RECURSIVE SUBSPACE PREDICTION OF LINEAR TIME-VARYING STOCHASTIC SYSTEMS*

Kentaro Kameyama and Akira Ohsumi**

Department of Mechanical & System Engineering,
Graduate School of Science and Technology, Kyoto Institute of Technology,
Matsugasaki, Sakyo, Kyoto 606-8585, Japan
E-mail: ohsumi@kit.ac.jp*; Tel/Fax: +81-75-724-7352*

Abstract: In this paper, a new subspace method for predicting time-invariant/varying stochastic systems is investigated in the 4SID framework. Using the concept of angle between past and current subspaces spanned by the extended observability matrices, the future subspace is predicted by rotating current subspace in the geometrical sense. In order to treat even time-varying system, a recursive algorithm is derived for implementation. The proposed algorithm is tested by simulation experiments.

Keywords: Subspace prediction; subspace methods; subspace identification; recursive algorithm; time-varying systems

1. INTRODUCTION

During the last two decades, 4SID-based system identification has been considerably developing, achieving a significant level of maturity and acceptability in control system applications. Nowadays, subspace identification is recognized to be very efficient to model multivariable systems, including estimations of the system states only from the set of input and output data. However, the prediction of system model has not been so much investigated in the framework of subspace identification.

In reality, most existing systems show time-varying and/or nonlinear behavior, and the nonlinear systems are sometimes treated as (high-order) linear time-varying systems from the practical point of view. Furthermore, most of actual phenomena show complex behaviors and their mathematical models are written by time-varying and/or nonlinear equations.

Thus, in order to obtain accurate mathematical models and to realize efficient control, the estimation and/or prediction of the system are significantly important. Especially, in order to realize the model predictive control the subspace prediction (SP) is very important (Favoreel, et al., 1999), and its algorithm must be necessarily recursive.

Motivated by the model predictive control which is a general name for a whole class of model-based control methods, in this paper an approach to predict unknown system dynamics will be developed in the 4SID framework with an idea of the angle between two subspace of the past and the future.

2. PROBLEM STATEMENT

Suppose that we are given a couple of input and output data sequences $\{u_k, y_k\}$ and the output data is generated from the discrete-time time-varying stochastic system:

$$x_{k+1} = A_k x_k + B_k u_k + w_k \tag{1}$$

$$y_k = C_k x_k + D_k u_k + v_k, \tag{2}$$

where $u_k \in R^m$, $y_k \in R^\ell$ and $x_k \in R^n$ are input, output and state vectors; $w_k \in R^n$ and $v_k \in R^\ell$ are zero-mean white Gaussian sequences with covariance matrices:

$$\mathcal{E}\left\{\begin{bmatrix} w_k \\ v_k \end{bmatrix}\begin{bmatrix} w_s \\ v_s \end{bmatrix}^T\right\} = \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}\delta_{ks}$$

(δ_{ks} : Kronecker delta).

The system order n is assumed to be known. Let k_1, k_2 ($k_1 < k_2$) be two (distinct) time instants, and let $Y_\alpha(k_\nu|k_\nu - N + 1) \in R^{\alpha\ell\times N}$ ($\nu = 1, 2$) be a block Hankel matrix constructed by arranging (column) output vectors $\boldsymbol{y}_\alpha(i) = [\, y^T(i-\alpha+1), \cdots, y^T(i)\,]^T$ from $i = k_\nu - N + 1$ to $i = k_\nu$, where α is the size of block rows. Similarly, let $U_\alpha(k_\nu|k_\nu - N + 1) \in R^{\alpha m\times N}$ be the block Hankel matrix constructed by the input data (e.g.,

---

* Part of this work is supported by the Japan Society for the Promotion of Science (JSPS) under Grant-in-Aid for Scientific Research (B)-16360046.

**Corresponding author.

Verhaegen and Dewilde, 1992; Van Overschee and De Moor, 1996).

Then, under the quasi-stationarity assumption (Ohsumi and Kawano, 2002b; Ohsumi, et al., 2003; Kameyama, et al., 2005) the input-output algebraic relationships with arguments k_1 and k_2 are given, respectively, as:

$$\begin{aligned} Y_\alpha(k_\nu|k_\nu - N + 1) &= \mathit{\Gamma}_\alpha(k_\nu)X_\alpha(k_\nu|k_\nu - N + 1) \\ &\quad + H_\alpha(k_\nu)U_\alpha(k_\nu|k_\nu - N + 1) \\ &\quad + \mathit{\Sigma}_\alpha(k_\nu)W_\alpha(k_\nu|k_\nu - N + 1) \\ &\quad + V_\alpha(k_\nu|k_\nu - N + 1) \quad (\nu = 1, 2), \end{aligned} \tag{3}$$

where $\mathit{\Gamma}_\alpha(\cdot) \in R^{\alpha\ell\times n}$ is the extended observability matrix; $X_\alpha(\cdot|\cdot) \in R^{n\times N}$ the matrix constructed by system states; $W_\alpha(\cdot|\cdot) \in R^{\alpha n\times N}$ and $V_\alpha(\cdot|\cdot) \in R^{\alpha\ell\times N}$ are system and observation noise matrices constructed similarly to $Y_\alpha(\cdot|\cdot)$; and $H_\alpha(\cdot)$, $\mathit{\Sigma}_\alpha(\cdot)$ are lower block triangular matrices consisting of system matrices $\{A_k, B_k, C_k, D_k\}$.

The subspaces spanned by the column vectors of extended observability matrices $\mathit{\Gamma}_\alpha(k_1)$ and $\mathit{\Gamma}_\alpha(k_2)$ form a relationship which is described by the concept of angles between subspaces.

Our problem is to derive a recursive algorithm for predicting the future subspace which is spanned by an extended observability matrix $\mathit{\Gamma}_\alpha(k_3)$ at a future step k_3 ($k_3 > k_2$) by applying the information about the angle to the past subspace. Hence, the SP problem can be stated as follows: Given a set of input and output data of the unknown linear time-varying system (1)-(2) up to the present time step k, predict the quadruple system matrices $(A_{k+\mu}, B_{k+\mu}, C_{k+\mu}, D_{k+\mu})$ at μ-step ahead (within a similarity transformation).

3. PRELIMINARIES

3.1 Angle between subspaces

Consider two matrices $A \in R^{p\times r}$ and $B \in R^{q\times r}$ ($p, q \le r$) with $\operatorname{rank} A = a_d$ and $\operatorname{rank} B = b_d$, respectively. Then, the angle between two subspaces spanned by column vectors of A and B is defined by a set of angles $\{\theta_i,\ i = 1, 2, \cdots, a_d \wedge b_d\}$ (where $a_d \wedge b_d = \min(a_d, b_d)$, $0 \le \theta_i \le \pi/2$) between principal vectors $a_i \in \operatorname{span}_{\text{col}}\{A\}$ and $b_j \in \operatorname{span}_{\text{col}}\{B\}$ ($i, j = 1, 2, \cdots, a_d \wedge b_d$), where $\operatorname{span}_{\text{col}}\{A\}$ denotes the subspace spanned by column vectors of A. The following is the definition of angle between subspaces (see Goulb and Van Loan (1996); Van Overschee and De Moor (1996)).

Definition: Given two matrices A and B mentioned above, choose first a pair of principal vectors $a_1 \in \operatorname{span}_{\text{col}}\{A\}$ and $b_1 \in \operatorname{span}_{\text{col}}\{B\}$ such that a_1 and b_1 minimize their angle θ_1. Next, choose unit vectors a_2 and b_2 which are orthogonal to a_1 and b_1, respectively, and minimize their angle θ_2. By repeating this procedure $a_d \wedge b_d$ times, obtain a set of vectors $\{a_1, \cdots, a_{a_d\wedge b_d}\}$ and $\{b_1, \cdots, b_{a_d\wedge b_d}\}$ called *principal vectors* for each subspace. Then, the angles $\theta_1, \cdots, \theta_{a_d\wedge b_d} \in [\,0, \pi/2\,]$ are called *principal angles* between two subspaces spanned by column vectors of A and B.

The principal vectors and angles of $\operatorname{span}_{\text{col}}\{A\}$ and $\operatorname{span}_{\text{col}}\{B\}$ can be calculated by performing the SVD as

$$A\left(A^TA\right)^\dagger A^TB\left(B^TB\right)^\dagger B^T = USV^T, \tag{4}$$

where $U \in R^{p\times p}$ and $V \in R^{q\times q}$ are orthogonal matrices; $S \in R^{p\times q}$ is the matrix consisting of singular values $\{\sigma_i,\ i = 1, 2, \cdots, p \wedge q\}$ as diagonal elements; and $\dagger$ denotes the Moore-Penrose pseudo-inverse. Then, the principal vectors of $\operatorname{span}_{\text{col}}\{A\}$ and $\operatorname{span}_{\text{col}}\{B\}$ are given as $u_i \in R^p$ and $v_j \in R^q$ ($i = 1, 2, \cdots, a_d \wedge b_d$; $j = 1, 2, \cdots, a_d \wedge b_d$), where u_i and v_j are the first a_d or b_d column vectors of U or V. The ith principal angle θ_i between u_i and v_i is obtained as the singular value σ_i with relationship:

$$\sigma_i = \cos\theta_i \qquad (i = 1, 2, \cdots, a_d \wedge b_d), \tag{5}$$

and other singular values are equal to zero. Furthermore, $\operatorname{span}_{\text{col}}\{A\} = \operatorname{span}_{\text{col}}\{U(:, 1 : a_d)\}$ and $\operatorname{span}_{\text{col}}\{B\} = \operatorname{span}_{\text{col}}\{V(:, 1 : b_d)\}$ by definition.

3.2 Rotation of vectors

Without loss of generality, let us consider the case of $p = q$. Let u_i and v_i be column vectors in p-dimensional subspace, $u_i = [\,u_{i1}, u_{i2}, \cdots, u_{ip}\,]^T$ and $v_i = [\,v_{i1}, v_{i2}, \cdots, v_{ip}\,]^T$. Then, the rotation operator $\mathcal{R}_{\theta_i}$ is defined such that the vector $v_i = \mathcal{R}_{\theta_i}u_i$ makes the angle θ_i with vector u_i. The rotation is realized by the following procedure.

First, define the orthonormal basis of the rotation plane on which u_i and v_i lie:

$$e_{i1} := \frac{u_i}{||u_i||^2} \tag{6}$$

$$e_{i2} := \frac{v_i - \langle v_i, e_{i1}\rangle\, e_{i1}}{||v_i - \langle v_i, e_{i1}\rangle\, e_{i1}||^2}\,, \tag{7}$$

where $\langle a, b\rangle$ denotes the inner product a^Tb of a and b. The rest of the vectors e_{ij} ($j = 3, \cdots, p$) of orthonormal basis of R^p are arbitrarily selected as

$$e_{ij}e_{ik}^T = I_p\delta_{jk} \tag{8}$$

(δ_{jk}: Kronecker delta; I_p: unit matrix of dimension p) for $j, k = 1, 2, \cdots, p$. Then, u_i is represented in terms of a basis $\{e_{i1}, \cdots, e_{ip}\}$ by the orthonormal basis as:

$$\begin{aligned} u_i &= a_{i1}e_{i1} + a_{i2}e_{i2} + \cdots + a_{ip}e_{ip} \\ &= a_{i1}e_{i1}, \end{aligned} \tag{9}$$

where $a_{ij} = \langle u_i, e_{ij} \rangle$ $(j = 1, 2, \cdots, p)$ and $a_{i2} = \cdots = a_{ir} = 0$. Similarly, v_i is written as

$$\begin{aligned} v_i &= b_{i1}e_{i1} + b_{i2}e_{i2} + \cdots + b_{ip}e_{ip} \\ &= b_{i1}e_{i1} + b_{i2}e_{i2} \end{aligned} \tag{10}$$

with $b_{ij} = \langle v_i, e_{ij} \rangle$ $(j = 1, 2, \cdots, p)$ and $a_{i3} = \cdots = a_{ip} = 0$. Then, the relationship between a_{ij} and b_{ij} is given as $(j = 1, 2)$:

$$\begin{bmatrix} b_{i1} \\ b_{i2} \end{bmatrix} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix} \begin{bmatrix} a_{i1} \\ a_{i2} \end{bmatrix}. \tag{11}$$

Substituting (9) and (11) into (10), we have

$$\begin{aligned} v_i &= \begin{bmatrix} e_{i1} \ e_{i2} \ e_{i3} \ \cdots \ e_{ip} \end{bmatrix} \\ &\quad \cdot \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & \cdots & 0 \\ \sin\theta_i & \cos\theta_i & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} e_{i1}^T \\ e_{i2}^T \\ e_{i3}^T \\ \vdots \\ e_{ip}^T \end{bmatrix} u_i \\ &=: \mathcal{R}_{\theta_i} u_i. \end{aligned} \tag{12}$$

4. PREDICTION OF FUTURE SUBSPACE

Given a set of input and output data up to the current time step k_2, we are interested in the estimation of all system matrices at some future time, say k_3, based on the currently obtained data set. Let k_1, k_2 and k_3 be past, current and future times, respectively $(k_1 < k_2 \leq k_3)$. In this paper, "time-varying" means that all system matrices as well as noise covariance matrices change slowly with time. Qualitatively speaking, the instinctive word "slowly" implies that all matrices change smoothly and continuously, and do never abruptly or randomly. Then the extended observability matrices can be computed at $k = k_\nu$ as (Ohsumi and Kawano, 2002b)

$$\Gamma_\alpha(k_\nu) = \begin{bmatrix} C_{k_\nu}^T \ (C_{k_\nu} A_{k_\nu})^T \ \cdots \ (C_{k_\nu} A_{k_\nu}^{\alpha-1})^T \end{bmatrix}^T \quad (\nu = 1, 2). \tag{13}$$

These matrices are also considered to change slowly. This implies that the change is took place by rotation and scaling of column vectors $\{\gamma_i(\cdot)\}_{i=1,2,\cdots,n}$ of the matrix $\Gamma_\alpha(\cdot)$, i.e.,

$$\Gamma_\alpha(k_\nu) = \begin{bmatrix} \gamma_1(k_\nu) \ \gamma_2(k_\nu) \ \cdots \ \gamma_n(k_\nu) \end{bmatrix}.$$

According to such observation, the angle between signal subspaces $\Gamma_\alpha(k_1)$ and $\Gamma_\alpha(k_2)$ is yielded as a result of rotation of each column vector $\gamma_i(k_1)$ during the interval $k_2 - k_1$ in $\alpha\ell$-dimensional vector space.

From (4) the principal vectors of $\mathrm{span}_{\mathrm{col}}\{\Gamma_\alpha(k_1)\}$ and $\mathrm{span}_{\mathrm{col}}\{\Gamma_\alpha(k_2)\}$ are calculated by

$$\begin{aligned} &\Gamma_\alpha(k_1) \left\{ \Gamma_\alpha^T(k_1) \Gamma_\alpha(k_1) \right\}^\dagger \Gamma_\alpha^T(k_1) \\ &\quad \cdot \Gamma_\alpha(k_2) \left\{ \Gamma_\alpha^T(k_2) \Gamma_\alpha(k_2) \right\}^\dagger \Gamma_\alpha^T(k_2) \\ &\quad\quad = U(k_1) S(k_1|k_2) V^T(k_2), \end{aligned} \tag{14}$$

where the column vectors of the matrices $U(k_1) = [u_1(k_1) \cdots u_n(k_1) \cdots u_{\alpha\ell}(k_1)]$ and $V(k_2) = [v_1(k_2) \cdots v_n(k_2) \cdots v_{\alpha\ell}(k_2)]$ consist of principal vectors of $\Gamma_\alpha(k_\nu)$ $(\nu = 1, 2)$. The angles $\{\theta_i(k_2|k_1)\}_{i=1,2,\cdots,n}$ made by principal vectors $\{u_i(k_1)\}$ and $\{v_i(k_2)\}$ are related to the first n singular values $\{\sigma_i(k_2|k_1)\}$ with relationship:

$$\sigma_i(k_2|k_1) = \cos\theta_i(k_2|k_1). \tag{15}$$

Here, let $\hat{v}_i(k_3|k_2)$ be the estimate of $v_i(k_3)$ based on the data up to the current time k_2. Then, (12) implies that this estimate can be computed based on $v_i(k_2)$ by

$$\hat{v}_i(k_3|k_2) = \mathcal{R}_{\theta_i(k_3|k_2)} v_i(k_2). \tag{16}$$

The rotation operator $\mathcal{R}_{\theta_i(k_3|k_2)}$ can be computed as follows. Since the rate of rotation during the interval $k_2 - k_1$ is given from (15) as

$$\Delta\theta_i(k_2|k_1) := \frac{\arccos\sigma_i(k_2|k_1)}{k_2 - k_1} \quad \text{[rad/step]}, \tag{17}$$

so that the extrapolated angle $\theta_i(k_3|k_2)$ is evaluated as

$$\theta_i(k_3|k_2) = \Delta\theta_i(k_2|k_1)(k_3 - k_2) + o(k_3 - k_2) \tag{18}$$

within the approximation of order $o(k_3 - k_2)$. Therefore, the estimate of the extended observability matrix at the future time k_3, $\hat{\Gamma}_\alpha(k_3|k_2)$, can be computed by

$$\hat{\Gamma}_\alpha(k_3|k_2) = \begin{bmatrix} \hat{v}_1(k_3|k_2) \ \hat{v}_2(k_3|k_2) \ \cdots \ \hat{v}_n(k_3|k_2) \end{bmatrix}. \tag{19}$$

Based on this predicted extended observability matrix the SP can be performed. This scheme is off-line and requires computations of SVD two times for obtaining the estimate of current subspace and the angle between past and current subspaces. Undoubtedly, this is lengthy. So, in the following section, we concentrate our attention on the recursive update of the main steps in the algorithm to reduce multiple SVDs.

5. RECURSIVE IMPLEMENTATION OF SUBSPACE PREDICTION

The recursive SP algorithm is derived incorporating the recursive subspace identification algorithm developed by Ohsumi and Kameyama with their colleagues (2003, 2005) (owing to limited space a brief review is omitted here). Up to the present time, there are mainly two kinds of recursive 4SID algorithms. One uses the forgetting factor to adapt 4SID algorithm to the identification of time-varying systems; while the other one uses the fixed size of input and output data. The latter one is rather congenial with the SP algorithm because its algorithm is constructed based on the same quasi-stationarity assumption.

Consider the LQ-factorization of the constructed data matrix:

$$\begin{bmatrix} U_\alpha(k_2|k_2-N+1) \\ U_\beta(k_2|k_2-N+1) \\ Y_\alpha(k_2|k_2-N+1) \end{bmatrix} = \begin{bmatrix} L_{11}(k_2) & 0 & 0 \\ L_{21}(k_2) & L_{22}(k_2) & 0 \\ L_{31}(k_2) & L_{32}(k_2) & L_{33}(k_2) \end{bmatrix} \begin{bmatrix} Q_1^T(k_2) \\ Q_2^T(k_2) \\ Q_3^T(k_2) \end{bmatrix}, \quad (20)$$

where $Y_\alpha(\cdot|\cdot)$, $U_\alpha(\cdot|\cdot)$ are block Hankel matrices (as appeared in (3)); $U_\beta(\cdot|\cdot)$ is an instrumental variable matrix constructed similarly from input data. Then, the estimate of the signal subspace is derived by performing the SVD of $L_{32}(k_2)$ or the eigendecomposition of $L_{32}(k_2)L_{32}^T(k_2)$, and $L_{32}(k_2)L_{32}^T(k_2)$ is renewed by the recursive 4SID algorithm using fixed input/output data size (Ohsumi, et al., 2003; Kameyama, et al., 2005).

Now, write the matrices $L_{32}(k_2)$ and $L_{32}(k_2)L_{32}^T(k_2)$ as

$$L_{32}(k_2) = \left[\, s_1(k_2) \ s_2(k_2) \ \cdots \ s_{\alpha\ell}(k_2) \,\right]$$
$$L_{32}(k_2)L_{32}^T(k_2) = \left[\, h_1(k_2) \ h_2(k_2) \ \cdots \ h_{\alpha\ell}(k_2) \,\right],$$

where $\{s_i(\cdot)\}$ and $\{h_i(\cdot)\}$ are column vectors; and further let $\{f_{ij}(\cdot)\}$ be the (i,j)-element of the matrix $L_{32}^T(k_2)$ $(f_{ij}(k_2) \neq 0)$. Then, the column vector of $L_{32}(k_2)L_{32}^T(k_2)$ is represented as

$$h_j(k_2) = f_{1j}(k_2)s_1(k_2) + f_{2j}(k_2)s_2(k_2) + \cdots + f_{\alpha\ell j}(k_2)s_{\alpha\ell}(k_2) \ (j = 1, \cdots, \alpha\ell). \quad (21)$$

Choosing n column vectors arbitrarily from $\{h_j(k_2)\}_{j=1,2,\cdots,\alpha\ell}$, and construct a matrix

$$L_b(k_2) = [\, h_i(k_2), \cdots, h_j(k_2) \,] \in R^{\alpha\ell\times n} \ (i \neq j). \quad (22)$$

Then, the following relation holds:

$$\mathrm{span_{col}}\{\mathit{\Gamma}_\alpha(k_2)\} \cong \mathrm{span_{col}}\{L_b(k_2)\} \subset \mathrm{span_{col}}\{L_{32}(k_2)L_{32}^T(k_2)\}. \quad (23)$$

As a result, the computation of the angle between $\mathit{\Gamma}_\alpha(k_1)$ and $\mathit{\Gamma}_\alpha(k_2)$ can be performed by that between $\mathit{\Gamma}_\alpha(k_1)$ and $L_b(k_2)$ as

$$\hat{\mathit{\Gamma}}_\alpha(k_1)\left\{\hat{\mathit{\Gamma}}_\alpha^T(k_1)\hat{\mathit{\Gamma}}_\alpha(k_1)\right\}^\dagger \hat{\mathit{\Gamma}}_\alpha^T(k_1) \cdot L_b(k_2)\left\{L_b^T(k_2)L_b(k_2)\right\}^\dagger L_b^T(k_2) = U(k_1)S(k_1|k_2)V^T(k_2), \quad (24)$$

and the estimate of $\mathrm{span_{col}}\{\hat{\mathit{\Gamma}}_\alpha(k_2)\}$ is given by the principal vectors of $L_b(k_2)$ as far as the angles between principal vectors at k_1 and k_2 hold the relation $\theta_i(k_2|k_1) < \pi/4$ (see Appendix A).

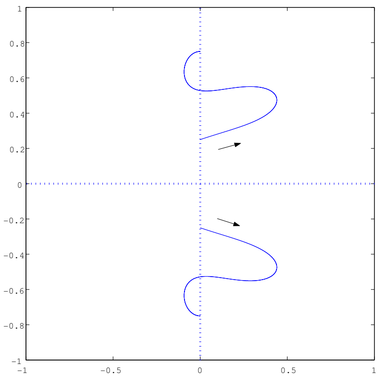

Fig.1. Conjugate pole loci of true time-varying system.

Consequently, the recursive SP algorithm is summarized as follows:

Subspace Prediction Algorithm

Step 1: Acquire a data set $\{u(k_2), y(k_2)\}$, and renew $L_{32}(k_2)L_{32}^T(k_2)$ according to the recursive algorithm proposed in Ohsumi, et al. (2003) or Kameyama, et al. (2005).

Step 2: Construct $L_b(k_2)$ and perform the SVD as (24).

Step 3: Predict the future subspace by the procedure mentioned in Section 4.

Step 4: Derive each unknown system matrices according to the 4SID framework.

6. NUMERICAL EXAMPLE

Consider a single-input, single-output two-dimensional time-varying stochastic system with matrices:

$$A_k = \frac{1}{2}\begin{bmatrix} \sin(2\pi k/1000) & 0.5 + \sin(2\pi k/4000) \\ -0.5 - \sin(2\pi k/4000) & \sin(2\pi k/2000) \end{bmatrix}$$

$$B_k = \begin{bmatrix} 2.0 \\ -1.0 \end{bmatrix}, \ C_k = \left[\, 1.0, \ 2.0 \,\right], \ D_k = 1.5.$$

Figure 1 shows the true loci of conjugate poles. The random noises w_k and v_k are mutually independent and have common covariance $\mathcal{E}\{w(k)w^T(j)\} = 0.1^2 I_2\delta_{kj}$ and $\mathcal{E}\{v(k)v^T(j)\} = 0.1^2\delta_{kj}$. The size of block Hankel matrices is specified as $\alpha = h = 5$. Five sets of experiments were performed by changing N (number of data for an identification) from $N = 25$ to $N = 150$ because reasonable N for the quasi-stationarity assumption must be chosen, and the results of the case $N = 75$ are depicted. Interval to calculate the rotation angles is decided as $k_2 - k_1 = N$. Figure 2 depict a couple of time evolutions of real and imaginary parts of

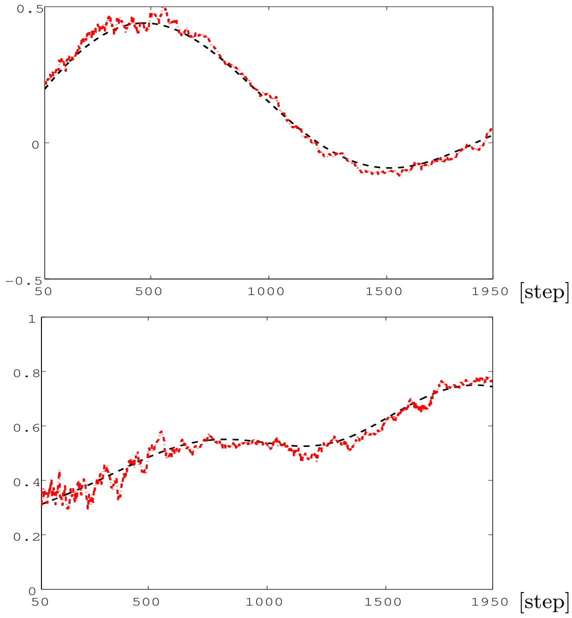

Fig.2. Time evolutions of real (top) and imaginary (bottom) parts of predicted conjugate poles (N=75, $L = 50$ (50-step ahead prediction)).

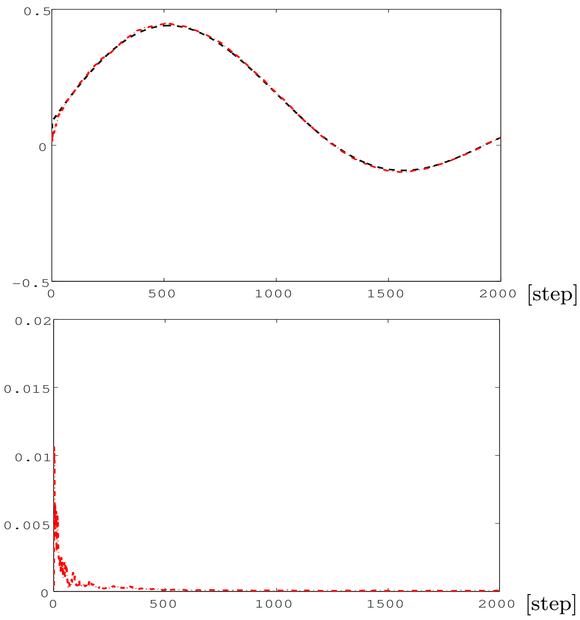

Fig.3. Sample mean (top) and variance (bottom) of the real part of predicted conjugate poles ($N = 75$, $L = 0$ (Estimation)).

typical one of 50-step ahead predicted conjugate poles.

Figures 3-5 show the results of 50 Monte Carlo experiments. Top and bottom pictures of each figure show sample mean and variance of the estimates of the real parts of poles for the case of $L(:= k_3 - k_2) = 0$ (estimation), $L = 50$ (50-step ahead prediction) and $L = 100$ (100-step ahead prediction). Predicted and true ones are depicted by chain (red) and broken (black) curves, respectively. Although both averaged sample variances

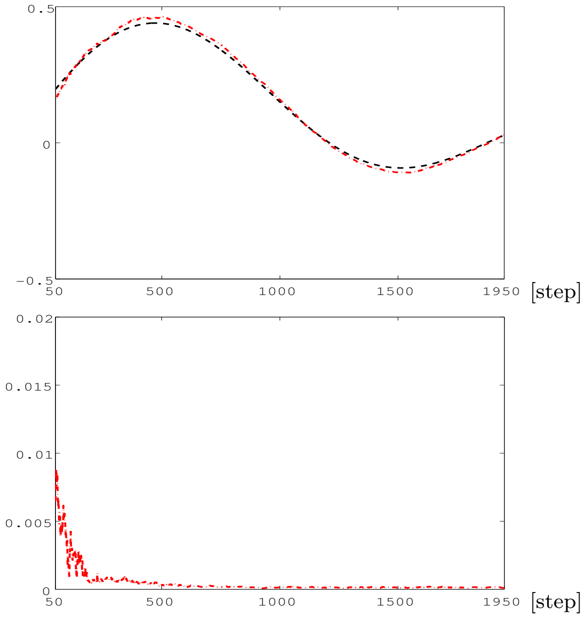

Fig.4. Sample mean (top) and variance (bottom) of the real part of predicted conjugate poles ($N = 75$, $L = 50$ (50-step ahead prediction)).

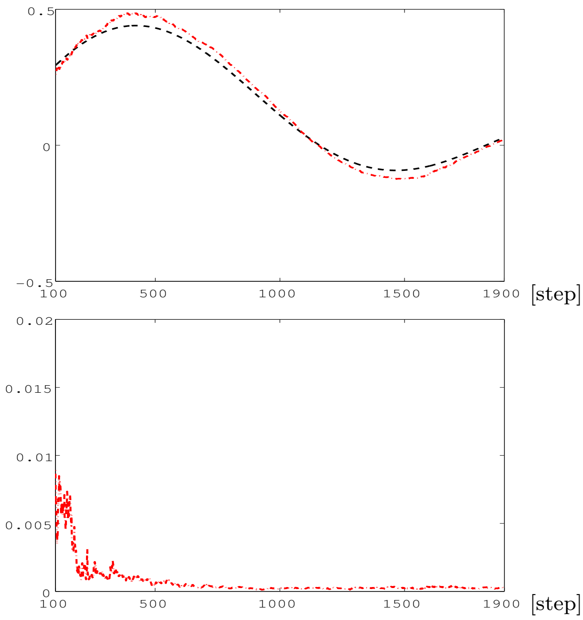

Fig.5. Sample mean (top) and variance (bottom) of the real part of predicted conjugate poles ($N = 75$, $L = 100$ (100-step ahead prediction)).

exhibit large value in the incipient stage, they become small less and less as time goes by.

In Fig. 6, results of 50-step prediction of a sudden change system are shown. From these we see that the proposed algorithm can be applied for such a system.

7. CONCLUSION

An approach to predict the time-varying system matrices of the linear systems has been proposed in the 4SID framework. The key of our approach is

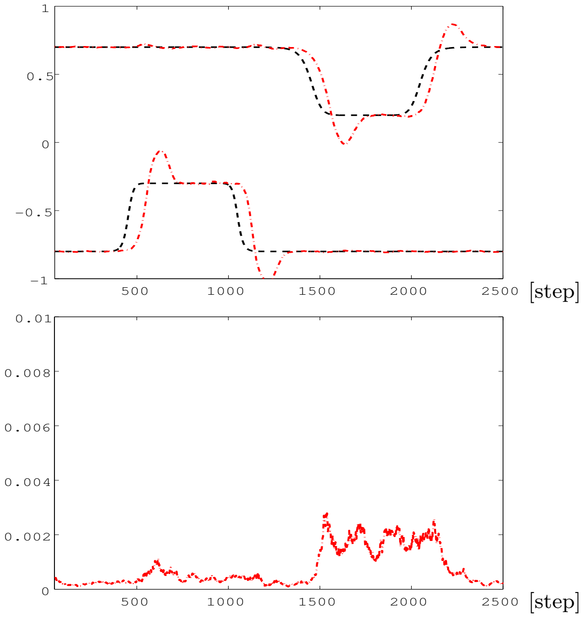

Fig.6. Sample mean (top) and variance (bottom) of the real part of predicted sudden change system ($N = 100$, $L = 50$ (50-step ahead prediction)).

the introduction of the idea of angles between two subspaces which is geometric and intuitive. Furthermore, the recursive implementation of subspace prediction was proposed and the efficacy has been confirmed by simulation experiments.

REFERENCES

Favoreel, W., De Moor, B., and Gevers, M. (1999). SPC: Subspace Predictive Control, *Proc. 14th IFAC World Congress*, Beijing, pp.235-240.

Goulb, G. H., and C. F. Van Loan (1996). *Matrix Computations* (2nd edition), John Hopkins Univ. Press, Baltimore.

Kameyama, K., A. Ohsumi, Y. Matsuüra, and K. Sawada (2005). Recursive 4SID-based Identification Algorithm with Fixed Input-Output Data Size, *Int. J. Innovational Computing & Information Control* (to appear).

Ohsumi, A., K. Kameyama, and K.-I. Yamaguchi (2002a). Subspace Identification for Continuous-time Stochastic Systems via Distribution-based Approach, *Automatica*, vol. 38, pp.63-79.

Ohsumi, A., and T. Kawano (2002b). Subspace identification for a class of time-varying continuous-time stochastic systems via distribution-based approach, *Proc. 15th IFAC World Congress*, Barcelona, vol.F, pp.241-246.

Ohsumi, A., Y. Matsuüra, and K. Kameyama (2003). Recursive subspace identification for continuous-/discrete-time stochastic systems, *Proc. 13th IFAC Symp. System Identification* (SYSID2003), Rotterdam, pp.911-916.

Van Overschee, P., and B. De Moor (1996). *Subspace System Identification for Linear Systems*, Kluwer Academic Pub., Boston.

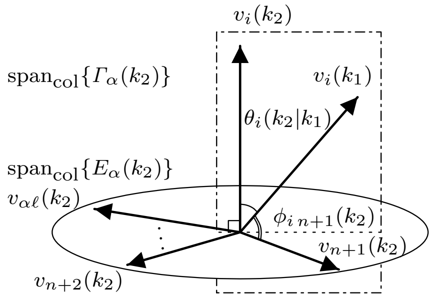

Fig.A.1. Relation between signal and noise subspaces.

Verhaegen, M., and P. Dewilde (1992). Subspace model identification Part 1: The output-error state-space model identification class of algorithms, *Int. J. Control*, vol. 56, no. 5, pp.1187-1210.

Appendix A. CONDITION FOR DERIVATION OF THE PRINCIPAL VECTORS

By the assumption that the noise sequences are mutually uncorrelated with input sequence, the subspace spanned by $L_b(k_2)$ is represented as

$$\text{span}_{\text{col}}\{L_b(k_2)\} = \text{span}_{\text{col}}\{\Gamma_\alpha(k_2)\} \oplus \text{span}_{\text{col}}\{E(k_2)\}, \qquad \text{(A.1)}$$

where $\oplus$ denotes the direct sum; and $\text{span}_{\text{col}}\{E(k_2)\}$ is the noise subspace. So, each column vector of $L_b(k_2)$ is represented as:

$$\begin{aligned} h_j(k_2) &= g_{1j}(k_2)v_1(k_2) + \cdots \\ &+ g_{nj}(k_2)v_n(k_2) + g_{n+1\,j}(k_2)v_{n+1}(k_2) \\ &+ \cdots + g_{\alpha\ell\,j}(k_2)v_{\alpha\ell}(k_2), \end{aligned} \qquad \text{(A.2)}$$

where $v_i(k_2) \in \text{span}_{\text{col}}\{\Gamma_\alpha(k_2)\}$ $(i = 1, \cdots, n)$ and $v_i(k_2) \in \text{span}_{\text{col}}\{E(k_2)\}$ $(i = n+1, \cdots, \alpha\ell)$ are principal vectors of the signal and noise subspaces, respectively (Fig. A); and $g_{ij}(k_2)$ $(i, j = 1, \cdots, \alpha\ell)$ are appropriate coefficients for the basis $v_i(k_2)$ $(i = 1, \cdots, \alpha\ell)$ in this representation. Then, for the angle between ith principal vector of the signal subspace and jth one of the noise subspace, $\phi_{ij}(k_2)$, holds the relation:

$$\phi_{ij}(k_2) = \frac{\pi}{2} - \theta_i(k_2) \ \ (0 \le \phi_{ij}(k_2) \le \pi/2). \quad \text{(A.3)}$$

On the other hand, the SVD in (24) yields principal vectors of signal subspace at time k_2 from $\text{span}_{\text{col}}\{L_b(k_2)\}$ so that the angle between principal vectors of $\text{span}_{\text{col}}\{L_b(k_2)\}$ and $\text{span}_{\text{col}}\{\hat{\Gamma}_\alpha(k_1)\}$ becomes minimum.

So, all $\{\theta_i(k_2|k_1)\}$ have to be smaller than $\phi_{ij}(k_2)$ to derive principal vectors of $\hat{\Gamma}_\alpha(k_2)$ as the first n column vectors of $V(k_2)$, i.e.,

$$\theta_i(k_2) < \phi_{ij}(k_2) = \frac{\pi}{2} - \theta_i(k_2) \Longleftrightarrow \theta_i(k_2) < \frac{\pi}{4}. \qquad \text{(A.4)}$$