ON DELAY-DEPENDENT STABILITY FOR A CLASS OF DISCRETE NONLINEAR STOCHASTIC SYSTEMS

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Abstract Global asymptotic stability conditions for discrete nonlinear scalar stochastic systems with state delay are obtained based on the convergence theorem for semimartingale inequalities, without assuming the Lipschitz conditions for nonlinear drift functions. The Lyapunov-Krasovskii and degenerate functionals techniques are used. The derived stability conditions are directly expressed in terms of the system coefficients. The obtained results are compared to some previously known asymptotic stability conditions for discrete nonlinear stochastic systems.

Keywords: Discrete stochastic time-delay system, asymptotic stability, Lyapunov-Krasovskii and degenerate functionals, martingale convergence theorems

1. INTRODUCTION

The stability and stabilizabilty problems for time-delay systems have been extensively studied in recent years due to direct applicability of the obtained results to various technical problems (Boukas and Liu, 2002; Gu and Niculescu, 2003; Richard, 2003). Initiated in the background works (Kolmanovskii and Nosov, 1986; Kolmanovskii and Myshkis, 1992; Hale and Verduyn-Lunel, 1993), the stability theory for linear time-delay systems is now actively developed. The Lyapunov-Krasovskii or Lyapunov-Razumikhin functionals are applied in the framework of the Lyapunov direct method to prove the stability conditions for a selected class of linear time-delay systems. Two types of stability conditions can be obtained: delay-independent, establishing stability for all possible delay values, or delay-dependent, corresponding to some restricted values of delay shifts. While the first type of conditions is comprehensive but conservative, the second one is more selective, flexible, and, as a consequence, preferable. Some examples of delay-dependent stability conditions can be found in (Kharitonov, 1999; Kharitonov and Melchor-Aguilar, 2000; Niculescu, 2001; Gu and Han, 2001; Fridman *et al.*, 2001) for various deterministic linear time-delay systems and in (Mao *et al.*, 1998; Liao and Mao, 2000; Xie and Xie, 2000; S.Xu and Chen, 2002; Kolmanovskii *et al.*, 2003) for stochastic ones. Note that it is frequently needed to make a special transformation of an original time-delay system to obtain such stability conditions. Nonetheless, virtually all known results involving delay-dependent stability conditions have been obtained for linear time-delay systems, with certain or even uncertain coefficients.

This paper concentrates on design of the stability conditions for nonlinear stochastic time-delay systems governed by nonlinear Ito scalar difference equations with state delay and a nontrivial diffusion term. The results are obtained using a modified Lyapunov-Krasovskii functional, so-called *degenerate functional*, which was introduced and described in details in (Kolmanovskii and Nosov, 1986; Kolmanovskii and Myshkis, 1992). Applications of degenerate functionals to design of the stability conditions for various classes of deterministic functional-

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differential equations can be found in (Kolmanovskii and Nosov, 1986; Kolmanovskii and Myshkis, 1992; Hale and Verduyn-Lunel, 1993). In the recent paper (Rodkina and Nosov, 2003), the degenerate functionals are used for obtaining delay-dependent stability conditions for deterministic scalar delay-differential equations. This paper extends the result of (Rodkina and Nosov, 2003), generalizing it to discrete stochastic nonlinear time-delay systems. The convergence theorem for semimartingale inequalities (Melnikov and Rodkina, 1993) serves as a key tool for obtaining stability conditions in terms of stochastic system coefficients, without any transformation of the original system itself. The results are compared to some previously known asymptotic stability conditions (Rodkina *et al.*, 2000) for discrete nonlinear stochastic systems, revealing that the conditions derived in this paper are less restrictive: specific examples on this subject are included. Another significant advance reached in this paper in comparison to (Rodkina and Nosov, 2003) is elimination of the Lipschitz condition for nonlinear drift functions, replacing it by the linear growth one. This means that the drift functions may have unbounded variation but must grow not faster than a linear map. The linear growth restriction seems to be quite reasonable, taking into account that the solution of an equation $x_{i+1} = x_i - x_{i-1}^{1+k}$, $x_{-1} = x_0 = a$, diverges to ∞ for any integer $k > 0$ with a sufficiently large a. Moreover, reduction to a one-step recursion, normally applied to linear systems (Kesten, 1973), leads to unnecessary complications in this case, since one scalar equation with a single delay of order k would be transformed to a system of k equations with delays. Finally note that design of a stabilizing controller for a class of nonlinear stochastic systems, based on the stability conditions given in this paper, would be a direct application of the obtained results. (see (Orlov *et al.*, 2002) for a similar scheme of stabilizing controller design for linear systems).

The paper is organized is follows. The basic definitions and necessary results for the theory of stochastic processes and, in particular, martingale-differences and semimartingales, are given in Section 2. The stability problem is stated and the stability conditions are derived for a scalar discrete nonlinear stochastic system with state delay in Section 3. The derived stability conditions are directly expressed in terms of the system coefficients. Nontrivial examples of nonlinear systems satisfying the obtained stability conditions are given. The obtained results are compared to some previously known asymptotic stability conditions for discrete nonlinear stochastic systems.

2. BASIC DEFINITIONS AND RESULTS

In this section, some basic definitions and results from the theory of stochastic processes are briefly reviewed (see ((Liptser and Shiryayev, 1989; Spreji, 2003) for details).

Let (Ω, F, P) be a complete probability space with a non-decreasing family of σ-algebras (filtrations) $F = \{\mathscr{F}_i\}, i = 0, 1, 2, \ldots$. A random sequence M_i is said to be an *$\mathscr{F}_i$-martingale*, if $\mathbf{E}|M_i| < \infty$ and $\mathbf{E}\left(M_i \middle| \mathscr{F}_j\right) = M_j$ for all $0 \le j < i$ and $i = 0, 1, 2, \ldots$. A random sequence is called a *semimartingale* if it admits the representation $X_i = X_0 + M_i + A_i$, where M_i is a martingale, $M_0 = 0$, A_i is a sequence with almost surely (*a.s.*) bounded variation, $A_0 = 0$, and X_0 is a random variable.

Consider a sequence ξ_i such that $\xi_0 = 0$ and $\xi_i = m_i - m_{i-1}$ for $i \ge 1$, where m_i is a martingale. Then, $\{\xi_i\}$ is called an *$\mathscr{F}_i$-martingale-difference*. The following lemma (cf. (Liptser and Shiryayev, 1989)) presents the Doob-Meyer decomposition for martingale-differences.

Lemma 1. Let $\{\xi_i\}$ be an $\mathscr{F}_i$-martingale- difference. Then there exists an $\mathscr{F}_i$-martingale- difference $\{\mu_i\}$ and an a.s. positive $\mathscr{F}_{i-1}$- measurable random sequence $\{\eta_i\}$ such that the relation

$$\xi_i^2 = \mu_i + \eta_i. \tag{1}$$

holds almost surely for each $i = 1, 2, \ldots$.

The next lemma, originally proved in (Melnikov and Rodkina, 1993), presents a modification of the martingale convergence theorems (cf. (Liptser and Shiryayev, 1989; Spreji, 2003)) in terms of inequalities for random sequences, which plays a key role in establishing the asymptotic stability conditions.

Lemma 2. Let

$$Z_i = Z_0 + B_i^1 - B_i^2 + M_i, \tag{2}$$

be a non-negative $\mathscr{F}_i$-measurable semimartingale, M_i be a $\mathscr{F}_i$-measurable martingale, Z_0 be a $\mathscr{F}_0$-measurable random variable, and let $A_i^1, A_i^2, B_i^1, B_i^2$ be almost surely (*a.s.*) non-decreasing $\mathscr{F}_{i-1}$- measurable random sequences such that $B_i^1 \le A_i^1$, $B_i^2 \ge A_i^2$ and $B_0^1 = B_0^2 = 0$. Assume that $\lim_{i\to\infty} A_i^1 < \infty$, *a.s.*. Then both $\lim_{i\to\infty} Z_i < \infty$ and $\lim_{i\to\infty} A_i^2 < \infty$ exist and are finite *a.s.*.

3. STABILITY CONDITIONS FOR DISCRETE NONLINEAR STOCHASTIC SYSTEMS WITH STATE DELAY

In this section, the asymptotic stability problem is considered for a scalar stochastic nonlinear difference equation with discrete delay $h = i - k$ and a nontrivial diffusion term

$$x_{i+1} = x_i - aN(x_i) - bN(x_{i-k}) + \sigma(i, x_{i-k}, \ldots, x_i)\xi_{i+1}, \tag{3}$$

with the initial values $\{x_{-k}, \ldots, x_0\} \in R$, where ξ_{i+1} is an $\mathscr{F}_{i+1}$-martingale-difference.

Assume that there exist such constants $K, \lambda > 0$ and a random variable $\bar{\eta}$ that the following conditions are satisfied almost surely

$$xN(x) > 0, \quad x \neq 0, \quad N(0) = 0, \qquad (4)$$

$$|N(x)| \leq K|x|, \quad \text{for any} \quad x \in \mathbf{R}, \qquad (5)$$

$$a + b > 0, \qquad (6)$$

$$\sigma^2(i, x_{i-k}, \ldots, x_i) \leq \lambda \sum_{j=i-k}^{i} N^2(x_j), \qquad (7)$$

$$\sup_{i=1,2,\ldots} \{\eta_i\} \leq \bar{\eta}, \qquad (8)$$

$$\alpha = \left[\frac{(a+b+2k|b|)}{2} + \frac{\lambda \bar{\eta}(k+2)}{2(a+b)} \right] K < 1. \quad (9)$$

The next theorem establishes asymptotic stability conditions for solutions of the equation (3)

Theorem 1. Let conditions (4)-(9) be satisfied. Then, $\lim_{i\to+\infty} x_i = 0$ holds a.s. for all solutions x of the equation (3) with arbitrary initial conditions $\{x_{-k}, \ldots, x_0\} \in R$.

Proof. Define a *degenerate* Lyapunov-Krasovskii functional $V = V_1 + V_2 + V_3$, where

$$V_1 = (V_1)_i = \left(x_i - b \sum_{j=i-k}^{i-1} N(x_j) \right)^2, \qquad (10)$$

$$V_2 = (V_2)_i = |b|(a+b) \sum_{j=i-k}^{i-1} \sum_{l=j}^{i-1} N^2(x_l), \qquad (11)$$

$$V_3 = (V_3)_i = \sum_{l=0}^{k-1} \eta_{i+1+l} \sum_{j=l+1}^{k} N^2(x_{i+l-j}), \quad (12)$$

instead of the frequently encountered Lyapunov function $V = x^2$. Note that the functional V is not negative but also is not positive definite.

Let us estimate increments of three parts of the functional V. First, note that

$$x_{i+1} - b \sum_{j=i+1-k}^{i} N(x_j) = x_{i+1} - b \sum_{j=i-k}^{i-1} N(x_j) - bN(x_i)$$
$$+ bN(x_{i-k}) = x_i - aN(x_i) - bN(x_{i-k}) + \sigma(\ldots)\xi_{i+1}$$
$$- b \sum_{j=i-k}^{i-1} N(x_j) - bN(x_i) + bN(x_{i-k}) =$$
$$x_i - b \sum_{j=i-k}^{i-1} N(x_j) - (a+b)N(x_i) + \sigma(\ldots)\xi_{i+1}. \quad (13)$$

Taking into account (1), the increment of V_1 is estimated as

$$(V_1)_{i+1} - (V_1)_i = \left(x_{i+1} - b \sum_{j=i+1-k}^{i} N(x_j) \right)^2$$
$$- \left(x_i - b \sum_{j=i-k}^{i-1} N(x_j) \right)^2$$
$$= \left(x_i - b \sum_{j=i-k}^{i-1} N(x_j) - (a+b)N(x_i) \right.$$
$$\left. + \sigma(\ldots)\xi_{i+1}^2 - \left(x_i - b \sum_{j=i-k}^{i-1} N(x_j) \right) \right)^2$$
$$= 2 \left(x_i - b \sum_{j=i-k}^{i-1} N(x_j) \right) (-(a+b)N(x_i)$$
$$+ \sigma(\ldots)\xi_{i+1} + \left(-(a+b)N(x_i) + \sigma(\ldots)\xi_{i+1}\right)^2$$
$$= -2(a+b)N(x_i) \left(x_i - b \sum_{j=i-k}^{i-1} N(x_j) \right)$$
$$+ (a+b)^2 N^2(x_i) + \sigma^2(\ldots)\eta_{i+1} + \rho_{i+1},$$

where ρ_i is a martingale-difference given by

$$\rho_{i+1} = 2 \left(x_i - b \sum_{j=i-k}^{i-1} N(x_j) \right) \sigma(\ldots)\xi_{i+1}$$
$$+ \sigma^2(\ldots)\mu_{i+1} - (a+b)N(x_i)\sigma(\ldots)\xi_{i+1},$$

and μ_i and η_i are the martingale-difference and a.s. positive random sequence, respectively, from the decomposition (1). Using the inequality (7) and the inequality

$$2N(x_i) \left(\sum_{j=i-k}^{i-1} N(x_j) \right) \leq kN^2(x_i)$$
$$+ \sum_{j=i-k}^{i-1} N^2(x_j) \qquad (14)$$

yields the following estimate for the increment of V_1

$$(V_1)_{i+1} - (V_1)_i \leq -2(a+b)N(x_i)x_i + (a+b)^2 N^2(x_i)$$
$$+ |b|(a+b)kN^2(x_i) + |b|(a+b) \sum_{j=i-k}^{i-1} N^2(x_j) +$$
$$\lambda \eta_{i+1} \sum_{j=i-k}^{i} N^2(x_j) + \rho_{i+1}. \qquad (15)$$

Next, the increment of V_2 has the form

$$(V_2)_{i+1} - (V_2)_i = |b|(a+b) \left[\sum_{j=i+1-k}^{i} \sum_{l=j}^{i} N^2(x_l) \right]$$
$$- |b|(a+b) \sum_{j=i-k}^{i-1} \sum_{l=j}^{i-1} N^2(x_l). \qquad (16)$$

Defining

$$\Lambda_{j,i} = \sum_{l=j}^{i} N^2(x_l)$$

and noting that

$$\Lambda_{j,i-1} = \Lambda_{j,i} - N^2(x_i), \; \Lambda_{i-k,i-1} = \sum_{l=i-k}^{i-1} N^2(x_l),$$
$$\Lambda_{i,i} = N^2(x_i),$$

the increment of $\Lambda_{j,i}$ in i is estimated as

$$\sum_{j=i+1-k}^{i}\Lambda_{j,i}-\sum_{j=i-k}^{i-1}\Lambda_{j,i-1}=\sum_{j=i+1-k}^{i}\Lambda_{j,i}$$
$$-\sum_{j=i-k}^{i-1}\Big(\Lambda_{j,i}-N^2(x_i)\Big)$$
$$=\sum_{j=i-k}^{i-1}\Lambda_{j,i}-\sum_{j=i-k}^{i-1}\Lambda_{j,i}-\Lambda_{i-k,i}+(k+1)N^2(x_i)$$
$$=-\Lambda_{i-k,i}+(k+1)N^2(x_i)=-\Lambda_{i-k,i-1}+kN^2(x_i)$$
$$=-\sum_{j=i-k}^{i-1}N^2(x_j)+kN^2(x_i). \qquad (17)$$

Substituting (17) into (16) yields the following estimate for the increment of V_2

$$(V_2)_{i+1}-(V_2)_i=|b|(a+b)$$
$$\left[kN^2(x_i)-\sum_{j=i-k}^{i-1}N^2(x_j)\right]. \qquad (18)$$

Finally, the increment of V_3 can be represented as

$$(V_3)_i=\lambda\sum_{l=0}^{k-1}V_{3,i,l},$$
$$\text{where } V_{3,i,l}=\eta_{i+1+l}\sum_{j=1+l}^{k}N^2(x_{i+l-j}).$$

Taking into account the equalities

$$V_{3,i,0}=\eta_{i+1}\sum_{j=1}^{k}N^2(x_{i-j})=\eta_{i+1}\sum_{j=i-k}^{i-1}N^2(x_j),$$
$$V_{3,i+1,0}=\eta_{i+2}\sum_{j=1}^{k}N^2(x_{i+1-j})=\eta_{i+2}N^2(x_i)$$
$$+\eta_{i+2}\sum_{j=2}^{k}N^2(x^2_{i+1-j})=\eta_{i+2}N^2(x_i)+V_{3,i,1},$$
$$V_{3,i+1,1}=\eta_{i+3}\sum_{j=2}^{k}N^2(x_{i+2-j})=\eta_{i+3}N^2(x_i)$$
$$+\eta_{i+3}\sum_{j=3}^{k}N^2(x_{i+2-j})=\eta_{i+3}N^2(x_i)+V_{3,i,2},$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$V_{3,i+1,l}=\eta_{i+l+2}N^2(x_i)+V_{3,i,l+1},$$
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$V_{3,i+1,k-2}=\eta_{i+k}N^2(x_i)+\eta_{i+k}N^2(x_{i-1})$$
$$=\eta_{i+k}N^2(x_i)+V_{3,i,k-1},$$
$$V_{3,i+1,k-1}=\eta_{i+k+1}N^2(x_i),$$

and the inequality (8) yields the following estimate for the increment of V_3

$$\Delta(V_3)_i=\lambda\sum_{l=0}^{k-1}V_{3,i+1,l}-\lambda\sum_{l=0}^{k-1}V_{3,i,l}$$
$$=\lambda N^2(x_i)\sum_{l=1}^{k}\eta_{i+1+l}+\lambda\sum_{l=1}^{k-1}V_{3,i,l}-\lambda\sum_{l=0}^{k-1}V_{3,i,l}$$
$$=\lambda N^2(x_i)\sum_{l=0}^{k}\eta_{i+1+l}-\lambda V_{3,i,0}$$
$$=\lambda N^2(x_i)\sum_{l=0}^{k}\eta_{i+1+l}-\lambda\eta_{i+1}\sum_{j=1}^{k}N^2(x_{i-j})$$
$$=\lambda N^2(x_i)\sum_{l=0}^{k}\eta_{i+1+l}-\lambda\eta_{i+1}\sum_{j=i-k}^{i-1}N^2(x_j)$$
$$=\lambda N^2(x_i)[\sum_{l=0}^{k}\eta_{i+1+l}+\eta_{i+1}]-\lambda\eta_{i+1}\sum_{j=i-k}^{i}N^2(x_j)$$
$$\le\lambda N^2(x_i)[(k+2)\overline{\eta}]-\lambda\eta_{i+1}\sum_{j=i-k}^{i}N^2(x_j). \qquad (19)$$

Using the obtained estimates (15), (18), (19) and applying the inequality (9) yield the estimate for the increment of the whole functional V

$$\Delta(V)_i\le-2(a+b)N(x_i)x_i+((a+b)^2+|b|(a+b)k$$
$$+|b|(a+b)k+\lambda\overline{\eta}(k+2))N^2(x_i)$$
$$+\rho_{i+1}\le-2(a+b)$$
$$\times(1-\left[\frac{(a+b)}{2}+\frac{|b|k}{2}+\frac{|b|k}{2}+\frac{\lambda\overline{\eta}(k+2)}{2(a+b)}\right]\frac{N(x_i)}{x_i})$$
$$\times N(x_i)x_i+\rho_{i+1}\le-2(a+b)(1-[\frac{(a+b+2|b|k)}{2}$$
$$+\frac{\lambda\overline{\eta}(k+2)}{2(a+b)}]K)N(x_i)x_i+\rho_{i+1}$$
$$\le-2(a+b)\,(1-\alpha)\,N(x_i)x_i+\rho_{i+1}. \qquad (20)$$

Summarizing both parts of (20) over i from 0 to n yields the formula

$$V_n\le V_0-A^2_n+A^1_n+m_{n+1}, \qquad (21)$$

where $A^2_n=2\sum_{i=0}^{n}(1-\alpha)(a+b)N(x_i)x_i$ and $A^1_n=0$ are almost surely non-decreasing processes, $m_{n+1}=\sum_{i=0}^{n}\rho_{i+1}$ is a martingale, and $P\{A^1_\infty<\infty\}=1$. Moreover, V_n is a nonnegative semimartingale. Applying Lemma 2 implies that

$$P\Big\{\{V\to\}\cap\{A^2_\infty<\infty\}\Big\}=1. \qquad (22)$$

where $\{V\to\}$ means that the limit of V exists and is finite as $i\to\infty$.

It can be proved now that $P\left\{\lim_{i\to\infty}x_i=0\right\}$. Indeed, suppose the opposite: there exist *a.s.* a finite random variable $\zeta_0(\omega)>0$ and a subsequence of random moments $i_k=i_k(\omega)$ such that $P(\Omega_1)=p_0$, where $\Omega_1=$

$\{\omega: |x_{t_k}|(\omega) > \zeta_0(\omega) > 0\}$. In view of continuity of the function N, there exists *a.s.* another finite random variable $\zeta_1(\omega) > 0$ such that $|x_{i_k} N(x_{i_k})|(\omega) > \zeta_1(\omega)$ if $\omega \in \Omega_1$. Let $k(n)$ be the number of elements in the subsequence $\{i_k\}$ belonging to the interval $[0,n]$. Then, for $\omega \in \Omega_1$,

$$\begin{aligned} A_n^2 &= 2\sum_{i=0}^{n}(1-\alpha)(a+b)N(x_i)x_i \\ &\geq 2(a+b)(1-\alpha)\sum_{k=1}^{n_1} N(x_{i_k})x_{i_k} \\ &\geq 2(a+b)(1-\alpha)\zeta_1 k(n) \to \infty, \end{aligned}$$

as $n \to \infty$, since $k(n) \to \infty$ as $n \to \infty$. Hence, $P\{A_\infty^2 = \infty\} \geq p_0 > 0$, which contradicts (22). Theorem 1 is proved.

4. COMPARISON TO PREVIOUSLY KNOWN RESULT

The purpose of this section is to compare the conditions (4)-(9) with the conditions previously obtained in (Rodkina *et al.*, 2000), where the asymptotic stability for solutions of the discrete nonlinear stochastic equation

$$\begin{aligned} x_{i+1} = \sum_{j=0}^{i} g_{i,j}\left(x_{i-j}\right) + f\left[i, x_i, x_{i-1}, \ldots, x_0\right] \\ + \sigma\left[i, x_i, x_{i-1}, \ldots, x_0\right]\xi_{i+1} \qquad (23) \end{aligned}$$

was established under the conditions

$$|g_{i,j}\left(x_{i-j}\right)| \leq |a_{i,j}||x_{i-j}|, \qquad (24)$$

$$\theta_i = \sum_{l=0}^{\infty}\left(\sum_{j=0}^{i+l}|a_{i+l,j}|\right)|a_{i+l,l}| < 1, \qquad (25)$$

$$\left|\sigma\left[i, x_i, x_{i-1}, \ldots, x_0\right]\right|^2 \leq \sum_{j=0}^{i}\lambda_{i,j}^2 L(x_{i-j}^2), \qquad (26)$$

$$\sum_{i=0}^{\infty}\sum_{l=0}^{\infty}\eta_{i+1+l}\lambda_{i+l,l}^2 < \infty. \qquad (27)$$

Here, $L(u)$ is a function growing not faster than $u + \ln u$ (see (Rodkina *et al.*, 2000)).

Consider the equation (3) with the function $N(u) = \frac{Cu^3}{1+u^2}$, $C = const$. To cast it in the form (23), the functions $g_{i,j}$ should be assigned as

$$g_{i,0}(u) = u - aN(u) = u - \frac{aCu^3}{1+u^2}, \quad g_{i,k}(u) = \frac{bCu^3}{1+u^2},$$
$$\text{and } g_{i,j}(u) \equiv 0, \text{ if } j \neq 0, k.$$

To satisfy (24), the function $g_{i,0}(u)$ is estimated as

$$|g_{i,0}(u)| = |u - \frac{aCu^3}{1+u^2}| = |\frac{u+u^3-aCu^3}{1+u^2}|$$
$$= |\frac{1+u^2(1-aC)}{1+u^2}||u| \leq 1|u|,$$

where the constant $a_{i,0} = 1$ cannot be reduced because of the limit

$$\lim_{u\to 0}|\frac{1+u^2(1-aC)}{1+u^2}| = 1.$$

Similarly, the function $g_{i,k}(u)$ is estimated as

$$|\frac{bCu^3}{1+u^2}| \leq |bC||u|,$$

where the constant $a_{i,k} = |bC|$ is also irreducible. All other constants $a_{i,j}$, $j \neq 0, k$ are equal to 0. Since

$$\begin{aligned} \sum_{l=0}^{\infty}\left(\sum_{j=0}^{i+l}|a_{i+l,j}|\right)|a_{i+l,l}| = \sum_{l=0}^{\infty}(1+|bC|)|a_{i+l,l}| \\ = (1+bC)^2 > 1, \end{aligned}$$

the condition (25) is not satisfied for $N(u) = \frac{Cu^3}{1+u^2}$, if $Cb \neq 0$, although the condition (5) holds.

Furthermore, the condition (26) takes the form (7), provided that the coefficients $\lambda_{i,j}$ are assigned as

$$\lambda_{i,j} \equiv \lambda_i \equiv \lambda, \quad j = 0, \ldots, k, \text{ and } \lambda_{i,j} \equiv 0, \quad \text{otherwise}.$$

However, if $\eta_i \equiv 1$, the condition (27) is not satisfied, since

$$\sum_{i=0}^{\infty}\sum_{l=0}^{\infty}\eta_{i+1+l}\lambda_{i+l,l}^2 = \lambda^2\sum_{i=0}^{\infty}\sum_{l=0}^{k}\eta_{i+1+l} = \lambda^2\sum_{i=0}^{\infty}k = \infty,$$

although the condition (7) holds.

Thus, the conditions (4)-(9) given in this paper noticeably weaken the asymptotic stability requirements for solutions of a discrete nonlinear stochastic equation (3) in comparison to the previously obtained set of conditions (Rodkina *et al.*, 2000).

5. CONCLUSIONS

The asymptotic stability problem has been considered for a scalar discrete nonlinear stochastic system governed by a difference equation with two drift terms, with and without state delay, and a nontrivial diffusion. No Lipshitz condition has been assumed for the nonlinear drift terms in the system. The global almost sure asymptotic stability conditions have been obtained and directly expressed in terms of the system coefficients. The Lyapunov- Krasovskii and degenerate functionals techniques have been used for establishing asymptotic stability in the framework of the Lyapunov direct method. The convergence theorem for semimartingale inequalities has served a key tool for obtaining stability conditions in terms of stochastic system coefficients, without any transformation of the original system itself. The obtained results have occurred to be less restrictive than some previously known asymptotic stability conditions for discrete nonlinear stochastic systems. The paper has introduced a systematic approach which would be applicable to design of the stability conditions for other classes of discrete nonlinear stochastic systems with state delay.

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