PARAMETRIZATION OF DECENTRALIZED OUTPUT FEEDBACK CONTROLLERS AND ROBUST STABILIZATION OF LARGE SCALE JUMP SYSTEMS [1]

P.V. Pakshin, * **I.N. Mitrofanov** *

* *Nizhny Novgorod State Technical University, Arzamas, 19, Kalinina Str., Arzamas, 607227, Russia. E-mail: pakshin@afngtu.nnov.ru*

Abstract: The paper considers a class of large scale control systems described by a finite set of linear systems with transitions between them determined by a homogeneous Markov chain. Each individual system of this family describes the plant state variables in the corresponding mode and is composed of a set of interconnected subsystems. At the moment of a discontinuous mode change the plant state vector can be changed by jump. A parametrization of the linear decentralized output feedback controllers that stabilize a given system of this class in the mean square is presented. Sufficient conditions for an output feedback controller to be robust stabilizing against the mode change parameter uncertainty are obtained. These conditions along with the parametrization result lead to an LMI-based algorithm for computation of the gain matrix of a robust stabilizing output feedback control law. An illustrative example is given.

Keywords: Large scale systems, jump process, robust stability, decentralized control, robust control, output feedback.

1. INTRODUCTION

One of the most important open questions in control theory is the output feedback problem (Cao *et al.*, 1998; Garcia *et al.*, 2003; Iwasaki and Skelton, 1995; Kučera and DeSouza, 1995; Syrmos *et al.*, 1997; Trofino-Neto and Kučera, 1993), despite the fact that this type of feedback represents the *simplest* closed loop control that can be realized in practice. There exist necessary and sufficient conditions for the output feedback stabilization (Cao *et al.*, 1998; Iwasaki and Skelton, 1995; Kučera and DeSouza, 1995; Trofino-Neto and Kučera, 1993) but these conditions are not readily implemented as numerical algorithms, except Cao *et al.* (1998) where an iterative LMI-based algorithm is proposed and Yu (2004). The major difficulty is due to non-convexity of the static output feedback solution set (Iwasaki and Skelton, 1995), which makes it a non-trivial computational task, analytical and computational alike.

For jumping systems we have especially weak development of analytical and computational solution methods of this problem (Mariton, 1990; Pakshin and Retinsky, 2003; Pakshin and Mitrofanov, 2004) and references therein; in the case of large scale jumping systems this problem has not been studied in the current literature. Boukas *et al.* (1997) consider large scale systems which are linear in the continuous plant state and whose mode dynamics are described via random jumps modeled by a discrete-state Markov chain. By use of decomposition and coordination leading to a two-level control system, the robustness in the sense of robust stability and guaranteed cost control is ensured for the partly unknown large scale linear system with Markovian jumps. Two different structures are proposed: decentralized and centralized one. In both

[1] This work was supported in part by the Russian Foundation for Basic Research.

the cases it is supposed that the state vector of each local subsystem is available to the controller.

This paper considers a similar class of large scale control systems described by a finite set of linear systems with transitions between them determined by a homogeneous Markov chain. Each individual system of this family describes the plant state variable in the corresponding mode and is composed of a set of interconnected subsystems. At the moment of a discontinuous mode change the plant state vector can be changed by jump. A parametrization of the linear decentralized output feedback controllers that stabilize a given system of this class in the mean square is presented. Sufficient conditions for an output feedback controller to be robust stabilizing against the mode change parameter uncertainty are obtained. These conditions along with the parametrization result and some ideas of (Boyd *et al.*, 1994; ElGhaoui and AitRami, 1996; AitRami and ElGhaoui, 1996) lead to an LMI-based algorithm for computation of the gain matrix of robust stabilizing output feedback control law. An illustrative example is given.

2. SYSTEM DESCRIPTION

Consider a decentralized system subject to random jumps composed of L interconnected subsystems and described by the following differential equations (Siljak, 1991):

$$\dot{x}_p^l(t) = A_p^l(r(t))x_p^l(t) + B_p^l(r(t))u_p^l(t) + \sum_{k=1}^{L} A_p^{lk}(r(t))x_p^k(t), \quad (1)$$

$$y_p^l(t) = C_p^l(r(t)x_p^l(t), \;\; t \geq 0, \quad (2)$$

$$x_p^l(\tau) = \Phi_{pij}^l x_p^l(\tau - 0), \quad (3)$$

where $x_p^l \in \mathbb{R}^{n_p^l}$ is the local plant state vector, $u^l \in \mathbb{R}^{m_p^l}$ is the local control vector, $y^l \in \mathbb{R}^{k_p^l}$ is the local plant output vector; $r(t)$ is a homogeneous Markov chain which state space is a set of integers $\mathbb{N} = \{1, 2, \ldots, \nu\}$ and transition matrix $P(\theta) = [P_{ij}(\theta)]_1^\nu = [\text{Prob}\{r(t+\theta) = j \mid r(t) = i\}]_1^\nu = \exp(\Pi\theta), \quad 0 \leq t \leq t+\theta, \quad \Pi = [\pi_{ij}]_1^\nu$ with $\pi_{ij} \geq 0, \; j \neq i, \pi_{ii} = -\sum_{j\neq i}^{\nu} \pi_{ij}$; $\tau > t_0$ is the moment of transition from $r(\tau - 0) = i$ to $r(\tau) = j$; $\Phi_{ij}^l, (i, j \in \mathbb{N})$ are $n_p^l \times n_p^l$ constant matrices, such that $\Phi_{ii}^l = I$. For each possible value of the process $r(t) \in \mathbb{N}$ we write $A_p^l(r(t)) = A_{pi}^l, \quad B_p^l(r(t)) = B_{pi}^l, \quad C_p^l(r(t)) = C_{pi}^l, \quad$ when $\quad r(t) = i$. These matrices have compatible dimensions and correspond to different modes of the system.

Consider a fixed-order decentralized dynamic output feedback controller in the form of the following equations:

$$\dot{x}_c^l(t) = A_{ci}^l x_c^l(t) + B_{ci}^l y_p^l(t), \;\; \text{if} \;\; r(t) = i, \quad (4)$$

$$u_p^l(t) = C_{ci}^l x_c^l(t) + D_{ci} y_p^l(t), \;\; \text{if} \;\; r(t) = i, \quad (5)$$

where $x_c^l \in \mathbb{R}^{n_c^l}$ and matrices $A_{ci}^l, \; B_{ci}^l, \; C_{ci}^l$ and D_{ci}^l have compatible dimensions.

The system (1)–(5) can be written as

$$\dot{x}^l(t) = A^l(r(t))x^l(t) + B^l(r(t))u^l(t) + \sum_{k=1}^{L} A^{lk}(r(t))x^k(t), \quad (6)$$

$$y^l(t) = C^l(r(t))x(t), \;\; t \geq 0, \quad (7)$$

$$x^l(\tau) = \Phi_{ij}^l x^l(\tau - 0), \quad (8)$$

$$u^l(t) = -G_i^l y^l(t), \; \text{if} \; r(t) = i, \quad (9)$$

where $x^l = [x_p^{lT} \;\; x_c^{lT}]^T \in \mathbb{R}^{n^l}, \;\; u^l \in \mathbb{R}^{m^l}, \;\; y^l \in \mathbb{R}^{k^l}, \;\; n^l = n_p^l + n_c^l, \;\; m^l = m_p^l + n_c^l, \;\; k^l = k_p^l + n_c^l \;\; \Phi_{ij}^l = \text{diag}[\Phi_{pij}^l \;\; 0] \;\; \text{if} \;\; r(\tau - 0) = i, \; r(\tau) = j$ and

$$A_i^l = \begin{bmatrix} A_{pi}^l & 0 \\ 0 & 0 \end{bmatrix}, B_i^l = \begin{bmatrix} B_{pi}^l & 0 \\ 0 & I_{n_c} \end{bmatrix}, C_i^l = \begin{bmatrix} C_{pi}^l & 0 \\ 0 & I_{n_c} \end{bmatrix},$$

$$A_i^{lk} = \begin{bmatrix} A_{pi}^{lk} & 0 \\ 0 & 0 \end{bmatrix}, G_i^l = -\begin{bmatrix} D_{ci}^l & C_{ci}^l \\ B_{ci}^l & A_{ci}^l \end{bmatrix}.$$

It is easy to see that this model gives a common description for system with both static and fixed-order dynamic output feedbacks.

The hybrid interconnected system (6)–(9) can be written in compact form

$$\dot{x}(t) = A(r(t))x(t) + B(r(t))u(t) + A_C(r(t))x(t), \quad (10)$$

$$y(t) = C(r(t))x(t), \;\; t \geq 0, \quad (11)$$

$$x(\tau) = \Phi_{ij} x(\tau - 0), \quad (12)$$

$$u(t) = -G_i y(t), \; \text{if} \; r(t) = i, \quad (13)$$

where $x = [x^{1T}, \ldots, x^{LT}]^T, \;\; u = [u^{1T}, \ldots, u^{LT}]^T$ and the block matrices are: $A_C = [A^{ij}]_1^L \;\; A = \text{diag}[A^1, \ldots, A^L], \;\; B = \text{diag}[B^1, \ldots, B^L], \;\; C = \text{diag}[C^1, \ldots, C^L], \;\; G = \text{diag}[G^1, \ldots, G^L], \;\; \Phi_{ij} = \text{diag}[\Phi_{ij}^1, \ldots, \Phi_{ij}^L]$.

We also define the nominal plant model as a set of isolated subsystems:

$$\dot{x}(t) = A(r(t))x(t) + B(r(t))u(t), \;\; t \geq 0, \quad (14)$$

which follows from (10), when $A_C(r_t) \equiv 0$.

3. PRELIMINARIES

For every $i \in \mathbb{N}$ the plant state space of the system (10) can be presented in the form of the following partition

$$\mathbb{R}^n = \text{Im}(C_i^{\text{T}}) \oplus \text{Ker}(C_i), \quad (15)$$

where $\mathrm{Im}(C_i^{\mathrm{T}})$ and $\mathrm{Ker}(C_i)$ are orthogonal subspaces. For any $x \in \mathbb{R}^n$ we can write

$$x = x_{\mathrm{I}} + x_{\mathrm{K}},$$

where $x_{\mathrm{I}} \in \mathrm{Im}(C_i^{\mathrm{T}})$ and $x_{\mathrm{K}} \in \mathrm{Ker}(C_i)$. Define the matrices

$$E_{\mathrm{I}}(i) = C_i^{+} C_i, \;\; E_{\mathrm{K}}(i) = I - E_{\mathrm{I}}(i), \qquad (16)$$

where C_i^{+} is the Moore-Penrose inverse of C_i. According to the partition (15) the matrices (16) are projection matrices on $\mathrm{Im}(C_i^{\mathrm{T}})$ and on $\mathrm{Ker}(C_i)$ correspondingly. These matrices are symmetric and unique. We use the notation $X^{\perp}$ for a full rank matrix orthogonal to X. The matrix $X^{\perp}$ exists if and only if X has linearly dependent rows and for a given X the matrix $X^{\perp}$ is not unique.

An important role in the sequel together with the output feedback control (13) plays also the state feedback decentralized control

$$u(t) = -K_i x(t), \;\; \text{if} \;\; r(t) = i \qquad (17)$$

with $K_i = \mathrm{diag}[K_i^1 \dots K_i^L]$.

Definition 1. *The control law (13) is said to be decentralized stabilizing output feedback (DSOF) control if there exists a positive definite matrix $H_{Ci} = H_{Ci}^T$ $(i \in \mathbb{N})$ such that the following inequalities hold*

$$(A_i + A_{Ci} - B_i G_i C_i)^T H_{Ci} + H_{Ci}(A_i + A_{Ci} - B_i G_i C_i) + \sum_{j=1}^{\nu} \pi_{ij} \Phi_{ij}^T H_{Ci} \Phi_{ij} < 0, \; i \in \mathbb{N}. \qquad (18)$$

Definition 2. *The control law (17) is said to be decentralized stabilizing state feedback (DSSF) control if there exists a positive definite matrix $H_{Ci} = H_{Ci}^T$ $(i \in \mathbb{N})$ such that the following inequalities hold*

$$(A_i + A_{Ci} - B_i K_i)^T H_{Ci} + H_{Ci}(A_i + A_{Ci} - B_i K_i) + \sum_{j=1}^{\nu} \pi_{ij} \Phi_{ij}^T H_{Ci} \Phi_{ij} < 0, \; i \in \mathbb{N}. \qquad (19)$$

Both DSOF control and DSSF control guarantee the exponential stability in the mean square (ESMS) (Mariton, 1990; Kats, 1998) of the closed loop system (10).

Define the following sets of block diagonal matrices

$$\mathcal{L}_o = \{H_i = H_i^T > 0, \exists G_i \;\text{ such that }\; (A_i - B_i G_i C_i)^T H_i + H_i(A_i - B_i G_i C_i) + \sum_{j=1}^{\nu} \pi_{ij} \Phi_{ij}^T H_j \Phi_{ij} < 0, \; i \in \mathbb{N}\},$$

$$\mathcal{L}_s = \{H_i = H_i^T > 0, \exists K_i \;\text{ such that }\; (A_i - B_i K_i)^T H_i + H_i(A_i - B_i K_i) + \sum_{j=1}^{\nu} \pi_{ij} \Phi_{ij}^T H_j \Phi_{ij} < 0, \; i \in \mathbb{N}\},$$

$$\mathcal{X} = \{X_i = X_i^T > 0, \;\; B_i^{\perp}(A_i X_i + X_i A_i^T + \sum_{j=1}^{\nu} \pi_{ij} X_i \Phi_{ij}^T X_j^{-1} \Phi_{ij} X_i) B_i^{\perp T} < 0, \;\; i \in \mathbb{N}\},$$

$$\mathcal{Y} = \{Y_i = Y_i^T > 0, \;\; C_i^{T\perp}(A_i^T Y_i + Y_i A_i + \sum_{j=1}^{\nu} \pi_{ij} \Phi_{ij}^T Y_j \Phi_{ij}) C_i^{T\perp T} < 0, \;\; i \in \mathbb{N}\},$$

$$\mathcal{U}(X_1, \dots, X_\nu) = \{R_i > 0, \; Q_i > 0, \; A_i^T X_i + X_i A_i - X_i B_i R_i^{-1} B_i^T X_i + \sum_{j=1}^{\nu} \pi_{ij} \Phi_{ij}^T X_j \Phi_{ij} + Q_i = 0, \; i \in \mathbb{N}\}$$

$$\mathcal{W}(Y_1, \dots, Y_\nu) = \{V_i > 0, \; W_i > 0, \; A_i Y_i + Y_i A_i^T - Y_i C_i^T V_i^{-1} C_i Y_i + \sum_{j=1}^{\nu} \pi_{ij} Y_i \Phi_{ij}^T Y_j^{-1} \Phi_{ij} Y_i + W_i = 0, \; i \in \mathbb{N}\}.$$

4. PARAMETRIZATION OF STABILIZING CONTROLLERS WITH STATIC OUTPUT FEEDBACK

In this section following the approach by Iwasaki and Skelton (1995) we obtain a characterization of a set of the matrices of Lyapunov stochastic functions $\mathcal{L}_o$ and a parametrization of the stabilizing static output feedback gains for the system (10)

Theorem 1. *Let a set of block diagonal matrices H_i $(i \in \mathbb{N})$ be given. Then the following statements are equivalent:*

$$H_i \in \mathcal{L}_o, \;\; i \in \mathbb{N}; \qquad (20)$$

$$H_i > 0, \; i \in \mathbb{N}, \; \mathcal{U}(H_1, \dots, H_\nu) \neq \emptyset, \;\; \textit{and} \;\; \mathcal{W}(H_1^{-1}, \dots, H_\nu^{-1}) \neq \emptyset; \qquad (21)$$

$$H_i^{-1} \in \mathcal{X} \;\; \textit{and} \;\; H_i \in \mathcal{Y}, \; i \in \mathbb{N}. \qquad (22)$$

All the stabilizing static output feedback gains for the nominal system (14) are given by

$$G_i = R_i^{-1} B_i H_i Q_i^{-1} C_i^T (C_i Q_i^{-1} C_i^T)^{-1} + \Theta_i^{\frac{1}{2}} \Lambda_i (C_i Q_i^{-1} C_i^T)^{-\frac{1}{2}}, \;\; i \in \mathbb{N}, \qquad (23)$$

where Λ_i $(i \in \mathbb{N})$ are arbitrary matrices such that $\| \Lambda_i \| < 1$, $H_i \in \mathcal{L}_o$, $\{R_i, Q_i\} \in \mathcal{U}(H_1, \dots H_\nu)$ and matrices $\Theta_i > 0$ $(i \in \mathbb{N})$ are defined by

$$\Theta_i = R_i^{-1} - R_i^{-1} B_i^T H_i Q_i^{-1} [Q_i - C_i^T C_i (C_i Q_i^{-1} C^T)^{-1} C_i] Q_i^{-1} H_i B_i R_i^{-1}. \qquad (24)$$

If the matrix (23) is such that LMI's (18) are feasible with respect to the LMI variable $H_{Ci} > 0$ then it is a gain matrix of DSOF control.

The theorem can be proved by the same way as the parametrization theorem for single jump systems (Pakshin and Mitrofanov, 2004).

5. PARAMETRIZATION OF DECENTRALIZED ROBUST STABILIZING CONTROLLERS WITH STATE FEEDBACK

In real control problems the transition probabilities between the modes are not exactly known. Suppose that the matrix $\Pi = \Pi(\delta)$ is an affine function of a vector parameter δ. That is, suppose that there exist real matrices $\Pi_0, \ldots, \Pi_N$ all of the same dimension as Π such that

$$\Pi(\delta(t)) = \Pi_0 + \delta_1 \Pi_1 + \ldots + \delta_N \Pi_N$$

for all $\delta \in \Delta$. Let the uncertain parameters $\delta_j,\ j = 1, \ldots, N$ take values in the interval $[\underline{\delta}_j,\ \bar{\delta}_j]$ i.e. $\delta_j \in [\underline{\delta}_j,\ \bar{\delta}_j]$. This means that the uncertainty of each independent parameter is assumed to be bounded between the two extremal values. Define the set of the corners of the uncertainty region as

$$\Delta_0 = \{\delta = (\delta_1, \ldots, \delta_N \mid \delta_j \in \{\underline{\delta}_j,\ \bar{\delta}_j\}, \quad j = 1, \ldots, N\}.$$

Definition 3. *The control law (13) is said to be robust decentralized stabilizing output feedback (RDSOF) control if there exist positive definite matrices $H_{Ci} = H_{Ci}^T$ $(i \in \mathbb{N})$ such that the inequalities (18) hold with $\Pi = \Pi(\delta)$ for all perturbations $\delta \in \Delta$. If in addition $G_i = G$ $(i \in \mathbb{N})$, then this control law is said to be nonswitching RDSOF control.*

Definition 4. *The control law (17) is said to be robust decentralized stabilizing state feedback (RDSSF) control if there exists a positive definite matrix $H_{Ci} = H_{Ci}^T$ $(i \in \mathbb{N})$ such that the inequalities (19) hold with $\Pi = \Pi(\delta)$ for all perturbations $\delta \in \Delta$. If in addition $K_i = K,\ i \in \mathbb{N}$, then this control law is said to be nonswitching RDSSF control.*

Denote $\mathcal{L}_{s\delta}$ and $\mathcal{X}_\delta$ the sets $\mathcal{L}_s$ and $\mathcal{X}$ with $\Pi = \Pi(\delta)$, where $\delta \in \Delta_0$.

Theorem 2. *Suppose that the plant state vector is available to the controller $(C_i = I)$. Then the following statements are equivalent:*

I. *there exists a gain matrix of RDSSF control for nominal system;*
II. $\mathcal{L}_{s\delta} \neq \emptyset$;
Let a set of matrices H_i $(i \in \mathbb{N})$ be given. Then the following statements are equivalent.
III. $H_i \in \mathcal{L}_{s\delta}, i \in \mathbb{N}$;
IV. *H_i is the unique positive definite solution of the set of coupled Riccati equations*

$$A_i^T H_i + H_i A_i - H_i B_i R_i^{-1} B_i^T H_i + Q_i(\delta) \sum_{j=1}^{\nu} \pi_{ij}(\delta) \Phi_{ij}^T H_j \Phi_{ij} = 0, \quad i \in \mathbb{N},\ \delta \in \Delta_o$$

for some $Q_i(\delta) > 0$ and $R_i > 0, \delta \in \Delta_o,\ i \in \mathbb{N}$;
V. *$H_i > 0$ and $H_i^{-1} \in \mathcal{X}_\delta$*

The gain matrix of RDSSF control in the form of (17) for the nominal system is given by

$$K_i = R_i^{-1} B_i^T P_i + R_i^{-\frac{1}{2}} \Lambda_i(\delta) Q_i(\delta)^{\frac{1}{2}} \quad i \in \mathbb{N}, \quad (25)$$

where the matrices $P_i,\ Q_i,\ R_i$ are the ones in IV. and $\Lambda_i(\delta)$ is a matrix such that $\parallel \Lambda_i(\delta) \parallel < 1$ and $\Lambda_i(\delta) Q_i(\delta)^{\frac{1}{2}}$ is independent of δ. *If the matrix (25) such that LMI's (19) with $\Pi = \Pi(\delta)$ $(\delta \in \Delta_0)$ are feasible with respect to LMI variable $H_{Ci} > 0$, then it is the gain matrix of RDSSF control for the system (10).*

The proof is based on well known results from convex analysis (Boyd *et al.*, 1994) and parametrization theorem (Pakshin and Mitrofanov, 2004).

6. ROBUST STABILIZATION VIA STATIC OUTPUT FEEDBACK

Developing some ideas by Trofino-Neto and Kučera (1993), Kučera and DeSouza (1995) in this section we obtain new necessary and sufficient conditions for robust stabilization via static output feedback of the system (10), (11)

Theorem 3. *The nominal system (14), (11) is robust stabilizable via static output feedback if and only if for some symmetric matrices $M_i(\delta)$ $(\delta \in \Delta_0)$, and $R_i > 0$ $(i \in \mathbb{N})$ there exist positive definite solutions $H_i = H_i^T$ of the system of the coupled Riccati equations*

$$A_i^T H_i + H_i A_i - H_i B_i R_i^{-1} B_i^T H_i + M_i(\delta) + \sum_{j=1}^{\nu} \pi_{ij}(\delta) \Phi_{ij}^T H_j \Phi_{ij} = 0 \quad (26)$$

and matrices L_i $(i \in \mathbb{N})$ of compatible dimensions, satisfying for $\delta \in \Delta_0$ the system of inequalities

$$(A_i - B_i K_i)^T H_i + H_i (A_i - B_i K_i) + \sum_{j=1}^{\nu} \pi_{ij}(\delta) \Phi_{ij}^T H_j \Phi_{ij} - (L_i E_I(i) - B_i^T H_i E_K(i))^T K_i - K_i^T (L_i E_I(i) - B_i^T H_i E_K(i)) < 0, \quad (27)$$

where

$$K_i = R_i^{-1} B_i^T H_i^T, \ i \in \mathbb{N}. \quad (28)$$

The robust stabilizing control for the nominal system (14) has the form of (13) where

$$G_i = R_i^{-1}(B_i^T H_i + L_i) C_i^+, \ i \in \mathbb{N}. \quad (29)$$

If the gain matrix (29) is such that the LMI's (18) with $\Pi = \Pi(\delta)$ $(\delta \in \Delta_0)$ *are feasible with respect to the LMI variable* $H_{Ci} > 0$*, then it is the gain matrix of RDSOF control.*

The proof is based on the results by Pakshin and Retinsky (2003) and Pakshin and Mitrofanov (2004).

Corollary 1. *The nominal system (14), (11) with* $C_i = C, \ (i \in \mathbb{N})$ *is robust stabilizable via nonswitching static output feedback if and only if for some symmetric matrices* $M_i(\delta)$ $(\delta \in \Delta_0)$*, and* $R_i > 0$ $(i \in \mathbb{N})$ *there exist positive definite solutions* $H_i = H_i^T$ *of the system of the coupled Riccati equations* (26) *and matrices* L_i $(i \in \mathbb{N})$ *of compatible dimensions, satisfying for* $\delta \in \Delta_0$ *the system of inequalities (27) and the following system of equations*

$$R_i^{-1}(B_i^T H_i + L_i) = R_{i+1}^{-1}(B_{i+1}^T H_{i+1} + L_{i+1}).$$

The robust stabilizing control for the nominal system (14) has the form of (13) where the gain matrix is given by (29) for an arbitrary fixed $i \in \mathbb{N}$*. If the gain matrix (29) is such that the LMI's (18) with* $\Pi = \Pi(\delta)$ $(\delta \in \Delta_0)$ *are feasible with respect to the LMI variable* $H_{Ci} > 0$*, then it is the gain matrix of nonswitching RDSOF control. If in addition* $\Phi_{ij} = I, \ (i, j \in \mathbb{N})$ *and the inequalities (18) admit a common solution* $H_{Ci} = H_C > 0$ $(i \in \mathbb{N})$*, then this control stabilizes the system (10) independently of the mode change process.*

The obtained results lead to the following algorithms for computing the stabilizing feedback gain matrices G_i $(i \in \mathbb{N})$.

Algorithm based on direct solution of coupled Riccati equations(CRE).

Step 1. Solve the system of CRE (26) by LMI optimization method (AitRami and ElGhaoui, 1996; El-Ghaoui and AitRami, 1996) and find the matrices $H_i = H_i^T > 0$ and $K_i, \ i \in \mathbb{N}$.

Step 2. If the LMI problem (27) is not feasible then correct the LQR parameters (weighting matrices) and go to step 1, else if this LMI problem is feasible find the matrices L_i and calculate the matrices G_i by the formula (29).

Algorithm based on parametrization of stabilizing solutions of CRE

According to Corollary 1 in the state feedback stabilizing gain matrices given by the formula (25). Because in this formula $\Lambda_i(\delta)$ is arbitrary matrix with $\| \Lambda_i(\delta) \| < 1$, we can suppose

$$\Lambda_i(\delta) = \frac{\rho R_i^{1/2} B_i^T \tilde{Q}_i Q_i^{1/2}(\delta)}{\max\limits_{\delta \in \Delta_0}(\| R_i^{1/2} B_i^T \tilde{Q}_i Q_i^{1/2}(\delta) \|)},$$

where $| \rho | < 1, \ \tilde{Q}_i = \max\limits_{\delta \in \Delta_0} Q_i(\delta)$ Define

$$H_i = P_i + \alpha_i \tilde{Q}_i, \quad (30)$$

where $\alpha_i = \rho(\max\limits_{\delta \in \Delta_0} \| R^{-1/2} B_i^T \tilde{Q}_i Q_i^{1/2}(\delta) \|)^{-1}$ According to equivalence II from Collorary 1 it is easy to see that in this case H_i satisfies (26) for some $M_i(\delta) = M_i^T(\delta)$ and (25) is equivalent to (28). Taking into account this fact we can formulate the algorithm as follows.

Step 1. Solve LMI's with respect to variable Y_i $(i \in \mathbb{N})$:

$$\begin{bmatrix} \Gamma_{11}(\delta) & \Gamma_{12}(\delta) \\ \Gamma_{12}^T(\delta) & \Gamma_{22} \end{bmatrix} < 0,$$

where

$$\Gamma_{11}(\delta) = B_{i\perp}(Y_i A_i^T + A_i Y_i + \pi_{ii}(\delta) Y_i) B_{i\perp}^T,$$
$$\Gamma_{12} = [B_{i\perp} Y_i \pi_{i1}^{1/2}(\delta) \Phi_{i1}^T \ldots B_{i\perp} Y_i \pi_{ii-1}^{1/2}(\delta) \Phi_{ii-1}^T$$
$$B_{i\perp} Y_i \pi_{ii+1}^{1/2}(\delta) \Phi_{ii+1}^T \ldots B_{i\perp} Y_i \pi_{i\nu}^{1/2}(\delta) \Phi_{i\nu}^T], \ \delta \in \Delta_0,$$
$$\Gamma_{22} = \mathrm{diag}[-Y_1 \ldots - Y_{i-1} \ - Y_{i+1} \ldots - Y_\nu].$$

Step 2. Find $Q_i(\delta)$ and R_i^{-1} as a solution of LMI's

$$A_i Y_i + Y_i A_i^T + Y_i Q_i(\delta) Y_i - B_i R_i^{-1} B_i^T + \sum_{j=1}^{\nu} \pi_{ij}(\delta) Y_i \Phi_{ij}^T Y(j)^{-1} \Phi_{ij} Y_i = 0, \ i \in \mathbb{N}, \delta \in \Delta_0.$$

Step 3. Find $P_i = Y_i^{-1}, \ i \in \mathbb{N}, \ \tilde{Q}_i = \max\limits_{\delta \in \Delta_0} Q_i(\delta), \ \rho_{\min} = \min \rho : |\rho| < 1, \ P_i + \alpha_i \tilde{Q}_i > 0.$

Step 4. Put $H_i = P_i, \ i \in \mathbb{N}, \ \rho = \rho_{\min}$.

Step 5. Find $K_i = R_i^{-1} B_i^T H_i$.

Step 6. If the LMI's (26) are feasible with respect to LMI variable L_i, then find G_i according to the formula (29) else put $\rho := \rho + \Delta\rho, \ H_i = P_i + \alpha_i \tilde{Q}_i$, if $| \rho | \geq 1$, then stop, else go to Step 5.

Step 7. If the LMI's (18) with $\Pi = \Pi(\delta)$ $(\delta \in \Delta_0)$ are feasible with respect to the LMI variable $H_{Ci} > 0$, then G_i is gain matrix of DSOF control, stop, else put $\rho := \rho + \Delta\rho, \ H_i = P_i + \alpha Q_i$, if $| \rho | \geq 1$, then stop, else go to Step 5.

7. AN EXAMPLE

Consider a dynamic system composed of two inverted pendulums connected with a spring (Siljak, 1991). This system is described by the following equations

$$\dot{x}^1 = \begin{bmatrix} 0 & 1 \\ \alpha & 0 \end{bmatrix} x^1 + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u^1 + \begin{bmatrix} 0 & 0 \\ -\gamma & 0 \end{bmatrix} x^1 + \begin{bmatrix} 0 & 0 \\ \gamma & 0 \end{bmatrix} x^2,$$

$$\dot{x}^2 = \begin{bmatrix} 0 & 1 \\ \alpha & 0 \end{bmatrix} x^2 + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u^2 + \begin{bmatrix} 0 & 0 \\ \gamma & 0 \end{bmatrix} x^1 + \begin{bmatrix} 0 & 0 \\ -\gamma & 0 \end{bmatrix} x^2,$$
$$y^1 = \begin{bmatrix} 1 & 0 \end{bmatrix} x^1,$$
$$y^2 = \begin{bmatrix} 1 & 0 \end{bmatrix} x^2,$$

where $x^l = [\theta^l \ \dot{\theta}^l]^T$, θ is the deviation angle of the l-th pendulum relative to the vertical line, u^l is the input force on the l-th pendulum $(l = 1, 2)$, $\alpha = g/l, \ \beta = 1/ml^2, \ \gamma = ka^2/ml^2$, m and l are the mass and the length of each pendulum, g is the acceleration of gravity, a is the distance from the pendulum axis to the point of spring attachment, k is stiffness coefficient of the spring. The mass of each pendulum can be changed in time taking any of the three discrete values: $m - \Delta m, \ m, \ m + \Delta m$ so that we have nine different modes of the system depending on the mass combination. It is supposed that the mode change process is a homogeneous Markov chain with unknown transition probabilities. The problem is to stabilize both the pendulums in their upper equilibrium states. Because the matrix of transition probabilities is unknown we try to find an output feedback nonswitching controller stabilizing the system independently of the mode change process. To form stabilizing input forces we use local dynamical controllers described by

$$\dot{x}_c^l = a_c^l x_c^l + b_c^l y^l,$$
$$u^l = c_c^l x_c^l + d_c^l y^l, \ l = 1, 2.$$

The numerical values of the parameters are the following: $l = 1\text{m}, \ m = 1\text{kg}, \Delta m = 0.2\text{kg}, k = 0.2\text{N/m}, g = 9.81\text{m/s}^2$. The algorithm based on direct CRE solution gives the following nonswitching output feedback matrix:

$$G = \begin{bmatrix} 58.0726 & -2.1871 & 0.0000 & 0.0000 \\ -52.5394 & 3.4058 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 57.8108 & -2.1833 \\ 0.0000 & 0.0000 & -51.9872 & 3.3395 \end{bmatrix}.$$

Typical impulse responses for the first pendulum in mode 1 are presented on Fig.1.

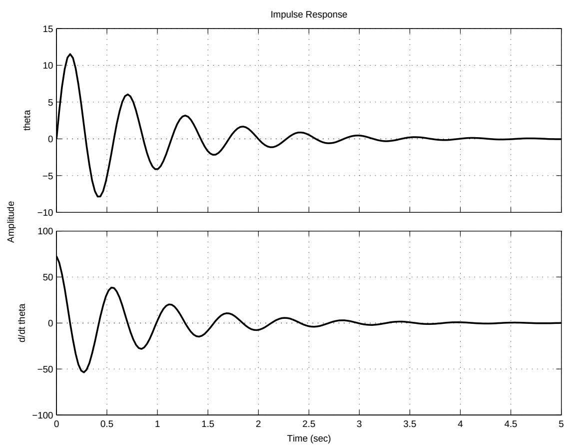

Fig. 1. Impulse responses for the first pendulum.

REFERENCES

AitRami, M. and L. ElGhaoui (1996). LMI optimization for nonstandard Riccati equation arising in stochastic control. *IEEE Trans. Automat. Control* **41**, 1666–1671.

Boukas, E.K., A. Swierniak, K. Simek and H. Yang (1997). Robust stabilization and guaranteed cost control of large scale linear systems with jumps. *Kybernetika* **33**, 121–131.

Boyd, S., L. ElGhaoui, E. Feron and V. Balakrishnan (1994). *Linear matrix inequalities in control and system theory*. SIAM. Philadelphia.

Cao, Y.Y., J. Lam and Y.X Sun (1998). Static output feedback stabilization: an ILMI approach. *Automatica* **34**, 1641–1645.

ElGhaoui, L. and M. AitRami (1996). Robust state feedback stabilization of jump linear systems via LMI's. *Int. J. Robust and Nonlinear Control* **6**, 1015–1022.

Garcia, G., B. Pradin, S. Tarbouriech and F. Zeng (2003). Robust stabilization and guaranteed cost control for discrete-time linear systems by static output feedback. *Automatica* **39**, 1635–1641.

Iwasaki, T. and R.E. Skelton (1995). Parametrization of all stabilizing controllers via quadratic lyapunov functions. *J. Optimization Theory and Appl.* **85**, 291–307.

Kats, I.Ya. (1998). *Lyapunov function method in problems of stability and stabilizability of random structure systems*. UGAPS. Ekaterinburg.

Kučera, V. and C.E. DeSouza (1995). A necessary and sufficient conditions for output feedback stabilizability. *Automatica* **31**, 1357–1359.

Mariton, M. (1990). *Jump linear systems in automatic control*. Marcel Dekker. New York.

Pakshin, P.V. and D.M. Retinsky (2003). Robust stabilization of jumping systems via static output feedback. In: *Proc. European Control Conf.*. Vol. CD ROM. pp. 1–6. EUCA. Cambridge,UK.

Pakshin, P.V. and I.N. Mitrofanov (2004). Parametrization of stabilizing controllers and robust stabilization via static output feedback for jump linear systems. In: *Prepr. 6th IFAC Symp. Nonlinear Control Systems*. Vol. 3. pp. 1151–1156. University of Stuttgart. Stuttgatt.

Siljak, D.D. (1991). *Decentralized control of complex systems*. Academic Press. New York.

Syrmos, V.L., C.T. Abdallah, P. Dorato and K. Grigoriadis (1997). Static output feedback - A survey. *Automatica* **33**, 125–137.

Trofino-Neto, A. and V. Kučera (1993). Stabilization via output feedback. *IEEE Trans. Automat. Control* **38**, 764–765.

Yu, J. (2004). A convergent algorithm for computing stabilizing static output feedback gains. *IEEE Trans. Automat. Control* **49**, 2271–2275.