ON THE ROOT INVARIANT REGIONS STRUCTURE FOR LINEAR SYSTEMS

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Abstract: D-decomposition technique is targeted to describe the stability domain in parameter space for linear systems, depending on parameters. The technique is very simple and effective for the case of one or two parameters. However the geometry of the arising parameter space decomposition into root invariant regions has not been studied in detail; it is the purpose of the present paper. We prove that the number of stability intervals for one real parameter is no more than $n/2$ (n being the degree of the characteristic polynomial) and provide an example, where this number is achieved. For one complex or two real parameters we estimate the number of root invariant regions (equal n^2-2n+3 for complex and $2n^2-2n+3$ for real case) and demonstrate that this upper bound is tight. The example with $n-1$ simply connected stability regions in $2D$ parameter plane is analyzed.

Keywords: Characteristic polynomials, stability analysis, stability domain, linear systems, Nyquist diagrams.

1. INTRODUCTION

Robust stability analysis is a trivial task if one has a full description of the stability domain in the parameter space. The approaches to this challenging problem can be traced to 19th century. Vishnegradsky (1876) investigated the stability domain for the third order polynomial with two uncertain parameters. Later Frazer and Duncan (1929) developed the method for the general case of the n^{th}-order polynomial; however it required to find all the roots of $n \times n$ Hurwitz determinant. The famous Nyquist diagram can be interpreted as a graphical tool for checking stability of a polynomial with one (real or complex) parameter. The final technique is due to Neimark (1948 and 1949), who developed his D-decomposition method for the stability domain analysis. The core of the approach is decomposition of the parameter space into root invariant regions; the boundaries of the regions are defined by a system of equations. This method in the Western literature is addressed in the books (Siljak, 1969; Ackermann, 2002). Its use for the design of low-order controllers for linear systems is very efficient, see e.g. (Bhattacharyya *et al.*, 2003).

Until recently, the geometry of D-decomposition was not well studied. In some particular cases one can validate that the stability domain is simply connected. However the structure of the stability domain can be much more complicated. For instance the examples were known with several stability intervals for a gain. In the recent paper (Nikolayev, 2002) there is an example, where the stability domain consists of $n-1$ simply connected regions for two uncertain parameters. Here several problems arise: how many root invariant regions are there in the parameter space? What is the maximal (minimal) number of the stability regions? In the present paper we address these problems for characteristic polynomials, depending linearly on one or two parameters. The boundary of the root invariant regions, generated by D-decomposition method, is an algebraic curve. The topological properties of algebraic curves refer to the 16^{th} Hilbert Problem (Ilyashenko and Yakovenko, 1995). Thus it is natural that we exploit some algebraic geometry tools for our analysis.

We deal with continuous and discrete-time systems. Here and elsewhere we denote by $P(s, \lambda)$ any continuous-time polynomial with an uncertain parameter λ and by $P(z, \lambda)$ any discrete-time polynomial. The first one is stable when all its roots have negative real parts, the second one is stable when all its roots are inside the unit circle. Using the mapping $s = \frac{z+1}{z-1}$ we can proceed from a continuous-time system to a discrete-time one and vice versa.

The paper is organized as follows. In Section 2 we explain the idea of the D-decomposition technique. In Section 3 we analyze one real parameter polynomial family. A theorem about the maximal number of root invariant intervals (and in particular the maximal number of stability intervals) is stated. The example showing the attainability of this upper bound is provided. Sections 4 and 5 are devoted to one complex parameter family and two real parameters family correspondingly. Several examples demonstrate that the geometry of D-decomposition can be fairly sophisticated. Some preliminary results are considered in (Gryazina, 2004).

2. D-DECOMPOSITION

Let $P(s, \lambda)$ be a polynomial of degree n with real coefficients $a_k(\lambda)$, where $\lambda \in \mathbb{R}^m$ is an uncertain parameter:

$$P(s, \lambda) = a_n(\lambda)s^n + a_{n-1}(\lambda)s^{n-1} + \ldots + a_0(\lambda).$$

If $P(s, \lambda)$ has k stable and $n - k$ unstable roots we say $\lambda \in D(k)$; thus $D(n)$ is a *stability domain*. Simply connected components of all $D(k)$s generate the decomposition of $\mathbb{R}^m$ into *root invariant regions*, our goal is to describe their boundaries. To abandon $D(k)$ λ should encounter one of the following situations: 1) the polynomial has an imaginary root, that is $P(j\omega, \lambda) = 0$ for some ω, 2) the polynomial has a zero root, i.e. $a_0(\lambda) = 0$, 3) the polynomial changes its degree, i.e. $a_n(\lambda) = 0$. Thus the boundary of each $D(k)$ can consist of the curves, generated by these three equations (in the first one $\omega \in (-\infty, \infty)$ is considered as a parameter):

$$P(j\omega, \lambda) = 0 \tag{1}$$

$$a_0(\lambda) = 0, \quad a_n(\lambda) = 0 \tag{2}$$

Note that equation (1) is equivalent to two real equations (for real and imaginary parts of $P(j\omega, \lambda)$). Equations (1), (2) define D-*decomposition* of the parameter space — they characterize the boundary of root invariant regions $D(k)$. We do not discuss here how to find k for each region (this issue will be addressed later).

The same technique can be used to construct a boundary of the regions with certain number of real roots. Indeed the number of real roots can change when a multiple real root arises, that is for some $s \in \mathbb{R}$

$$\begin{array}{c} P(s, \lambda) = 0 \\ P'(s, \lambda) = 0 \end{array} \tag{3}$$

Similar equations can define the domain of *aperiodic stability*, that is the set of parameters which guarantee that all roots are stable and real.

Example (Vyshnegradsky, 1876). Consider a cubic polynomial reduced to Vyshnegradsky's form (that is with $a_3 = 1, a_0 = 1$):

$$P(s, \lambda_1, \lambda_2) = s^3 + \lambda_2 s^2 + \lambda_1 s + 1. \tag{4}$$

Solving the system of equations (1) (note that $a_3 \neq 1, a_0 \neq 1$) we have the parametrized curve $\lambda_1(\omega) = \omega^2, \lambda_2(\omega) = 1/\omega^2$, or after elimination of ω, nonparametric formula for the boundary of stability domain is $\lambda_1\lambda_2 = 1$, while stability domain itself is defined by inequality $\lambda_1\lambda_2 > 1, \lambda_1 > 0, \lambda_2 > 0$. For the same example equations (3) become $\lambda_1(s) = -\frac{2}{s} + s^2$, $\lambda_2(s) = \frac{1}{s^2} - 2s$, where s is a real parameter. Figure 1 depicts stability domain and the regions with a certain number of real roots (they are marked by digits).

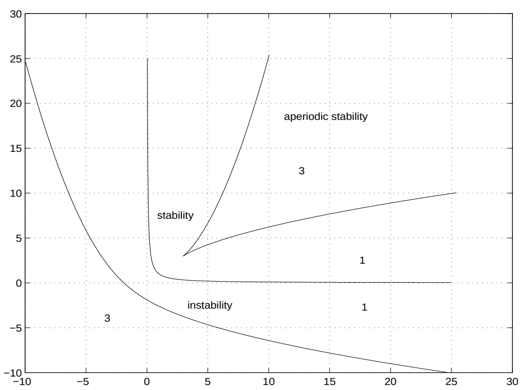

Fig. 1. D-decomposition for cubic polynomial

We provided the basic D-decomposition technique for continuous-time polynomials. It can be extended with minor changes to discrete-time polynomials $P(z, \lambda)$. The imaginary axis should be replaced by the unit circle, thus the equation of D-decomposition (1) reads

$$P(e^{j\omega}, \lambda) = 0. \tag{5}$$

Note that there is no analog of (2) because degree dropping does not affect instability of discrete-time polynomials. Similarly more general Γ-stability can be treated (a polynomial is Γ-*stable*, if all its roots lie in a domain Γ of the complex plane). Then the boundary of Γ should be taken instead of the imaginary axis or the unit circle.

3. ONE REAL PARAMETER

Consider the one real parameter polynomial family

$$P(s,\lambda) = \{a(s) + \lambda b(s),\ \lambda \in \mathbb{R}\}, \qquad (6)$$

where $a(s) = a_n s^n + a_{n-1}s^{n-1} + ... + a_1 s + a_0$ and $b(s) = b_n s^n + b_{n-1}s^{n-1} + ... + b_1 s + b_0$ are given polynomials with real coefficients of degree n. When λ varies, the number of stable roots of $P(s,\lambda)$ can vary as well; we call such values of λ *critical values*. Below we estimate their number.

Theorem 1. In the polynomial family (6) there exist no more than $n+1$ root invariant intervals and no more than $\lceil \frac{n}{2} \rceil$ stability intervals.

Proof Critical values are the solutions of (1). Represent it as:

$$P(j\omega,\lambda) = U_0 + \lambda U_1 + j\omega(V_0 + \lambda V_1) = 0,$$

where

$$\begin{aligned} U_0 &= a_0 - a_2\omega^2 + a_4\omega^4 + ... \\ V_0 &= a_1 - a_3\omega^2 + a_5\omega^4 + ... \\ U_1 &= b_0 - b_2\omega^2 + b_4\omega^4 + ... \\ V_1 &= b_1 - b_3\omega^2 + b_5\omega^4 + ... \end{aligned}$$

A solution of two linear equations with one variable $\lambda = \lambda(\omega)$ exists if and only if

$$U_0 V_1 - U_1 V_0 = 0. \qquad (7)$$

Left-hand side of (7) is equal to $Im(\lambda(\omega))$, the $n-1$ order polynomial in ω^2. So there exists no more than $n-1$ different real values of $\lambda = -\dfrac{U_0}{U_1} = -\dfrac{V_0}{V_1}$ which are critical ones. Equation (2) provides two extra critical values. One is $\lambda = -a_n/b_n$, while another is $\lambda = -a_0/b_0$. Distinct $n+1$ points divide the λ axis into $n+1$ root invariant intervals (intervals $\lambda \longrightarrow +\infty$ and $\lambda \longrightarrow +\infty$ we regard as the same interval). Since two neighboring intervals can't be both stability intervals, there can be no more than $\lceil \frac{n}{2} \rceil$ stability intervals. $\diamond$

We suggest an algorithm to calculate the number of stable roots in every root invariant interval which is purely algebraic, not graphical.

Algorithm 1. i. Order the values $\lambda(\omega_i)$, ω_i being solutions of equation (7), and extra two values $\lambda = -a_n/b_n, \lambda = -a_0/b_0$ such that $\lambda_1 < \lambda_2 < ... < \lambda_s$.

ii. When $\lambda < \lambda_1$ $P(s,\lambda)$ has the same number of stable roots as $b(s)$ has and it is easy to obtain this number.

iii. We proceed by increasing λ. When λ passes one of the λ_i values one or two roots become stable or unstable. For any of two extra critical values only one real root moves. For all other ω_i two conjugate roots cross imaginary axis. If $\left.\dfrac{d\ Im(\lambda(\omega))}{d\omega}\right|_{\omega_i} < 0$ the roots become stable and if the derivative is positive − unstable. So adding and subtracting the proper number of roots we get the stable root number for every root invariant region.

The result is also valid for discrete-time case. Let us consider an example with the maximal number of root invariant intervals.

Example 1 The uncertain polynomial

$$P(z,\lambda) = z^n + \lambda z^{n-1} + \varepsilon z^{n-2} + \alpha \qquad (8)$$

where $1 < \varepsilon < 1 + \frac{2}{(n-2)^2}$, $\alpha = 1 - \varepsilon - \frac{1}{n^2}$ has $\lfloor \frac{n}{2} \rfloor$ stability intervals. In Fig. 2 digits mark the number of stable roots in every interval for $n = 8,\ \varepsilon = 1.01,\ \alpha = -0.026$.

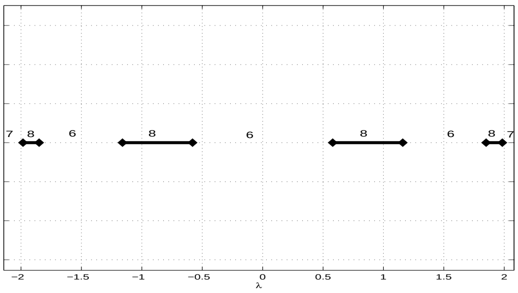

Fig. 2. $\frac{n}{2}$ stability intervals in Example 1

The critical values are defined by

$$\lambda = -e^{j\omega} - \varepsilon e^{-j\omega} - \alpha e^{-(n-1)j\omega},\ Im(\lambda) = 0.$$

The last equation

$$(\varepsilon - 1)\sin\omega + \alpha \sin(n-1)\omega = 0$$

has n solutions in the segment $[0,\pi]$ because $|\alpha| > |\varepsilon - 1|$. When $\lambda < \lambda_1$ the polynomial has one unstable root and the signs of the derivatives alternate in critical points. That is why we can obtain the maximal number of the stability intervals in this example.

Example 2 Let for $n = 4m$

$$P(s,\lambda) = s^n + \lambda s + 1.$$

Then $P(j\omega,\lambda) = \omega^n + \lambda j\omega + 1$, and $\mathrm{Re}P(j\omega,\lambda) \neq 0$ for all λ. Thus there are no critical values of ω, and the entire real axis is the single root invariant region for the polynomial $P(s,\lambda)$ (indeed it has $2m$ stable and $2m$ unstable roots for any λ). Thus the least number of root invariant regions is one. Minor variation of the example $P(s,k) = \lambda(s^n + 1) + s$ provides real axis with the exception of the origin as the root invariant region: for any $\lambda \neq 0, P(s,\lambda)$ has $2m$ stable and $2m$ unstable roots.

Theorem 1 can be stated in terms of the Nyquist criterion. Consider a plant with the transfer function $H(s) = \dfrac{a(s)}{b(s)}$ closed by P-controller with gain k. The closed-loop characteristic polynomial is $ka(s) + b(s)$. Substituting $\lambda = -\frac{1}{k}$, we get the polynomial family (6).

Theorem 1'. The Nyquist diagram $H(j\omega)$ has no more than $n+1$ intersections $\mu_i, i = 1,\ldots,n+1$ with the real axis. The interval (μ_i, μ_{i+1}) is a stability region for k $(\frac{1}{\mu_{i+1}} < k < \frac{1}{\mu_i})$ if

$$2(m_+ - m_-) + p + \delta = 0,$$

where m_+ (m_-) is the number of bottom-up (top-down) intersections of $H(j\omega)$ with the real axis below μ_i, p is the number of unstable roots for $b(s)$ and

$$\delta = \begin{cases} 1, & -\dfrac{a_0}{b_0} < \mu_i < -\dfrac{a_n}{b_n}; \\ -1, & -\dfrac{a_n}{b_n} < \mu_i < -\dfrac{a_0}{b_0}; \\ 0, & \text{otherwise.} \end{cases}$$

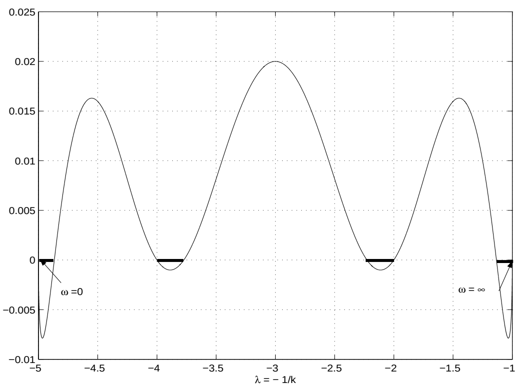

Fig. 3. The Nyquist diagram

Consider the continuous-time version of Example 1:

$$\begin{aligned} a(s) =&\ (s+1)^n + \varepsilon(s+1)^{n-2}(s-1)^2 + \\ &+ \alpha(s-1)^n, \\ b(s) =&\ (s+1)^{n-1}(s-1); \end{aligned}$$

Figure 3 depicts its Nyquist plot for n=8. The system $\lfloor \frac{n}{2} \rfloor$ times acquires and loses stability.

4. ONE COMPLEX PARAMETER

A complex counterpart of polynomial family (6) is

$$P(s,\lambda) = \{a(s) + \lambda b(s),\ \lambda \in \mathbb{C}\}, \qquad (9)$$

where $a(s)$ and $b(s)$ are given real polynomials of degree n.

The curve

$$\lambda(\omega) = -\frac{a_0 + a_1 j\omega + \ldots + a_n (j\omega)^n}{b_0 + b_1 j\omega + \ldots + b_n (j\omega)^n}, \quad \omega \in (-\infty, \infty) \qquad (10)$$

forms the root invariant regions boundary. It is bounded when the polynomial $b(s)$ has no roots on the imaginary axis.

Theorem 2. For the polynomial family (9) there exist no more than $(n-1)^2 + 2$ root invariant regions.

Proof. The number of regions depends on the number of self-crossing points of the boundary curve. A curve without self-crossing points divides the parameter plane into two regions, and every simple self-crossing point appends an extra region. So the idea of the proof is based on the computation of the number of self-crossing points of the algebraic curve (10). Self-crossing points are specified by

$$\lambda(\omega_1) = \lambda(\omega_2), \quad \omega_1 \neq \omega_2.$$

It is equivalent to the system of equations

$$\begin{aligned} \sum_{i=0}^{n-2} \sum_{l=1}^{\frac{n}{2}} (-1)^{i+l} c_{ik} \omega_1^i \omega_2^i (\omega_2^{2l} - \omega_1^{2l}) &= 0 \\ \sum_{i=0}^{n-1} \sum_{l=0}^{\frac{n}{2}} (-1)^{i+l+1} c_{im} \omega_1^i \omega_2^i (\omega_2^{2l+1} - \omega_1^{2l+1}) &= 0, \end{aligned} \qquad (11)$$

where $c_{ik} = a_i b_k - a_k b_i,\ k = i + 2l,\ m = i + 2l + 1$. Now we exploit the following result (Ackermann, 2002 (Appendix); Walker, 1950).

Theorem (Bezout) Two bivariate polynomials $P(x,y) = p_1 x^{\alpha_1} y^{\beta_1} + \ldots + p_k x^{\alpha_k} y^{\beta_k},\ deg(P) = n$ $Q(x,y) = q_1 x^{\gamma_1} y^{\delta_1} + \ldots + q_l x^{\gamma_l} y^{\delta_l},\ \ deg(Q) = m$ (where $deg(P(x,y)) \doteq \max_i (\alpha_i + \beta_i)$) have no more than mn common real zeros.

Due to this theorem, system (11) can have $(2n-2)$ $(2n-1)$ solutions; but this is a conservative estimate. The first equation is an identity when $\omega_1 + \omega_2 = 0$. It leads to $n-1$ self-crossing points on the real axis λ. Notice that two different solutions (α, β) and (β, α) describe the same self-crossing point. To avoid this degeneracy we change the variables $\omega \Rightarrow d$:

$$\begin{aligned} \omega_1 \omega_2 &= d_1 \\ \omega_1 + \omega_2 &= d_2 \neq 0. \end{aligned}$$

In these variables we get no more than $(n-1)(n-2)$ solutions. Thus the total number of self-crossing points does not exceed $(n-1)+(n-2)(n-1)=(n-1)^2$, and hence the number of root invariant regions is less or equal $(n-1)^2+2$. $\diamond$

The steps of the proof allow to suggest an algebraic algorithm (extension of Algorithm 1) to calculate the number of stable roots in every root invariant region $D(k)$ and to check the existence and the number of stability domains; we omit it here.

Example 3 The uncertain polynomial

$$P(z) = z^n + \lambda z^{n-1} + \alpha, \ \lambda \in \mathbb{C} \tag{12}$$

has $(n-1)^2+1$ root invariant regions for $\alpha > 1$ and 2 invariant regions for $\alpha < 1/(n-1)$. Fig. 4 provides D-decomposition of the complex plane for this example with $n = 6$ and $\alpha = 1.5$.

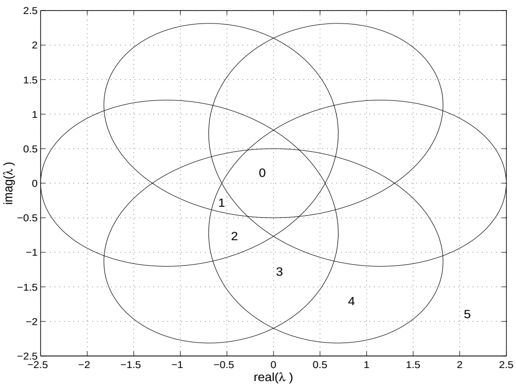

Fig. 4. Root invariant regions for Example 3

In Example 3 there is no stability regions. The question what is the maximal number of stability regions remains open.

Note that the minimal number of root invariant regions is one, as the following example confirms.

Example 4 D-decomposition for the polynomial $s^n + \lambda$, where $n = 2m$, $\lambda \in \mathbb{C}$, is given by

$$\lambda(\omega) = -(j\omega)^n = -(-1)^m \omega^n, \ \omega \in (-\infty, \ \infty),$$

i.e. ray $[0, (-1)^{m+1}\infty)$. The complex plane with the exception of this ray is the single root invariant region with m stable and m unstable roots.

5. TWO REAL PARAMETERS

Consider the polynomial family with two real parameters:

$$P(s, \lambda_1, \lambda_2) = \{a(s) + \lambda_1 b(s) + \lambda_2 c(s), \ \lambda_1, \lambda_2 \in \mathbb{R}\}, \tag{13}$$

where $a(s)$, $b(s)$ and $c(s)$ are given polynomials with real coefficients of degree n.

The structure of D-decomposition is a bit different if compared with the previous section. As usual it contains a complex root boundary curve specified as

$$P(j\omega, \lambda_1, \lambda_2) = 0.$$

Solving it with respect to the parameters we have:

$$\lambda_1 = -\frac{\Delta_1}{\Delta}, \ \ \lambda_2 = -\frac{\Delta_2}{\Delta}, \tag{14}$$

where:

$$\Delta = \begin{vmatrix} U_b & U_c \\ V_b & V_c \end{vmatrix}; \ \ \Delta_1 = \begin{vmatrix} U_a & U_c \\ V_a & V_c \end{vmatrix}; \ \ \Delta_2 = \begin{vmatrix} U_b & U_a \\ V_b & V_a \end{vmatrix};$$

$$U_a = a_0 - a_2\omega^2 + a_4\omega^4 + ..., \ \ U_b = b_0 - b_2\omega^2 + b_4\omega^4 + ...,$$
$$V_a = a_1 - a_3\omega^2 + a_5\omega^4 + ..., \ \ V_b = b_1 - b_3\omega^2 + b_5\omega^4 + ...,$$
$$U_c = c_0 - c_2\omega^2 + c_4\omega^4 + ...$$
$$V_c = c_1 - c_3\omega^2 + c_5\omega^4 + \dots .$$

This curve starts at the line $a_0 + \lambda_1 b_0 + \lambda_2 c_0 = 0$ and terminates at the line $a_n + \lambda_1 b_n + \lambda_2 c_n = 0$. The lines are called singular. For some particular ω such that $\Delta = \Delta_1 = \Delta_2 = 0$ we have not a point but an extra singular line in the parameter space. All these curves and lines generate D-decomposition of $\mathbb{R}^2$.

Theorem 3. Polynomial family (13) has no more than $2n(n-1)+3$ root invariant regions in the (λ_1, λ_2) parameter plane.

The proof is similar to the proof of Theorem 2.

Example 5 The following example demonstrates that the number of root invariant regions N can achieve $O(n^2)$. Let

$$P(s, \lambda) = a(s^2) + s(\lambda_1 b(s^2) + \lambda_2 c(s^2) + \alpha),$$

where $a(t), b(t), c(t)$ are polynomials of degree $m, m-1, m-1$ correspondingly (thus $P(s,\lambda)$ has degree $n = 2m$), $a(t)$ has m negative real roots $-t_i^2, i = 1, \dots, m$. Then D-decomposition equation is $P(j\omega, \lambda) = U(\omega^2) + j\omega V(\omega^2) = 0$ and we get two equations $U(\omega^2) = a(-\omega^2) = 0$, $\omega V(\omega^2) = \omega(\lambda_1 b(-\omega^2) + \lambda_2 c(-\omega^2) + \alpha) = 0$. The first equation does not depend on λ, it has n real roots $\omega_i = \pm t_i$. Hence D-decomposition is generated by singular straight lines $\lambda_1 b(\omega_i^2) + \lambda_2 c(\omega_i^2) + \alpha = 0$, their total number equals m. The plane is divided into $(m^2+m)/2+1$ regions by m straight lines of generic position (this well-known fact can be confirmed by induction), thus $N = n^2/4 + o(n^2)$. In the example below we do not intend to achieve the largest number of root invariant regions, but our goal is to demonstrate, how extraordinary D-decomposition can look for the polynomials of this form. Let $m = 4$,

$a(t) = (1+t)(2+t)\dots(8+t)$, $b(t) = (1+t)(3+t)(5+t)(7+t)$, $c(t)=(2+t)(4+t)(6+t)(8+t)$, $\alpha = 105$. Then we have six orthogonal lines $\lambda_1 = -7$, $\lambda_1 = 1$, $\lambda_1 = 11\frac{2}{3}$, $\lambda_2 = -7$, $\lambda_2 = 1$, $\lambda_2 = 11\frac{2}{3}$.

It is not clear yet what is the maximal number of stability regions. Below is an example, where the stability domain consists of $n-1$ simply connected regions.

Example 6 (Nikolaev, 2002). The polynomial with a_{n-1}, a_0 coefficients being the parameters

$$P(z) = z^n + a_{n-1}z^{n-1} + (1+\varepsilon)z^{n-2} + a_0, \quad 0 < \varepsilon < 2/(n-2) \tag{15}$$

has $n-1$ stability regions.

Figure 5 depict the parameter space decomposition into the root invariant regions for $n = 5$. The behavior of the complex root boundary curve is rather complicated. The curve runs to infinity for some particular values of ω and has loops. The stability regions are inside these loops.

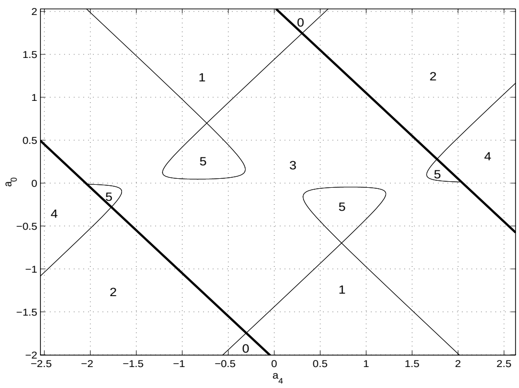

Fig. 5. Root invariant regions for Example 6

Besides a_{n-1}, a_0 parameters there is a free parameter ε. The behavior of the stability regions while ε increases is of interest. The stability regions become smaller and simultaneously shrink for one particular ε value. This critical value is $\varepsilon = \frac{2}{n-2}$ and there is no stability for larger ε.

6. CONCLUSIONS

The root invariant regions geometry in the parameter space can be quite diverse. We prove that for the one-parameter polynomial family the D-decomposition divides the real axis into no more than $n+1$ parts. Thus, there exists no more than $\lceil \frac{n}{2} \rceil$ stability intervals. In other words, the Nyquist diagram has no more than $n+1$ intersections with the real axis and there exists no more than $\lceil \frac{n}{2} \rceil$ stability intervals for the gain. We construct an example with the maximal number of the stability intervals and this example has an obvious geometric interpretation. For the case of one complex parameter the maximal possible number of root invariant regions is $n^2 - 2n + 3$, and this upper bound is tight. Similar results are valid for two real uncertain parameters. We study the discrete-time system from (Nikolayev, 2002) (the parameters are two coefficients of the polynomial), where the stability domain consists of $n-1$ simply connected regions. In particular, we show that all these regions simultaneously shrink for one particular parameter value.

This results can be helpful for design of low order controllers and for detailed robustness analysis (compare Kiselev and Polyak, 1999). The main limitation of the proposed approach is the low dimensionality of the parameter space (one or two).

The extension of D-decomposition technique for the matrix case to construct the stability domain in the parameter space for systems with scalar gain and DIDO systems will be presented in the separate paper.

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