APPLICATION OF GENETIC ALGORITHMS IN OPTIMAL EXCITATION AND CONTROLLER DESIGN

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Abstract: Genetic Algorithms (GAs) are used in a set of covariance based optimum input signal algorithms using a proposed architecture suitable for on-line system identification. The optimal signals are computed recursively using a predictive filter. The relationships among these algorithms are investigated and compared based on a set of simulations. In addition, a nested GA is proposed for intelligent LQR controller design. The GAs are used to find the minimum distance to uncontrollability of a given system and to maximize that minimum distance by finding the optimal coefficients in the weighting matrices of the LQR controller.

Keywords: Identification, Genetic Algorithms, Input Signals, Intelligent Control, Controllability.

1. INTRODUCTION

Intelligent techniques such as Neural Networks (NN), Fuzzy Logic (FL), and Genetic Algorithms (GA) have become ever more popular in controls engineering. In particular GAs attract a lot of attention to master cumbersome optimization problems. The lack of having to deal with derivatives and the possibility to find the global optimum point are a few of the features responsible for the popularity of GAs. For the past few decades, system identification has established itself as a valuable tool for facilitating the control design task of a vast array of different systems. One key objective in system identification is to automate the modeling process for unknown or partially unknown systems. While different algorithms are being proposed, one common problem is the experimental design. In particular, the question is addressed how to excite the system such that sufficient information can be collected for the system identification algorithm to be successful. This can be formulated as an optimum problem and therefore a short discussion on the application of GA to identification and control is a topic of this paper. For system identification, generally one tries to excite all system modes so that they are represented in the collected input/output data. Since the system is – at least at the beginning of the experiment – to a certain degree unknown, one generally uses, theoretically, white gaussian noise with zero mean. In practice the whiteness of the signal cannot be achieved due to finite power supply. One of the first studies undertaken to define optimal inputs for system identification was by Levin (1960), who considered the reference input to fulfill some constraint, defined by the process dynamics or power limitation on the reference input, or a range limitation. This topic has been studied intensively ever since its introduction by Levin and is generally in the field of optimum input signals. Mehra (1974)

presented the problem of optimal input in a statistical fashion. In particular, the covariance matrix is used to define the optimum criteria. This is a logical extension from using the Cramer-Rao lower bound in the estimation problem. Schoen (2002) proposed an optimal input design algorithm that is also based on the covariance of the information matrix. The properties of this matrix are exploited in order to direct the new input energy to modes that are underrepresented in the information matrix. The optimum input signal is computed using a Genetic Algorithm. GAs are evolutionary algorithms that simulate Darwin's survival of the fittest principle. The initial population of candidate solutions is randomly generated and represented as chromosomes in the form of genes. These chromosomes are evaluated based on an objective function and ranked in terms of its fitness. A subset of the next generation of candidate solutions is selected based on their performance with the objective function. The remaining set of the new generation is generated by a mating process, where the best performing candidate solutions comprise the subset of the parents. In addition to the mating process, a mutation rate is also imbedded in the generation of the new population. The mutation rate enables the search for the optimum solution to overcome local minimums and – provided enough randomness is included in the GA – locate the global minimum/optimum. This process of selection, mating, and mutation is repeated a number of times until the best performing candidate solution converges to some stationary value.

In this paper, we will examine these covariance based optimal input design algorithms based on a proposed algorithm architecture applied to all algorithms. In particular, a comparison among the existing algorithms with the one proposed by the author of this paper is investigated. The architecture uses a GA approach for the computation of the optimum, which will enable a comparison. In the second part, a LQR controller is proposed using a nested GA.

The organization of this paper is as follows. In Section 2 the general architecture of the algorithm for optimal input design is introduced. Section 3 provides a detailed mathematical formulation of the existing and proposed optimal input signal algorithms. Section 4 introduces the intelligent LQR controller. Section 5 includes the simulation results with some discussion, and Section 6 states the conclusion and problems for further research.

2. STRUCUTURE OF ALGORITHM IMPLEMENTATION

The implementation of the optimum input signal algorithms is done using the architecture depicted in Fig 1. The plant is first excited for a finite duration by a random, white, gaussian input sequence. This is to ensure that all modes of the system are at least to some extend represented in the information matrix.

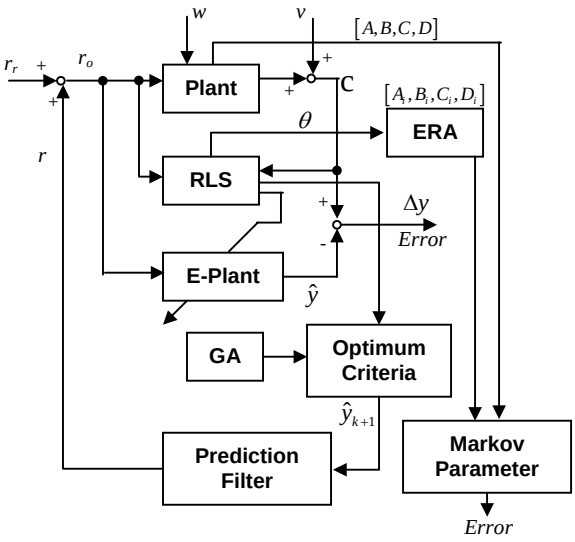

Fig. 1. Proposed recursive architecture for use of GA in optimal input signal computation.

The plant and the measurement signals are corrupted by a white noise process. The plant parameters are estimated using a recursive least-squares (RLS) approach. The future optimal input signals are computed using a genetic algorithm. This computation is facilitated by a predictive filter, which is based on the estimated plant parameters. To access the accuracy and establish a base for comparison for the different optimality criteria, the open-loop system Markov parameters are computed for each iteration using the state-space matrices obtained from the eigensystem realization algorithm (ERA).

3. MATHEMATICAL FORMULATION OF OPTIMAL INPUT SIGNAL

A linear, time-invariant discrete time system can be modeled using a linear finite difference model, or ARX (Auto-Regressive with eXogenous input) model of order *p*:

$$\vec{y}_k = \sum_{i=1}^{p} \vec{a}_i \vec{y}_{k-i} + \sum_{i=1}^{p} \vec{b}_i \vec{r}_{k-i} + \vec{\varepsilon}_k \qquad (1)$$

where $\vec{\varepsilon}_k$ is the residual between the estimated output $\hat{\vec{y}}$ and the actual output $\vec{y}$ at current time step *k*. $\vec{a}_i$ and $\vec{b}_i$ are the parameter coefficient matrices of the ARX model and $\vec{r}_i$ is the input to the system. Defining a parameter matrix vector $\vec{\Theta}$ and an information matrix $\vec{\Phi}$:

$$\vec{\Theta} = \begin{bmatrix} \vec{a}_1 & \vec{b}_1 & \vec{a}_2 & \vec{b}_2 & \cdots & \vec{a}_p & \vec{b}_p \end{bmatrix} \qquad (2)$$

$$\vec{\Phi} = \begin{bmatrix} \vec{y}_p^T & \vec{r}_p^T & \vec{y}_{p-1}^T & \vec{r}_{p-1}^T & \cdots & \vec{y}_1^T & \vec{r}_1^T \\ \vec{y}_{p+1}^T & \vec{r}_{p+1}^T & \vec{y}_p^T & \vec{r}_p^T & \cdots & \vec{y}_2^T & \vec{r}_2^T \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \vec{y}_{L-1}^T & \vec{r}_{L-1}^T & \vec{y}_{L-2}^T & \vec{r}_{L-2}^T & \cdots & \vec{y}_{L-p}^T & \vec{r}_{L-p}^T \end{bmatrix} \qquad (3)$$

where L is the current data length. The output vector $\vec{\xi}$ can be given as

$$\vec{\xi} = \begin{bmatrix} \vec{y}_{p+1} & \vec{y}_{p+2} & \cdots & \vec{y}_L \end{bmatrix}^T . \qquad (4)$$

Then, one can write the error equation $\vec{\varepsilon} = \vec{\xi} - \vec{\Phi}\vec{\Theta}$ and form a quadratic cost function $J = \vec{\varepsilon}^T \vec{\varepsilon}$, which upon minimization yields the least-squares estimate of the parameter vector $\hat{\vec{\Theta}}$:

$$\hat{\vec{\Theta}} = \left(\vec{\Phi}^T \vec{\Phi}\right)^{-1} \vec{\Phi}^T \vec{\xi} . \qquad (5)$$

For online system identification one needs a recursive form of the estimation problem in order to avoid computational speed constraints. Assuming one has obtained a new set of data at the discrete time *k+1*, the future output of the system can be given by the ARX model

$$\vec{y}_{k+1} = \sum_{i=1}^{p} \vec{a}_i \vec{y}_{k+1-i} + \sum_{i=1}^{p} \vec{b}_i \vec{r}_{k+1-i} + \varepsilon_{k+1} . \qquad (6)$$

For time-invariant systems, or systems that vary slowly over time, one can define

$$\vec{\Theta}_{k+1} = \vec{\Theta}_k$$

$$\vec{\Phi}_{k+1} = \begin{bmatrix} \vec{y}_p^T & \vec{r}_p^T & \vec{y}_{p-1}^T & \cdots & \vec{r}_1^T \\ \vec{y}_{p+1}^T & \vec{r}_{p+1}^T & \vec{y}_p^T & \cdots & \vec{r}_2^T \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vec{y}_{L-1}^T & \vec{r}_{L-1}^T & \vec{y}_{L-2}^T & \cdots & \vec{r}_{L-p}^T \\ \vec{y}_L^T & \vec{r}_L^T & \vec{y}_{L-1}^T & \cdots & \vec{r}_{L-p+1}^T \end{bmatrix} = \begin{bmatrix} \vec{\Phi}_k \\ \vec{\phi}_{k+1} \end{bmatrix} \qquad (7)$$

where $L+1$ is equal to the new data length. The corresponding output vector can be given using Equation (7) as:

$$\vec{\xi}_{k+1} = \begin{bmatrix} \vec{y}_{p+1} & \vec{y}_{p+2} & \cdots & \vec{y}_L & \vec{y}_{L+1} \end{bmatrix}^T = \begin{bmatrix} \vec{\xi}_k \,|\, \vec{y}_{L+1} \end{bmatrix}^T . \qquad (8)$$

Using the matrix inversion lemma and defining the inverse correlation matrix $\vec{P}_k$ as

$$\vec{P}_k = \left(\vec{\Phi}_k^T \vec{\Phi}_k\right)^{-1} , \qquad (9)$$

the well-known recursive formula for the inverse correlation matrix can be given as:

$$\vec{P}_{k+1} = \vec{P}_k - \vec{P}_k \vec{\phi}_{k+1}^T \frac{\vec{\phi}_{k+1} \vec{P}_k}{1 + \vec{\phi}_{k+1} \vec{P}_k \vec{\phi}_{k+1}^T} , \qquad (10)$$

which does not involve the computation of the inverse of large data matrices and is therefore suitable for on-line methods. Note that the inverse of the $\vec{P}$ - matrix represents the Fisher's information matrix $\vec{M}$. Using the scalar measures defined by Mehra (1974), the optimality based on the covariance matrix $\vec{P}$ can be given as

i) *A-Optimality:* $\min\left[tr\left\{ \vec{P}_{k+1} \right\} \right]$, (11)

which will have the effect of minimizing the variances of the estimated model parameters.

ii) *D-Optimality:* $\min\left[\det\left\{ \vec{P}_{k+1} \right\} \right]$, (12)

which will minimize the generalized variance of the covariance matrix.

iii) *E-Optimality:* $\min\left[\max\left\{ \lambda\left(\vec{P}_{k+1} \right) \right\} \right]$, (13)

where λ is an eigenvalue of the covariance matrix.

Noting that a poorly excited mode of the system to be identified is represented in the $\vec{P}$ - matrix by a large element, Schoen (2002) proposed to use this observation for an optimal input signal algorithm. Recognizing the poorly excited mode by its element representation in the $\vec{P}$ -matrix allows one to direct the future input to the system to excite this particular mode. The computation for exciting the poorly excited mode is done by minimizing the magnitude of this particular element in the $\vec{P}$ -matrix of the next discrete time step. The formulation for this algorithm is summarized by defining the sub-matrix $\vec{S}$

$$\vec{S}_{L-i,L-j} = \begin{bmatrix} \vec{y}_{L-i} \vec{y}_{L-j}^T & \vec{y}_{L-i} \vec{r}_{L-j}^T \\ \vec{r}_{L-i} \vec{y}_{L-j}^T & \vec{r}_{L-i} \vec{r}_{L-j}^T \end{bmatrix} \in \Re^{(no+ni)x(no+ni)} ,$$

where *no* is the number of outputs of the system and *ni* the number of inputs to the system. One can write the definition for the (*i,j*)'s element of the $\vec{P}_{k+1}$ - matrix as:

$$\vec{P}_{i,j}^{(k+1)} = \vec{P}_{i,j}^{(k)} - \sum_{v=1}^{p} \left\{ \sum_{u=1}^{p} \vec{P}_{i,u}^{(k)} \vec{S}_{L-u+1,L-v+1}^{(k+1)} \right\} \vec{P}_{v,j}^{(k)} + \left\{ 1 + \vec{\phi}^{(k+1)} \vec{P}^{(k)} \vec{\phi}^{(k+1)^T} \right\}^{-1} . \qquad (14)$$

Suppose the largest element in the future $\vec{P}$ -matrix is contained in the sub-matrix $\vec{P}_{i,j}^{(k+1)}$, one can minimize the magnitude of this element for the future $\vec{P}$ - matrix by choosing a new $\vec{y}_{L+1}$:

iv) Mode-Optimality:

Minimize $\vec{P}_{i,j}^{(k+1)}$. (15)

With regard to the continuously computed optimum outputs $\vec{y}_L$, which minimize Equations (11), (12), (13), or (15), the required input needs to be calculated. Note that Equation (6) relates the next output with the current input. A predictive ARX filter can be designed where the future output is related to the current input (Chinvorarat, *et al.,* 1999), given here as a one step ahead predictive filter:

$$\hat{\vec{y}}_{k+1} = \sum_{i=1}^{p} \vec{a}_i^{(1)} \vec{y}_{k-i} + \sum_{j=0}^{p} \vec{b}_j^{(1)} \vec{r}_{k-j} , \qquad (16)$$

where the predictive parameter coefficient matrices are defined as

$$\begin{aligned} &\vec{a}_1^{(1)} = \vec{a}_1 \vec{a}_1 + \vec{a}_2 \qquad \vec{b}_0^{(1)} = \vec{a}_1 \vec{b}_0 + \vec{b}_1 \\ &\cdots \qquad\qquad\qquad\qquad \cdots \\ &\vec{a}_{p-1}^{(1)} = \vec{a}_1 \vec{a}_{p-1} + \vec{a}_p \quad \vec{b}_{p-1}^{(1)} = \vec{a}_1 \vec{b}_{p-1} + \vec{b}_p \\ &\vec{a}_p^{(1)} = \vec{a}_1 \vec{a}_p \qquad\qquad \vec{b}_p^{(1)} = \vec{a}_1 \vec{b}_p \end{aligned} \qquad (17)$$

The superscript in the brackets indicates how many steps ahead will be predicted with the coefficient matrices. Note, the predictive parameter coefficient matrices are a function of the estimated parameter coefficient matrices given in Equation (2), which can be calculated with Equation (5) or recursively with

$$\hat{\vec{\Theta}}_{k+1} = \hat{\vec{\Theta}}_k + \vec{P}_k \vec{\phi}_{k+1}^T \left[1 + \vec{\phi}_{k+1} \vec{P}_k \vec{\phi}_{k+1}^T \right]^{-1} \left\{ \vec{y}_{L+1} - \vec{\phi}_{k+1} \hat{\vec{\Theta}}_k \right\} . \qquad (18)$$

For a system without a direct transmission line, Equation (17) and (18) can be used together with the optimization of Equations (11), (12), (13) or (15) to compute the desired input to the system:

$$r_k = \frac{1}{\vec{b}_1} \left\{ \hat{\vec{y}}_{k+1} - \sum_{i=1}^{p} \vec{a}_{i1}^{(1)} \vec{y}_{k-i} - \sum_{i=1}^{p} \vec{b}_i^{(1)} \vec{r}_{k-i} \right\} . \qquad (19)$$

The identification method used for the optimum input calculation is the Observer Kalman-filter System Identification method (Juang, 1994).

4. INTELLIGENT LQR CONTROLLER DESIGN

In many cases the identification serves for the purpose of updating a controller, which may in turn also affect the performance of the input signal with regard to the identification results. One way to enhance the system identification results is to improve the identifiability of the system using a controller. The identification of the augmented system can then be accomplished using the Closed-Loop system Identification (CLID) algorithm proposed by Huang, *et al.* (1994). Consider the observability and controllability matrices multiplied out for a given system:

$$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix} =$$

$$\begin{bmatrix} CB & \dots & CA^{n-1}B \\ CAB & \dots & CA^{n}B \\ \vdots & \ddots & \vdots \\ CA^{n-1}B & \dots & CA^{2n-2}B \end{bmatrix} = \begin{bmatrix} H_1 & H_2 & \dots & H_n \\ H_2 & H_3 & \dots & H_{n+1} \\ \vdots & \vdots & \ddots & \vdots \\ H_n & H_{n-1} & \dots & H_{2n-2} \end{bmatrix}.$$

The last matrix is recognized as the Hankel matrix with the open-loop system Markov parameters as its entries. Since this matrix is the basis for the eigensystem realization algorithm (ERA), which employs an SVD of this matrix, the identifiability of the system is enhanced by improving the controllability and the observability. For this paper, we focus only on the controllability. We leave the detailed investigation on the effect to the identifiability for future work and only propose an intelligent controller design. The objective therefore is to enhance the controllability of the system using an intelligent LQR controller. This controller can be derived as follows: Considering a continuous time, linear time-invariant, or slowly time varying system

$$\dot{\vec{x}} = A\vec{x} + B\vec{u}\,, \qquad (20)$$

where $A \in R^{n*n}$ and $B \in R^{n*m}$. The system (A,B) is controllable if

$$rank\left(\left[A - sI, B\right]\right) = n \text{ for } \forall s \in C\,. \qquad (21)$$

Page (1981) defines the distance to uncontrollability as the spectral norm distance of the pair (A,B) from the set of all uncontrollable pairs:

$$d(A,B) = \min\left\{\left\|\left[E,H\right]\right\| : (A+E, B+H)\,uncontrollable\right\}$$

Elsner and He (1991) refined this as

$$d\left(A,B\right) = \min_{s\in C} \sigma_n\left(\left[A - sI, B\right]\right) = \min_{s\in C} \sigma\left(s\right), \quad (22)$$

where $\sigma_n(G)$ is the n^{th} singular value of the $(n*(n+m))$ matrix G. This implies that the problem of finding the distance to uncontrollability is the problem of minimizing $\sigma(s)$ over the complex plane. Elsner and He (1991) show that the function

$$f\left(s\right) = v_n^H\left(s\right)\begin{pmatrix} u_n\left(s\right) \\ 0 \end{pmatrix} \qquad (23)$$

is the partial derivative of $G(s)$ with respect to the components of *s*. The normalized left singular vector $\vec{u}_n(s)$ and the normalized right vector $\vec{v}_n(s)$ are the n^{th} column of U and V respectively, which results from the singular value decomposition $\left[A - sI, B\right] = U\Sigma V^H$, and H denotes the complex (Hermitian) transpose. In Elsner and He (1991) the function $f(s)$ is used to find the minimum distance to the uncontrollable region by use of an iterative Newton method. The disadvantage of this method is that a good starting value for the iteration is needed and that no guaranty is given that the global optimum is found. We propose an alternative approach by employing a simple genetic algorithm with a small mutation rate and sufficient randomness embedded into the selection, the pairing, mating operations to ensure the ability to find the global optimum point. The search area for the smallest singular value can be defined as (Elsner and He, 1991)

$$I_{real} = \left[\lambda_{\min}\left(\frac{A+A^T}{2}\right) \le x^* \le \lambda_{\max}\left(\frac{A+A^T}{2}\right)\right] \quad (24)$$

and

$$I_{imag} = \left[\lambda_{\min}\left(\frac{A-A^T}{2i}\right) \le y^* \le \lambda_{\max}\left(\frac{A-A^T}{2i}\right)\right] \quad (25)$$

where $s^* = x^* + iy^*$ defines the optimum point in the complex *s* plane. The minimization problem is formulated as follows:

Minimize $\quad \sigma_n\left(\left[A - (x+iy)I, B\right]\right) \qquad (26)$

subjected to $\quad I_{real_{\min}} \le x^* \le I_{real_{\max}}$

and $\quad I_{imag_{\min}} \le y^* \le I_{imag_{\max}}$.

To implement this in to a LQR controller design, consider the closed-loop system given by $(A+BF)$, where $F \in R^{m*n}$ is the feedback gain matrix.
Using Equation 22,

$$d\left(A+BF, B\right) = \min_{\mathrm{Re}(\lambda)\ge 0} \sigma_n\left(\left[A_c - \lambda I\right]\right) \quad (27)$$

where $A_c = A + BF$, the controller can be formulated as follows: Find F such that the minimum distance to uncontrollability is maximized. Expressed in matrix form:

$$Max\left\{Min\left\{\sigma_n\left(\left[A+BF-sI, B\right]\right)\right\}\right\} \qquad (28)$$

subjected to

$$I_{real_{\min}} \le x^* \le I_{real_{\max}} \;\; I_{imag_{\min}} \le y^* \le I_{imag_{\max}}\,. \qquad (29)$$

Since the actuation power is limited, an additional constraint can be imposed for actuator saturation:

$$\Psi_l \le r_{L+1} \le \Psi_u \qquad (30)$$

where ψ_l and ψ_u are the lower and upper limits of the actuator. The implementation of computing the controller parameters is done using a nested genetic algorithm. The weighting matrices Q and R of the linear quadratic regulator formulation

$$J_{LQR} = \sum\left\{\vec{x}^T Q\vec{x} + \vec{u}^T R\vec{u}\right\} \qquad (31)$$

are found using a GA formulation that employs the minimum distance to the uncontrollable region as the cost or objective function, given by Equation (28) and (29). Equation (28) subjected to constraint (29) is

also solved using a GA, with the search areas defined by Equations (24) and (25). The distance to the uncontrollable region is computed for each candidate weighting matrices Q and R. The gain matrix F is computed from these two weighting matrices.

5. SIMULATION RESULTS

For the numerical simulation a human respirator system model is used. The model used is based on Grodins, *et al. (*1954). For details on the system model and its linearization characteristics see Schoen (2002). The application of respiratory systems, its identification, and the application to control systems is manifold. The model given by Grodins, *et al.* (1954) was slightly altered by Sano and Kikucki (1985) to introduce an adaptive feedback control system for incubator oxygen treatments. The controller determines the optimum oxygen concentration of the mixed gas, which is forced into the incubator so that the partial oxygen pressure in the arterial blood flow is kept in a certain range. This is a necessary treatment for newborns who suffer from respiratory distress. In general, system identification in biomedical engineering is very helpful wherever modeling is difficult due to the lack of physical information or describing laws. The problem with system identification for biomedical systems is the danger to the system by exciting it to extract the dynamic features. Generally, input signals are severely constraint for such systems. The continuous time state-space description for the pulmonary ventilation is reproduced here for convenience and given as follows:

$$A = \begin{bmatrix} 0 & 1 \\ -1.8861 & -11.826145 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 760 \end{bmatrix} \text{ and}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \qquad (32)$$

For the numerical simulation the model given in Equation (32) was discretized using the zero-order hold method on the inputs with a sampling time of 0.025s. The ARX model and the predictive ARX filter represented the system best with a model order of 15. The first 150 input data points were generated as white gaussian noise with zero mean. After that, the input design algorithm was switched on using the different optimality definitions. The system was identified at every time step in order to track the error deviation between the true and reconstructed open-loop Markov parameters. The error percentage was calculated using the following equation:

$$\sum_{i=1}^{p} \frac{\left\| \hat{C}\hat{A}^{i-1}\hat{B} - CA^{i-1}B \right\|_F}{\left\| CA^{i-1}B \right\|_F} \qquad (33)$$

where $\hat{A}, \hat{B}, \hat{C}$ denote the identified system matrices and the subscript F denotes the Frobenius norm. For the simulation results, the measurement and process noise were set to 1% noise variance. All simulations were carried out with an input limitation of ± 1, representing the actuator saturation. Fig. 2 indicates that all four optimum input signal schemes reduce the error percentages of the open-loop Markov parameters compared to the random input signal.

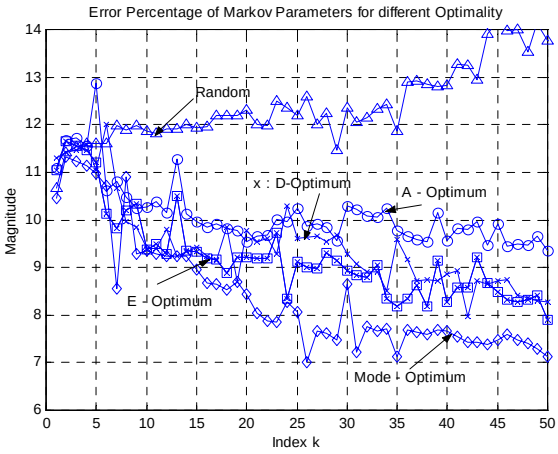

Fig. 2. Error percentages of open-loop system Markov parameters for all five input signals.

The difference in performance between the various optimal inputs is not significant for this example, the application and implementation in the embedded optimal evaluation on the other hand has value, in particular for biomedical systems. Consider certain known modes that are mapped to be harmful to the system. The Mode optimal computation permits evaluation of what modes are being excited, and therefore allows the exclusion of these potentially harmful signals. One would expect that for all four optimum input signal algorithms the error percentage should be a monotone-declining curve. Note that the spikes in the error percentage plot are associated with the fact that the predictive filter is based on the current information matrix, which is the objective for improvement. Another comparison of the performance of the identified system is to look at the correlation of the output of the estimated parametric models and the analytical model. The error can be given, see Fig. 1, as $\Delta y = y - \hat{y}$. Table 1 lists the correlation coefficient and the standard deviation of this error for the various optimal input signal algorithms and a random input. The calculations are based on a data set of 200 points, and an ARX model order of $p = 15$. At a first glance, one could conclude that the Mode-optimality performs well compared to the other algorithms, while the D-optimality is numerically close in terms of performance, at least numerically based on these results represented in Table 1 and the graph depicted in Fig. 2. Physically, these two algorithms operate in two different coordinate spaces. One can map the Mode-optimal algorithm and the A-, E-, and D-optimal algorithms into a common coordinate space by a series of similarity transformations of the covariance matrix. Using this mapping, one can easily show that, with the exception of the Mode optimal algorithm, all these algorithms are ultimately based on the eigenvalues of the current covariance matrix. The D-optimal, and for that matter the E-, and A-optimal algorithms, do not detect emerging modes that have not yet developed any influence on the eigenvalues of the covariance matrix.

Table 1: Correlation coeff. and Standard Deviation

	A-opt	E-opt	D-opt	M-opt	Rand.
Correlat.	0.969	0.9688	0.9697	0.9704	0.960
Std.	13.79	13.70	13.51	13.36	15.55

These emerging modes will not be promoted by the A-, D-, and E- optimal algorithms. On the other hand, the Mode optimal algorithm does not discriminate based on the influence of a mode to the eigenvalues, and thus is able to allow development of emerging, under-represented modes during the system identification process. If the promoted mode develops together with the constructed covariance matrix into having influence to the eigenvalues of the covariance matrix, the Mode optimal input algorithm will have an advantage over the other algorithms.

With respect to the results of the proposed intelligent controller, simulations were performed with the model given by Equation (32). The distance as defined by Equation (22) to the uncontrollable region of this open-loop model is 0.2697. Using the nested GA to compute the optimal weight matrices of the LQR formulation, the distance can be increased to 0.2841, an increase of over 5%. The nested GA uses 25 generations for the convergence of the optimal weighting matrices, and 10 iterations for the computation of the minimum distance to uncontrollability. Both GAs use a mutation rate of 4%. The eigenvalues of the open-loop system and the closed-loop system are

$$\lambda_{1_{ol}} = -2.1425 \qquad \lambda_{1_{cl}} = -22.0153$$

$$\lambda_{2_{ol}} = -9.6837 \qquad \lambda_{2_{cl}} = -184.5028.$$

The cost function and its convergence for finding the minimum distance is depicted in Fig. 3a. A similar plot can be established for the cost of the outer GA loop to find the optimal weights for the LQR controller. The response of the compensated and uncompensated system is depicted in Fig. 3b. The two graphs also depict the rise time, which is for the compensated system around 0.05 seconds and for the uncompensated system about 1.2 seconds.

6. CONCLUSIONS

This paper presents a new algorithm architecture for optimal input signal implementation for an on-line identification algorithm. The Mode-optimal input algorithm has the potential to perform better than the other covariance based optimal input algorithms. A nested GA is used to find the optimal LQR weighting matrices. The optimization is based on the objective to increase the minimum distance to uncontrollability with sight to increase the identifiability. The proposed nested GA works well to define the intelligent LQR controller. Future work is necessary for imbedding an observability objective into the controller design and test this with a closed-loop system identification algorithm along with the proposed optimal input architecture.

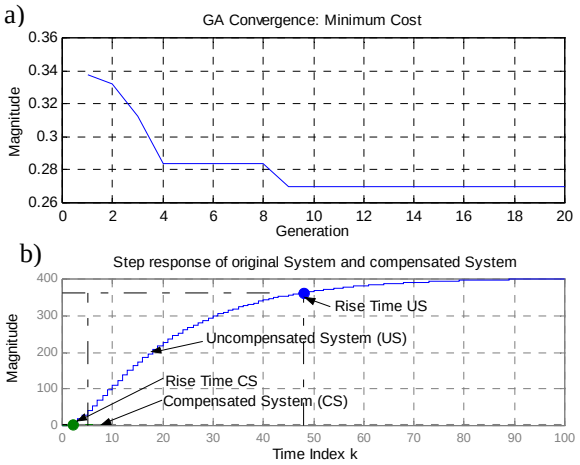

Fig. 3. Part a) Convergence plot for GA using minimum distance to uncontrollability. Part b) Step response of uncompensated and compensated system.

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