varpi_i$

Following the above mentioned arguments, we can also describe the unknown dynamics of exosystem (2) as

$$\begin{aligned} \dot{x}_{im} = & -A_{im} x_{im} - W_{im} S_{im}(x_{im}) \\ & - W_{ih} S_{ih}(x_m) + \omega_{i0}(x_{im}, x_m) \end{aligned} \quad (4)$$

where A_{im} is a $n_i \times n_i$ matrix with positive eigenvalues. W_{im} and W_{ih} are $n_i \times L_{im}$ and $n_i \times L_{ih}$ matrices of adjustable synaptic weights . $S_{im}(x_{im})$ is a $L_{im}-$dimensional vector, $S_{ih}(x_m)$ is a $L_{ih}-$dimensional vector, they are smooth monotone functions which select Sigmoid function; $\omega_{i0}(x_{im}, x_m)$ is the modelling error term, it suffices following assumption.

Assumption 2: There exist appropriately small positive constants ϖ_{i0} such that $|\omega_{i0}(x_i, x_m)| \leq \varpi_{i0}$

Define the tracking error e_i of the ith subsystem

$$e_i = x_i - x_{im} \quad (5)$$

Differentiating (5) with respect to time. Define $A_{im} = A_i + \Delta A_i$, then we obtain

$$\begin{aligned} \dot{e}_i = & -A_i e_i - \tilde{W}_{i1} S_{i1}(x_i) - \tilde{W}_{i2} S_{i2}(x_i) u_i \\ & - \tilde{W}_{iN} S_{iN}(x) - \tilde{W}_{im} S_{im}(x_{im}) - \tilde{W}_{ih} S_{ih}(x_m) \\ & + \hat{W}_{i1} S_{i1}(x_i) + \hat{W}_{i2} S_{i2}(x_i) u_i + \hat{W}_{iN} S_{iN}(x) \\ & + \hat{W}_{im} S_{im}(x_{im}) + \hat{W}_{ih} S_{ih}(x_m) + \Delta A_i x_{im} \\ & + \omega_i(x_i, u_i, x) - \omega_{i0}(x_{im}, x_m) \end{aligned} \quad (6)$$

where $\hat{W}_{i1}$, $\hat{W}_{i2}$, $\hat{W}_{iN}$, $\hat{W}_{im}$, $\hat{W}_{ih}$ are estimates of the unknown weight values W_{i1}, W_{i2}, W_{iN}, W_{im}, W_{ih}, respectively. The parameter errors $\tilde{W}_{i1}$, $\tilde{W}_{i2}$, $\tilde{W}_{iN}$, $\tilde{W}_{im}$, $\tilde{W}_{ih}$ are defined as $\tilde{W}_{i1} = \hat{W}_{i1} - W_{i1}$, $\tilde{W}_{i2} = \hat{W}_{i2} - W_{i2}$, $\tilde{W}_{iN} = \hat{W}_{iN} - W_{iN}$, $\tilde{W}_{im} = \hat{W}_{im} - W_{im}$, $\tilde{W}_{ih} = \hat{W}_{ih} - W_{ih}$.

From assumption 1, 2 and $x_{im} \in L_\infty$, we obtain

$$\begin{aligned} & |\Delta A_i x_{im} + \omega_i(x_i, u_i, x) - \omega_{i0}(x_{im}, x_m)| \\ \leq & \ |\Delta A_i||x_{im}| + |\omega_i(x_i, u_i, x)| + |\omega_{i0}(x_{im}, x_m)| \\ \leq & \ \varepsilon_i \end{aligned}$$

where ε_i is an unknown bound.

Thus the tracking control problem can be transformed that we will design the controller of state feedback and appropriate update law to guarantee the uniform ultimate boundedness of the tracking error.

3. CONTROLLER DESIGN

Consider the tracking error equation (6), taking the following control laws

$$u_i = u_{i1} + u_{i2} + u_{i3} + u_{i4} \tag{7}$$

$$u_{i1} = \frac{S_{i2}^T(x_i)\hat{W}_{i2}^T\hat{W}_{i1}S_{i1}(x_i)}{\lambda_{i1}M_i} \tag{8}$$

$$u_{i2} = \frac{S_{i2}^T(x_i)\hat{W}_{i2}^T\hat{W}_{iN}S_{iN}(x)}{\lambda_{i2}M_i} \tag{9}$$

$$u_{i3} = \frac{S_{i2}^T(x_i)\hat{W}_{i2}^T\hat{W}_{im}S_{im}(x_{im})}{\lambda_{i3}M_i} \tag{10}$$

$$u_{i4} = \frac{S_{i2}^T(x_i)\hat{W}_{i2}^T\hat{W}_{ih}S_{ih}(x_m)}{\lambda_{i4}M_i} \tag{11}$$

where λ_{i1}, λ_{i2}, λ_{i3}, λ_{i4} are positive design parameters, $M_i = 1 + \|\hat{W}_{i2}\|^2\|S_{i2}(x_i)\|^2$.

Taking the following adaptive laws

$$\dot{\hat{W}}_{i1} = -\gamma_{i1}\hat{W}_{i1} + k_ie_iS_{i1}^T(x_i) \tag{12}$$

$$\dot{\hat{W}}_{i2} = -\gamma_{i2}\hat{W}_{i2} + k_ie_iu_i^TS_{i2}^T(x_i) \tag{13}$$

$$\dot{\hat{W}}_{iN} = -\gamma_{i3}\hat{W}_{iN} + k_ie_iS_{iN}^T(x) \tag{14}$$

$$\dot{\hat{W}}_{im} = -\gamma_{i4}\hat{W}_{im} + k_ie_iS_{im}^T(x_{im}) \tag{15}$$

$$\dot{\hat{W}}_{ih} = -\gamma_{i5}\hat{W}_{ih} + k_ie_iS_{ih}^T(x_m) \tag{16}$$

where k_i, γ_{i1}, γ_{i2}, γ_{i3}, γ_{i4}, γ_{i5} are positive design parameters. So the following theorem 1 can be seen to hold.

Theorem 1: Consider the error equation (6). The control laws (7)-(11) together with the adaptive laws (12)-(16) guarantee the uniform ultimate boundedness of $|e_i|$, $\|\hat{W}_{i1}\|$, $\|\hat{W}_{i2}\|$, $\|\hat{W}_{iN}\|$, $\|\hat{W}_{im}\|$, $\|\hat{W}_{ih}\|$ with respect to the set $\upsilon_i = \{V_i(t) : V_i \le \frac{\mu_i}{c_i}\}$. where

$$c_i = min\{\frac{2(k_{i1} - \frac{1}{4})}{k_i}, \gamma_{i1}, \gamma_{i2}, \gamma_{i3}, \gamma_{i4}, \gamma_{i5}\}$$
$$\mu_i = \varepsilon_i^2k_i^2 + \frac{\gamma_{i1}}{2}\|W_{i1}\|^2 + \frac{\gamma_{i2}}{2}\|W_{i2}\|^2 + \frac{\gamma_{i3}}{2}\|W_{iN}\|^2 + \frac{\gamma_{i4}}{2}\|W_{im}\|^2 + \frac{\gamma_{i5}}{2}\|W_{ih}\|^2$$
$$k_{i1} > \frac{1}{4}$$

Proof: Firstly consider the ith subsystem, take the Lyapunov function candidate as follow:

$$V_i = \frac{k_i}{2}e_i^Te_i + \frac{1}{2}tr\{\tilde{W}_{i1}^T\tilde{W}_{i1}\} + \frac{1}{2}tr\{\tilde{W}_{i2}^T\tilde{W}_{i2}\} + \frac{1}{2}tr\{\tilde{W}_{iN}^T\tilde{W}_{iN}\} + \frac{1}{2}tr\{\tilde{W}_{im}^T\tilde{W}_{im}\} + \frac{1}{2}tr\{\tilde{W}_{ih}^T\tilde{W}_{ih}\}. \tag{17}$$

Differentiating (17) with respect to time, where $A_i = a_iI_i$, substituting (7)–(11) and (12)–(16) into it we get

$$\begin{aligned}\dot{V}_i \le & -k_ia_i|e_i|^2 + k_ie_i^T\hat{W}_{i1}S_{i1}(x_i) + k_ie_i^T\hat{W}_{iN}S_{iN}(x) \\ & + \frac{k_ie_i^T\hat{W}_{i2}S_{i2}(x_i)S_{i2}^T(x_i)\hat{W}_{i2}^T\hat{W}_{i1}S_{i1}(x_i)}{\lambda_{i1}M_i} \\ & + \frac{k_ie_i^T\hat{W}_{i2}S_{i2}(x_i)S_{i2}^T(x_i)\hat{W}_{i2}^T\hat{W}_{iN}S_{iN}(x)}{\lambda_{i2}M_i} \\ & + \frac{k_ie_i^T\hat{W}_{i2}S_{i2}(x_i)S_{i2}^T(x_i)\hat{W}_{i2}^T\hat{W}_{im}S_{im}(x_{im})}{\lambda_{i3}M_i} \\ & + \frac{k_ie_i^T\hat{W}_{i2}S_{i2}(x_i)S_{i2}^T(x_i)\hat{W}_{i2}^T\hat{W}_{ih}S_{ih}(x_m)}{\lambda_{i4}M_i} \\ & + k_ie_i^T\hat{W}_{im}S_{im}(x_{im}) + k_ie_i^T\hat{W}_{ih}S_{ih}(x_m) \\ & + \varepsilon_ik_i|e_i| - \gamma_{i1}tr\{\hat{W}_{i1}^T\tilde{W}_{i1}\} \\ & - \gamma_{i2}tr\{\hat{W}_{i2}^T\tilde{W}_{i2}\} - \gamma_{i3}tr\{\hat{W}_{iN}^T\tilde{W}_{iN}\} \\ & - \gamma_{i4}tr\{\hat{W}_{im}^T\tilde{W}_{im}\} - \gamma_{i5}tr\{\hat{W}_{ih}^T\tilde{W}_{ih}\}\end{aligned} \tag{18}$$

where

$$tr\{\hat{W}_{i1}^T\tilde{W}_{i1}\} = \frac{1}{2}\|\hat{W}_{i1}\|^2 + \frac{1}{2}\|\tilde{W}_{i1}\|^2 - \frac{1}{2}\|W_{i1}\|^2 \tag{19}$$

$$tr\{\hat{W}_{i2}^T\tilde{W}_{i2}\} = \frac{1}{2}\|\hat{W}_{i2}\|^2 + \frac{1}{2}\|\tilde{W}_{i2}\|^2 - \frac{1}{2}\|W_{i2}\|^2 \tag{20}$$

$$tr\{\hat{W}_{iN}^T\tilde{W}_{iN}\} = \frac{1}{2}\|\hat{W}_{iN}\|^2 + \frac{1}{2}\|\tilde{W}_{iN}\|^2 - \frac{1}{2}\|W_{iN}\|^2 \tag{21}$$

$$tr\{\hat{W}_{im}^T\tilde{W}_{im}\} = \frac{1}{2}\|\hat{W}_{im}\|^2 + \frac{1}{2}\|\tilde{W}_{im}\|^2 - \frac{1}{2}\|W_{im}\|^2 \tag{22}$$

$$tr\{\hat{W}_{ih}^T\tilde{W}_{ih}\} = \frac{1}{2}\|\hat{W}_{ih}\|^2 + \frac{1}{2}\|\tilde{W}_{ih}\|^2 - \frac{1}{2}\|W_{ih}\|^2 \tag{23}$$

substituting (19)-(23) into (18) we obtain

$$
\begin{aligned}
\dot{V}_i \leq & -k_i a_i |e_i|^2 + k_i(1+\frac{1}{\lambda_{i1}})|e_i|\|\hat{W}_{i1}\||S_{i1}(x_i)| \\
& + k_i(1+\frac{1}{\lambda_{i2}})|e_i|\|\hat{W}_{iN}\||S_{iN}(x)| \\
& + k_i(1+\frac{1}{\lambda_{i3}})|e_i|\|\hat{W}_{im}\||S_{im}(x_{im})| \\
& + k_i(1+\frac{1}{\lambda_{i2}})|e_i|\|\hat{W}_{ih}\||S_{ih}(x_m)| + \varepsilon_i k_i |e_i| \\
& - \frac{\gamma_{i1}}{2}\|\tilde{W}_{i1}\|^2 - \frac{\gamma_{i1}}{2}\|\hat{W}_{i1}\|^2 + \frac{\gamma_{i1}}{2}\|W_{i1}\|^2 \\
& - \frac{\gamma_{i2}}{2}\|\tilde{W}_{i2}\|^2 - \frac{\gamma_{i2}}{2}\|\hat{W}_{i2}\|^2 + \frac{\gamma_{i2}}{2}\|W_{i2}\|^2 \\
& - \frac{\gamma_{i3}}{2}\|\tilde{W}_{iN}\|^2 - \frac{\gamma_{i3}}{2}\|\hat{W}_{iN}\|^2 + \frac{\gamma_{i3}}{2}\|W_{iN}\|^2 \\
& - \frac{\gamma_{i4}}{2}\|\tilde{W}_{im}\|^2 - \frac{\gamma_{i4}}{2}\|\hat{W}_{im}\|^2 + \frac{\gamma_{i4}}{2}\|W_{im}\|^2 \\
& - \frac{\gamma_{i5}}{2}\|\tilde{W}_{ih}\|^2 - \frac{\gamma_{i5}}{2}\|\hat{W}_{ih}\|^2 + \frac{\gamma_{i5}}{2}\|W_{ih}\|^2 \quad (24)
\end{aligned}
$$

Select $k_i = \frac{k_{i1}+k_{i2}+k_{i3}+k_{i4}+k_{i5}}{a_i}$, $k_{i1}, k_{i2}, k_{i3}, k_{i4}, k_{i5} > 0$. According to definition of $S_{i1}(x_i)$, $S_{iN}(x)$, $S_{im}(x_{im})$, $S_{ih}(x_m)$. We get $|S_{i1}(x_i)| \leq s_{i1}$, $|S_{iN}(x)| \leq s_{iN}$, $|S_{im}(x_{im})| \leq s_{im}$, $|S_{ih}(x_m)| \leq s_{ih}$. where $s_{i1}, s_{iN}, s_{im}, s_{ih}$ are known positive constant. Choose design parameter λ_{i1}, λ_{i2}, λ_{i3}, λ_{i4}. $\lambda_{i1} \geq \frac{k_i s_{i1}}{\sqrt{2k_{i2}\gamma_{i1} - k_i s_{i1}}}$, $\lambda_{i2} \geq \frac{k_i s_{iN}}{\sqrt{2k_{i3}\gamma_{i3} - k_i s_{iN}}}$, $\lambda_{i3} \geq \frac{k_i s_{im}}{\sqrt{2k_{i4}\gamma_{i4} - k_i s_{im}}}$, $\lambda_{i4} \geq \frac{k_i s_{ih}}{\sqrt{2k_{i5}\gamma_{i5} - k_i s_{ih}}}$

then (24) becomes

$$
\begin{aligned}
\dot{V}_i = & -(k_{i1} - \frac{1}{4})|e_i|^2 - \frac{\gamma_{i1}}{2}\|\tilde{W}_{i1}\|^2 - \frac{\gamma_{i2}}{2}\|\tilde{W}_{i2}\|^2 \\
& - \frac{\gamma_{i3}}{2}\|\tilde{W}_{iN}\|^2 - \frac{\gamma_{i4}}{2}\|\tilde{W}_{im}\|^2 - \frac{\gamma_{i5}}{2}\|\tilde{W}_{ih}\|^2 \\
& + \varepsilon_i^2 k_i^2 + \frac{\gamma_{i1}}{2}\|W_{i1}\|^2 + \frac{\gamma_{i2}}{2}\|W_{i2}\|^2 \\
& + \frac{\gamma_{i3}}{2}\|W_{iN}\|^2 + \frac{\gamma_{i4}}{2}\|W_{im}\|^2 + \frac{\gamma_{i5}}{2}\|W_{ih}\|^2 \quad (25)
\end{aligned}
$$

Let

$$
c_i = min\{\frac{2(k_{i1} - \frac{1}{4})}{k_i}, \gamma_{i1}, \gamma_{i2}, \gamma_{i3}, \gamma_{i4}, \gamma_{i5}\} \quad (26)
$$

(25) becomes

$$
\dot{V}_i \leq -c_i V_i + \mu_i \quad (27)
$$

where

$$
\begin{aligned}
\mu_i = & \varepsilon_i^2 k_i^2 + \frac{\gamma_{i1}}{2}\|W_{i1}\|^2 + \frac{\gamma_{i2}}{2}\|W_{i2}\|^2 \\
& + \frac{\gamma_{i3}}{2}\|W_{iN}\|^2 + \frac{\gamma_{i4}}{2}\|W_{im}\|^2 + \frac{\gamma_{i5}}{2}\|W_{ih}\|^2 \\
k_{i1} > & \frac{1}{4}
\end{aligned}
$$

choosing $V_i > \frac{\mu_i}{c_i}$, then

$$
\dot{V}_i < 0 \quad (28)
$$

Thus, we can prove the uniform ultimate boundedness of V_i with respect to the set $\upsilon_i = \{V_i(t) : V_i \leq \frac{\mu_i}{c_i}\}$.

If V_i is outside υ_i then $\dot{V}_i \leq 0$. on the other hand, If V_i is inside υ_i then V_i is bounded by $\frac{\mu_i}{c_i}$.

For nonlinear composite system, we employ Lyaounov function as follows

$$
V = \sum_{i=1}^{N} V_i \quad (29)
$$

Differentiating (29) with respect to time we obtain

$$
\dot{V} \leq \sum_{i=1}^{N}(-c_i V_i + \mu_i) \quad (30)
$$

choosing $V_i > \frac{\mu_i}{c_i}$, then

$$
\dot{V} < 0 \quad (31)
$$

So V is uniformly ultimately bounded. According to definition of V, we can conclude that $|e_i|$, $\|\hat{W}_{i1}\|$, $\|\hat{W}_{i2}\|$, $\|\hat{W}_{iN}\|$, $\|\hat{W}_{im}\|$ and $\|\hat{W}_{ih}\|$ also are uniformly ultimately bounded.

from (7)-(11) and theorem 1, we obtain

$$
|u_i| \leq (\frac{s_{i1}}{\lambda_{i1}} + \frac{s_{iN}}{\lambda_{i2}} + \frac{s_{im}}{\lambda_{i3}} + \frac{s_{ih}}{\lambda_{i4}})\frac{\mu_i}{c_i} \quad (32)
$$

namely, we get the set U_i

$$
U_i = \{u_i : |u_i| \leq (\frac{s_{i1}}{\lambda_{i1}} + \frac{s_{iN}}{\lambda_{i2}} + \frac{s_{im}}{\lambda_{i3}} + \frac{s_{ih}}{\lambda_{i4}})\frac{\mu_i}{c_i}\}
$$

Corollary 1: the control laws (7)-(11) are uniformly ultimately bounded with respect to the set U_i

4. PERFORMANCE ANALYSIS

From (27) we get

$$
0 \leq V_i(t) = \frac{\mu_i}{c_i} + [V_i(0) - \frac{\mu_i}{c_i}]e^{-c_i t} \quad (33)
$$

from (17) and (33)

$$
|e_i(t)| = \begin{cases} \sqrt{\frac{2}{k_i}V_i(0)}, & \text{if } V_i(0) > \frac{\mu_i}{c_i} \\ \sqrt{\frac{2}{k_i}\frac{\mu_i}{c_i}}, & \text{if } V_i(0) \leq \frac{\mu_i}{c_i} \end{cases} \quad (34)
$$

given any positive constant $R_i > \sqrt{\frac{2\mu_i}{k_i c_i}}$ there exist a finite T_{i0}

$$
T_{i0} = \begin{cases} -\frac{1}{c_i}\ln\frac{\frac{k_i}{2}R_i^2 - \frac{\mu_i}{c_i}}{V_i(0) - \frac{\mu_i}{c_i}}, & \text{if } e_i(0) > R_i \\ 0, & \text{if } e_i(0) \leq R_i \end{cases}
$$

such that $e_i(t)$ enters the ball B_{R_i} at time $t \leq T_{i0}$. And the ultimate bound of $e_i(t)$ namely $\sqrt{\frac{2\mu_i}{k_i c_i}}$ is independent of initial condition.

Theorem 2: For the closed loop system (6)-(16), the mean-square values of $|e_i|$, $\|\tilde{W}_{i1}\|$, $\|\tilde{W}_{i2}\|$, $\|\tilde{W}_{iN}\|$, $\|\tilde{W}_{im}\|$, $\|\tilde{W}_{ih}\|$ are bounded by

$$\left(\frac{1}{t}\int_0^t |e_i(\tau)|^2 d\tau\right)^{\frac{1}{2}} = \begin{cases} \sqrt{\dfrac{2\mu_i}{k_i c_i} + \dfrac{2\left[V_i(0) - \frac{\mu_i}{c_i}\right]}{k_i}}, \\ \text{if } V_i(0) > \dfrac{\mu_i}{c_i} \\ \sqrt{\dfrac{2}{k_i}\dfrac{\mu_i}{c_i}}, \quad \text{if } V_i(0) \le \dfrac{\mu_i}{c_i} \end{cases} \quad (35)$$

$$\left(\frac{1}{t}\int_0^t \|\tilde{W}_{i1}(\tau)\|^2 d\tau\right)^{\frac{1}{2}} \le D_i = \begin{cases} \sqrt{\dfrac{2\mu_i}{c_i} + 2\left[V_i(0) - \dfrac{\mu_i}{c_i}\right]} \\ , \quad \text{if } V_i(0) > \dfrac{\mu_i}{c_i} \\ \sqrt{\dfrac{2\mu_i}{c_i}}, \text{if } V_i(0) \le \dfrac{\mu_i}{c_i} \end{cases} \quad (36)$$

$$\left(\frac{1}{t}\int_0^t \|\tilde{W}_{i2}(\tau)\|^2 d\tau\right)^{\frac{1}{2}}, \left(\frac{1}{t}\int_0^t \|\tilde{W}_{iN}(\tau)\|^2 d\tau\right)^{\frac{1}{2}},$$
$$\left(\frac{1}{t}\int_0^t \|\tilde{W}_{im}(\tau)\|^2 d\tau\right)^{\frac{1}{2}}, \left(\frac{1}{t}\int_0^t \|\tilde{W}_{ih}(\tau)\|^2 d\tau\right)^{\frac{1}{2}} \le D_i \quad (37)$$

proof: integrating (34) over [0,t] we get

$$\int_0^t |e_i(\tau)|^2 d\tau \le \frac{2\mu_i}{k_i c_i} t + \frac{2}{k_i c_i}[V_i(0) - \frac{\mu_i}{c_i}]\left(1 - e^{-c_i t}\right) \quad (38)$$

from which we distinguish two possible cases.

case 1: Let $V_i(0) \le \frac{\mu_i}{c_i}$ and (38) becomes

$$\left(\frac{1}{t}\int_0^t |e_i(\tau)|^2 d\tau\right)^{\frac{1}{2}} = \sqrt{\frac{2}{k_i}\frac{\mu_i}{c_i}}$$

case 2: Let $V_i(0) > \frac{\mu_i}{c_i}$. from (38) we obtain

$$\left(\frac{1}{t}\int_0^t |e_i(\tau)|^2 d\tau\right)^{\frac{1}{2}} = \sqrt{\frac{2\mu_i}{k_i c_i} + \frac{2\left[V_i(0) - \frac{\mu_i}{c_i}\right]}{k_i}}$$

Thus, we have proven (35). Similarly, we can prove (36) and (37) after observing that from (33)

$$\|\tilde{W}_{i1}\|^2, \|\tilde{W}_{i2}\|^2, \|\tilde{W}_{iN}\|^2, \|\tilde{W}_{im}\|^2, \|\tilde{W}_{ih}\|^2$$
$$= \frac{\mu_i}{c_i} + 2\left[V_i(0) - \frac{\mu_i}{c_i}\right] e^{-c_i t}$$

5. SIMULATION

In this section, we consider the simple composite system

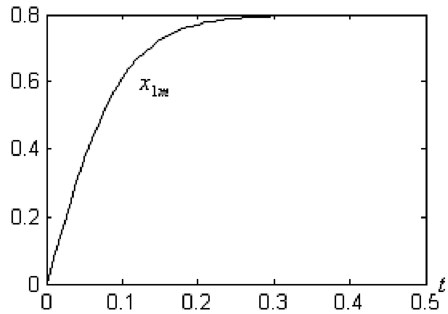

Fig. 1. The trajectory of x_{1m}

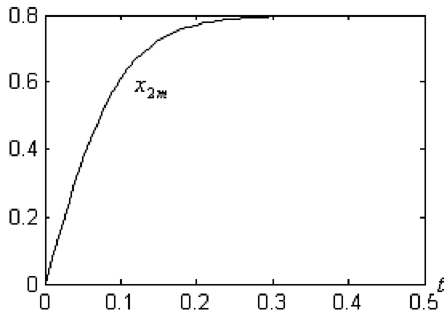

Fig. 2. The trajectory of x_{2m}

$$\dot{x}_1 = -x_1 + 0.2sinx_1 + x_1u_1 + x_2$$
$$\dot{x}_2 = -x_2 + 0.2sinx_2 + x_2u_2 + x_1$$

The problem is to develop a control law that forces the practical system states to follow the given reference bounded trajectory x_m. Fig. 1,2 show the reference signals.

Since the practical system states are unknown. A third order dynamic neural network is used as a model of the practical system . namely

$$\dot{x}_1 = -a_1x_1 + \sum_{i=1}^{3} w_{11i}s_{11}^i(x_1) + \sum_{j=1}^{2} w_{12j}s_{12}^j(x_1)u_1 + \sum_{i=1}^{3} w_{1Ni}s_N^i(x_2)$$

$$\dot{x}_2 = -a_2x_2 + \sum_{i=1}^{3} w_{21i}s_{21}^i(x_2) + \sum_{j=1}^{2} w_{22j}s_{22}^j(x_2)u_2 + \sum_{i=1}^{3} w_{2Ni}s_N^i(x_1)$$

Similarly, A third order dynamic neural network is used as a model of the reference system, namely

$$\dot{x}_{1m} = -a_{1m}x_{1m} + \sum_{i=1}^{3} w_{1mi}s_{1m}^i(x_{1m}) + \sum_{i=1}^{3} w_{1hi}s_{1h}^i(x_{2m})$$

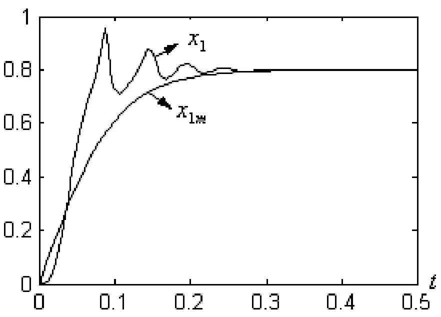

Fig. 3. The trajectory of x_1 and x_{1m}

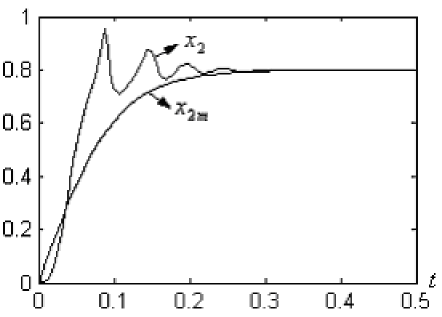

Fig. 4. The trajectory of x_2 and x_{2m}

$$\dot{x}_{2m} = -a_{2m}x_{2m} + \sum_{i=1}^{3} w_{2mi}s_{2m}^{i}(x_{2m}) + \sum_{i=1}^{3} w_{2hi}s_{2h}^{i}(x_{1m})$$

The initial values are chosen as follows

$$x_1 = x_2 = x_{1m} = x_{2m} = 0,$$

$$w_{12j} = w_{22j} = -0.1, j = 1, 2$$

$$w_{11i} = w_{21i} = w_{1Ni} = w_{2Ni} = -0.1$$

$$w_{1mi} = w_{2mi} = w_{1hi} = w_{2hi} = -0.1, i = 1, 2, 3$$

we take the parameters

$$\lambda_{i1} = \lambda_{i2} = \lambda_{i3} = \lambda_{i4} = 4, i = 1, 2$$

$$\gamma_{i1} = \gamma_{i2} = \gamma_{i3} = \gamma_{i4} = \gamma_{i5} = 0.001, i = 1, 2$$

$$a_1 = a_2 = 8, a_{1m} = a_{2m} = 4, k_1 = k_2 = 800$$

The simulation results are presented in Figs.3 and Figs.4. Figs.3 shows the trajectory of the states x_1 and x_{1m}; Figs.4 shows the trajectory of the states x_2 and x_{2m}, from the above figures it obviously shows that the practical system states converge to the reference trajectory after short time, the simulation results show that the controller design method is valid.

6. CONCLUSIONS

We discussed the problem of the tracking control for a class of nonlinear composite system that can be modelled by dynamical neural networks. More specifically, we aim at designing a controller that will force the actual system states to follow a given bounded reference trajectory. Lyapunov stability theory was used to guarantee a uniform ultimate boundedness property for the tracking error and all other signals in the closed loop, the controller derived is smooth. In addition, the performance criteria of the mean-square performance are provided to quantify the control performance of proposed method. Numerical simulation example is used to illustrate and clarify the theoretical results.

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