AN IDENTIFICATION METHOD OF SEEK-INDUCED VIBRATION MODES IN HARD DISK DRIVES

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Abstract: A method to identify the transfer function of HDD head/actuator system modes that have gains that are smaller than the rigid-body mode is presented. Prony's method is used to identify the frequency and damping of the modes, and the measured actuator current and PES in the time-domain are used to identify the gain of the transfer function of the modes. Overall accuracy is verified by comparing the measured PES and the estimated PES by the obtained transfer function. Seek simulations are performed that demonstrate that modes which cannot be identified by frequency domain analysis have a significant influence on the seek-settling characteristics. It is also shown that although a smooth actuator current helps to reduce the high frequency vibrations at settling, it degrades the effect of vibrations due to the coil and other low frequency modes, such as the dynamics of the flexible printed circuit attached to the actuator.

Keywords: Actuators, Command signal, Fourier analysis, Frequencies, Least squares, Minimization, Servomechanisms, Position Errors, Trajectories

1. INTRODUCTION

As the data capacity of hard disk drives (HDD) increases, the servo-mechanical designs of actuators are becoming more important. The servo controller is designed to meet the track-following performance by using the disturbance spectra and the mechanical characteristics of the actuator. The actuator characteristics can be approximated by the rigid-body mode and the resonances which have frequencies that are normally higher than 4-5 kHz. The HDD actuator characteristics are usually identified by the frequency response analysis of the head position when the actuator is excited by a frequency sweep or random excitation signal. This technique is effective for identifying modes at such high frequencies, and the servo transfer functions can be well predicted by simulations. However, for the seek characteristics, especially at seek settling, behaviors are harder to predict by simulations because other dynamics of the head/actuator system are involved in the seeking.

Fig. 1 shows a flexible printed circuit which is attached to the actuator and carries electrical signals. It has its own vibration modes in the frequency range from 100 Hz to 1 kHz. The coil, which is located at the other end of the actuator from the head, also has its own dynamics around 1 or 2 kHz. These modes are concerns for the seek-settling characteristics, because they affect the settling behavior. A better design optimization method has been explored (Wickert, 2003; Brakes and Wickert, 2004) , but it still requires hardware build and experiments with the servo loop closed in order to confirm the seek settling characteris-

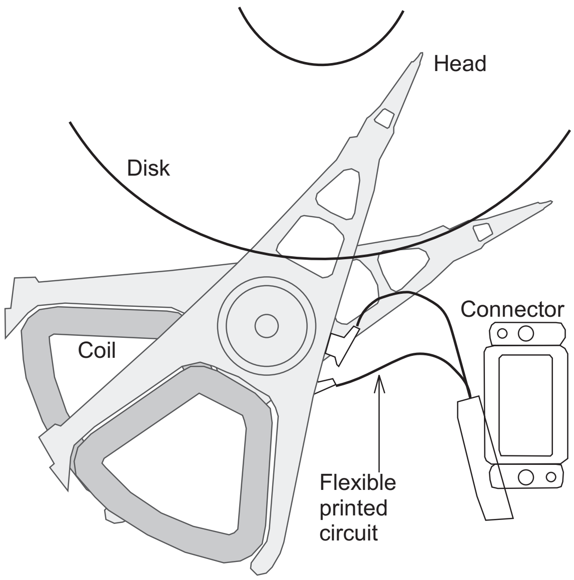

Fig. 1. Actuator and flex circuit cable in a HDD

tics. The reason for this difficulty is that the gain of the rigid-body mode is higher than the gain of these modes and so the rigid-body mode masks the response of these modes during the frequency response measurements.

A simple method to identify these modes is the frequency spectrum analysis of the position error signal (PES) after settling, which is widely used. However, it does not consider the actuator current waveform, and sometimes the identified frequency is disturbed by the frequency content of the current waveform. Many methods of time-domain identification have been developed in the area of signal processing (Ljung, 1987) , but they are not suitable for the analysis of modes which affect seek settling. In a seek operation of a disk drive, the major component of the control output is due to the trajectory from the starting track to the target track. The PES at settling is much smaller than the trajectory signals. Conventional identification methods generally do not provide enough resolution to analyze modes affecting the settling.

This paper presents a method to identify the transfer function of these modes which have gains that are smaller than the rigid-body mode by using a time-domain analysis technique. The technique is based on Prony's method (Therrien and Velasco, 1995; Numasato, et al., 1999; Trudnowski, et al., 1999; Ruiz-Vega, et al., 2002). However, since Prony's method is designed to analyze the impulse response, it is extended in this paper to identify the transfer function of modes. The seek-settling waveforms are simulated for different seek trajectory designs to confirm the effects.

2. PRONY'S METHOD

Prony's method is designed to identify the amplitude, damping, and frequency of modes of the impulse response waveform. When the actuator dynamics can be expressed as the following equation

$$P(s) = \frac{k_0}{s^2} + \sum_{r=1}^{n} \frac{a_r \omega_r}{s^2 + 2\zeta_r \omega_r s + (1+\zeta_r^2)\omega_r^2}, \quad (1)$$

the impulse response of the second term can be expressed as

$$h(t) = \sum_{r=1}^{n} a_r e^{-\zeta_r \omega_r t} \sin(\omega_r t). \quad (2)$$

Assuming $t = kT_s$, the impulse response in discrete-time is

$$h(k) = \sum_{r=1}^{n} a_r \mathrm{Im}\left[x_r^k\right], \quad (3)$$

where $x_r = \exp(-\zeta_r \omega_r T_s - j\omega_r T_s)$ and T_s is the sampling frequency. When a polynomial which has roots of x_r and its complex conjugate x_r^* is introduced, the following relationship is obtained

$$\prod_{r=1}^{n} (z - x_r)(z - x_r^*) = \sum_{r=0}^{2n} b_r z^k = 0, \quad (4)$$

where $b_{2n} = 1$ and b_r are real parameters of the coefficients of the polynomial. When b_r is multiplied for every term of $h(k)$, then the total sum about k will be from the previous equation

$$\sum_{k=0}^{2n} b_k h(k) = \mathrm{Im}\left[\sum_{r=0}^{n} a_r \sum_{k=0}^{2n} b_r x_r^k\right] = 0. \quad (5)$$

Since $b_{2n} = 1$ and $h(2n) + \sum_{r=1}^{2n-1} b_r h(k) = 0$, the value of b_k can be analytically solved by the impulse responses $h(k)$.

In practical applications, since the measured signal is noisy, the following least-square (LS) method is introduced to improve the accuracy for longer data lengths. The cost function to minimize is

$$J = \sum_{i=0}^{M} \left(\sum_{k=0}^{2n-1} b_k h(k+i) + h(2n+i) \right)^2. \quad (6)$$

Since the partial derivative is equal to zero at the optimal value,

$$\frac{\partial J}{\partial b_k} = \sum_{i=0}^{M} 2h(k+i) \left(\sum_{k=0}^{2n-1} b_k h(k+i) + h(2n+i) \right) = 0,$$

the following matrix equation is obtained

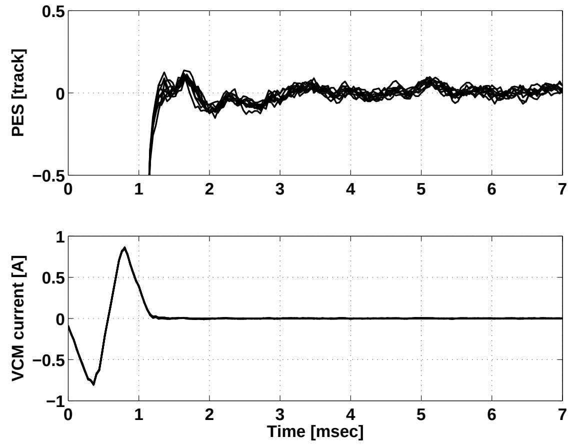

Fig. 2. Actuator current and PES measured in a HDD

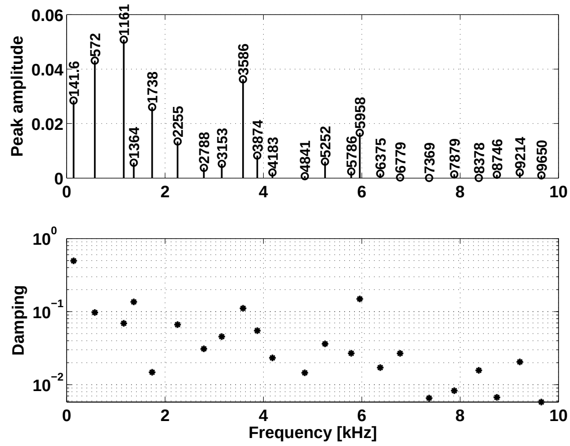

Fig. 3. Identified amplitude and damping of modes as a function of their frequency

$$\begin{bmatrix} -H_{0,2n} \\ \vdots \\ -H_{2n-1,2n} \end{bmatrix} = [H_{j,k}] \begin{bmatrix} b_0 \\ \vdots \\ b_{2n-1} \end{bmatrix}$$

$$H_{j,k} = \sum_{i=0}^{M} h(i+j)h(i+k). \tag{7}$$

The values of b_k are solved by this equation and x_r, ζ_r, and ω_r are obtained from eq. (4). The amplitude a_r can also be obtained by solving for the minimum least squares with the cost function

$$J = \sum_{i=0}^{M} \left(h(k) - \sum_{r=0}^{R} a_r h_r(k) \right)^2 .$$

Fig. 2 is the waveform of the actuator current and PES signal measured in a HDD. The PES signal after settling is analyzed by this method to obtain the amplitude, damping, and frequency of each mode as in Fig. 3.

However, since the input excitation is assumed to be an impulse, this amplitude does not correspond to the transfer function of $P(s)$. If the servo controller generates a control input which has large components at particular frequencies, the PES will have larger signals of that component and the amplitude a_r will increase. Therefore, it is not easy to identify whether the modes are due to mechanical resonances or due to servo control characteristics by looking only at the PES at the seek settling.

3. TRANSFER FUNCTION IDENTIFICATION

3.1 Rigid-body mode

The method described in the previous section is used to identify modes by assuming that the PES after settling is similar to the impulse response of the actuator. In order to evaluate the effect of seek-induced vibration modes, it is necessary to identify the gain relationship between the rigid-body mode and the vibration modes, because the settling vibration is evaluated by the PES amplitude when the rigid-body mode moves the head a certain distance. The effect of the rigid-body mode is expressed by the following double summation

$$y(i) = \sum_{j=0}^{i} \sum_{k=0}^{j} \left(b_0 u(k) - b_1 \right), \tag{8}$$

where $u(i)$ and $y(i)$ are the input and the output of the actuator, and b_0 is the gain of the actuator. The term b_1 is added to cancel the effect of the DC bias force. The parameters which minimize the following cost function can be obtained by the LS method

$$J = \sum_{i=0}^{M} \left(y(i) - b_0 \sum_{j=0}^{i} \sum_{k=0}^{j} u(k) - b_1 \sum_{j=0}^{i} \sum_{k=0}^{j} 1 \right)^2 .$$

The resulting gain is

$$\begin{bmatrix} b_0 \\ b_1 \end{bmatrix} = \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=0}^{M} \left(y(i) \sum_{j=0}^{i} \sum_{k=0}^{j} u(k) \right) \\ \sum_{i=0}^{M} \left(y(i) \sum_{j=0}^{i} j \right) \end{bmatrix},$$

where

$$U_{11} = \sum_{i=0}^{M} \left(\sum_{j=0}^{i} \sum_{k=0}^{j} u(k) \sum_{j=0}^{i} \sum_{k=0}^{j} u(k) \right)$$

$$U_{12} = U_{21} = \sum_{i=0}^{M} \left(\sum_{j=0}^{i} j \sum_{j=0}^{i} \sum_{k=0}^{j} u(k) \right)$$

$$U_{22} = \sum_{i=0}^{M} \left(\sum_{j=0}^{i} j \sum_{j=0}^{i} j \right).$$

The rigid-body mode gain k_0 in eq. (1) can be obtained by substituting $b_0 = k_0 T_s^2/2$.

3.2 Transfer function of vibration modes

After the frequency and the damping of each mode are identified by Prony's method, the gain of the transfer function can be estimated. Assuming the transfer function gain for each mode is g_r, the control output is formulated by the control input $u(n)$ with the following equation

$$y(n) = \sum_{k=0}^{\infty} h(k)u(n-k) = \sum_{r=0}^{R} g_r y_r(n), \qquad (9)$$

where

$$h(k) = \sum_{r=0}^{R} g_r h_r(n)$$

and

$$y_r(k) = \sum_{k=0}^{\infty} h_r(n)u(n-k).$$

The optimal gain for each mode minimizes the following square error:

$$J = \sum_{i=N}^{M} \left(y(i) - \sum_{r=0}^{R} g_r y_r(i) \right)^2, \qquad (10)$$

where N is the settling point and M is the last available sample. The purpose of this error function is to minimize the error between the measured PES after settling and the estimated PES from the current. The optimal gain g_r can be obtained by minimizing the error function J. Then the result is

$$\begin{bmatrix} g_0 \\ \vdots \\ g_R \end{bmatrix} = \begin{bmatrix} Y_{0,0} & \cdots & Y_{0,R} \\ \vdots & \ddots & \vdots \\ Y_{R,0} & \ddots & Y_{RR} \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=N}^{M} y(i)y_0(i) \\ \vdots \\ \sum_{i=N}^{M} y(i)y_R(i) \end{bmatrix},$$

where

$$Y_{j,k} = \sum_{i=N}^{M} y_j(i)y_k(i). \qquad (11)$$

Fig. 4 shows the Fourier spectrum of the PES after the settling and the actuator current. Fig. 5 shows the estimated transfer function which is obtained from the PES after settling and the actuator control signal. The straight line above the peaks represents the characteristics of the rigid-body mode. It indicates that the modes below 2 kHz which have gains that are lower than the rigid-body mode have a strong influence on the PES after settling. Some of the peaks of the PES spectrum correspond to the mechanical modes of the actuator. However, since the PES is not captured for a long enough time and the modes quickly decay, the resolution of the frequency is not accurate. On the other hand, the actuator current spectrum shows some peaks, but they do not seem to correspond to the PES spectrum. This indicates that the conventional Fourier analysis of the control input and the output does not work effectively for the mode identification. Fig. 6 shows two traces of the PES. One is the measured PES after seek settling and the other is estimated by the transfer function identification method previously described. Since they match reasonably well, this indicates that Prony's method and its extension give us good estimate of the transfer function of the actuator dynamics. There are many modes around 4-5 kHz in Fig. 5 where the PES spectrum in Fig. 4 does not have peaks. They are an indication of airflow-induced flutter inside

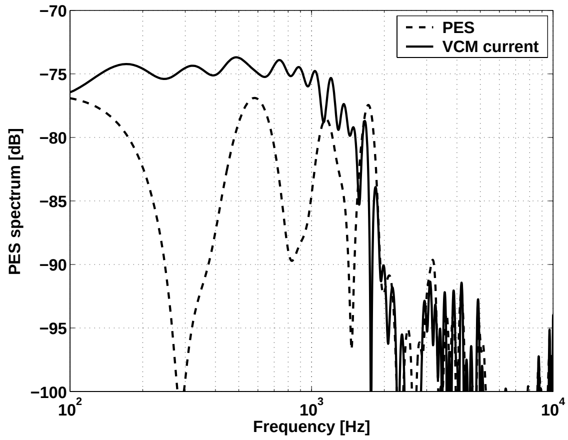

Fig. 4. Frequency spectrum of PES and actuator current

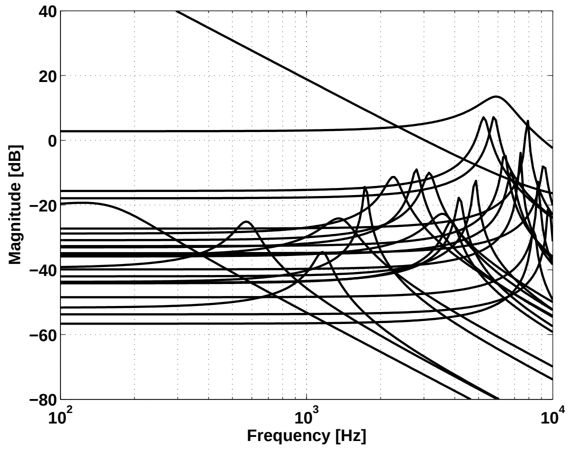

Fig. 5. Estimated transfer function of each mode for $P(s)$

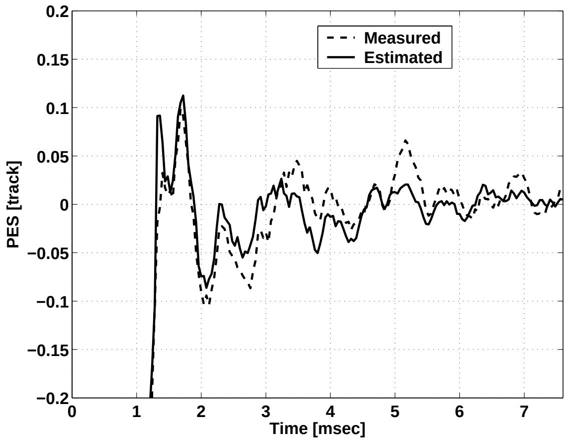

Fig. 6. Settling PES of measured and estimated by the transfer function and current

the drive which is not correlated to the actuator control signal.

4. SEEK CONTROL SIMULATION

Since the PES after seek-settling can be estimated by the method described in the previous section, the optimal control input during seeking can also be evaluated. Fig. 7 shows the block diagram of the seek control structure for the two-degrees-of-freedom control (TDOF) (Ishikawa et al., 1996). It has been used to improve the seek control behavior. An actuator model is used in the TDOF structure and the head trajectory is controlled to match the one which is generated by the model. The feedback controller works to suppress the error between the actual and the model and the error due to disturbances. The PES deviation from the nominal trajectory for this structure can be formulated by the following equation.

$$E(s) = S(s)\,(P(s) - P_m(s))\,F(s), \qquad (12)$$

where $S(s)$ is the sensitivity function shown in Fig. 8. The variables $P(s)$ and $P_m(s)$ are the actuator dynamics and the nominal model, respectively. $F(s)$ is the command signal which is equal to the actuator control signal when there is no model mismatch. Since the model error $P(s) - P_m(s)$ corresponds to the second term of eq. (1), the PES deviation from the nominal trajectory is the sum of the responses of the modes which have been identified by the previous section. It should be also noted that since the sensitivity function is involved, some of the responses are amplified depending on their frequency.

In order to evaluate the effect of the modes which have gains that are below the gain of the rigid-body mode, seek simulations have been performed for different actuator control waveforms. There

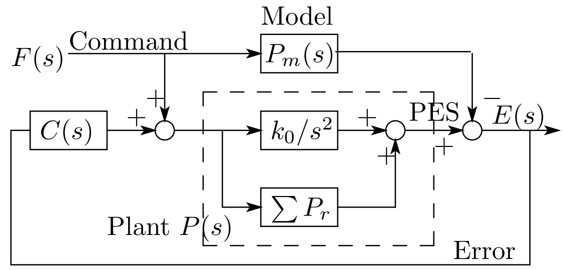

Fig. 7. Two-degrees-of-freedom control

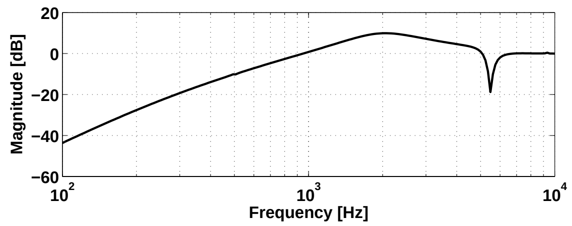

Fig. 8. Typical sensitiviy function of HDD servo controller

has been extensive work by many researchers in terms of optimal seek trajectories for HDDs (Singer and Seering, 1990; Mizoshita, et al., 1996; Ishikawa, et al., 1996), but in this paper the following two simplified waveforms have been used for the comparison. One is the sinusoidal waveform given by

$$\begin{aligned} f(t) &= \frac{2\pi K}{T_{sk}^2}\sin(\frac{2\pi t}{T_{sk}}) \quad (0 \le t \le T_{sk}) \\ &= 0 \qquad (t > T_{sk}). \end{aligned}$$

The actuator control signal is normalized by reaching the seek distance $K = 200$ tracks at seek time $T_{sk} = 1.2$ msec. Fig. 9 shows the simulation results of the input control signal and the response of the mode at 1738 Hz which appeared in Fig. 5. The actual response is the total sum of the responses of all modes, but the response of the mode with the largest gain is picked for this simulation. The waveforms for other seek lengths are also plotted. It can be observed that even though the majority of the frequency content is lower than 1 kHz in the control signal, the 1738 Hz mode is easily excited. When the seek time is 2 msec, the majority of the frequency content is concentrated at 500 Hz. Even in this case, the settling vibration is still observed in the PES and it is not negligibly small. This is even more serious when the number of tracks per inch (TPI) increases, because the PES error becomes larger relative to the track pitch.

Fig. 10 shows the second case which has transition points that are smoothed by multiplying a window function curve as follows

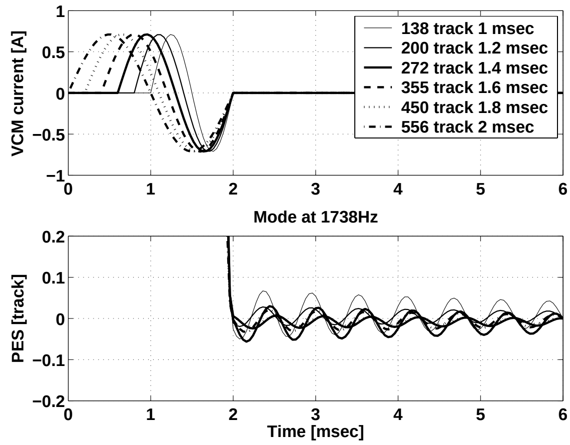

Fig. 9. Seek settling with sinusoidal current

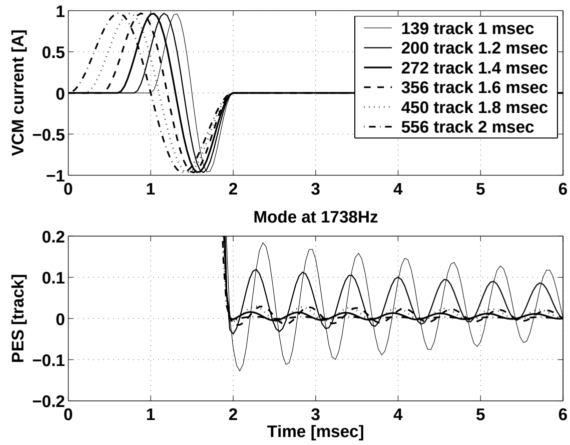

Fig. 10. Seek settling with smooth current

$$f(t) = \frac{9\pi K}{8T_{sk}^2} \sin(\frac{\pi t}{T_{sk}}) \sin(\frac{2\pi t}{T_{sk}}) \quad (0 \leq t \leq T_{sk})$$
$$= 0 \qquad (t > T_{sk}).$$

The PES settling vibration is larger in this case, because the amplitude of the control input increases and the amplitude of the low frequency component also increases. This is due to the penalty of the additional smoothing. It should be noted that the smooth waveform of the control input is effective for the reduction of the effects of high frequency vibration modes, but it is not appropriate for these low frequency modes. This effect will become a bigger issue when the track density increases, because although high frequency modes are suppressed by the notch filters, the low frequency modes cannot be suppressed unless the servo control bandwidth is increased. Therefore, the seek control trajectory has to be determined not only with respect to the high frequency actuator modes, but also for the low frequency modes.

5. CONCLUSIONS

A method to identify transfer functions of HDD actuator modes which are smaller than the rigid-body mode is developed. It is an extension of Prony's method in which the measured actuator control signal and PES in the time-domain are used. According to the results of seek simulations, it is clarified that the modes which cannot be identified by the frequency domain analysis have a big influence on the seek-settling characteristics. It is also identified that although the smooth actuator current helps to reduce the high frequency vibrations at settling, it degrades the effect of vibrations due to the low frequency modes. For a HDD with higher track density, it is important to lower these modes by appropriate design of the actuator system and/or to increase the servo bandwidth in proportion to the track density.

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