STABILITY OF LINEAR NEUTRAL SYSTEMS WITH MIXED DELAY AND POLYTOPIC UNCERTAINTY

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Abstract: This paper is concerned with the stability of uncertain linear systems with mixed neutral and discrete delays. The uncertainty under consideration is of polytopic type. A new analysis approach which combines the descriptor system transformation and relaxation matrix is proposed to derive some less conservative stability criteria. The criteria are dependent on both neutral delay and discrete delay. Numerical examples are given to indicate improvements over some existing results.

Keywords: Stability, neutral delay, discrete delay, uncertainty

1. INTRODUCTION

Due to the increase in high performance of VLSI systems, delay circuits become very important. The two types of delay circuits which are widely used in the literature are the partial element equivalent circuits (PEEC's) (Bellen *et al.*, 1999; Cullum *et al.*, 2000) and the distributed networks containing lossless transmission lines (Brayton, 1966). One can use the so-called neutral systems to model these kinds of delay circuits. During the past few years, the delay-dependent stability problem, which means that the stability conditions include the delay information, of linear neutral systems has attracted considerable attention. The goal is to obtain the maximum allowed upper bound on the delay that guarantees the stability of a linear neutral system. Therefore, the admissible allowed upper bound on the delay is the main "performance index" for measuring the conservatism of the conditions obtained. In the time-domain, the direct Lyapunov method is a powerful tool for studying the stability of linear neutral systems. The Lyapunov-Krasovskii approach is widely employed to consider the stability problem for the systems. In the following some more recent results regarding discrete-delay dependent stability conditions are briefly mentioned.

There are two kinds of linear neutral systems. One kind of system is that the neutral delay is the same as the discrete delay while the other kind of system is one with *different* neutral delay and discrete delay. For linear neutral systems

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with the same neutral delay and discrete delay, some delay-dependent stability criteria are proposed in the literature (Han, 2001; Han, 2002; Lien *et al.*, 2000; Yue and Han, 2004*a*). For linear neutral systems with *different* neutral delay and discrete delay, depending on whether the stability conditions include the information of neutral delay or not, the existing criteria can be classified into two types; that is *neutral-delay-independent* and discrete-delay-dependent stability criteria and *neutral-delay-dependent* and discrete-delay-dependent stability criteria (Han, 2001). Most of the existing results mainly focus on the *neutral-delay-independent* and discrete-delay-dependent stability criteria (Chen *et al.*, 2000; Han, 2004*b*; Han, 2004*a*; Niculescu, 2001*b*; Yue and Won, 2003). To date, only very few result (He *et al.*, 2004) regarding *neutral-delay-dependent* and discrete-delay-dependent stability criteria are available. Therefore, there is still much work to be done along this direction since the neutral delay also has a sigificant effect on the stability of the considered systems, which can be seen from a simple scalar example, $\ddot{y}(t) + \dot{y}(t) + y(t) = a\left[\ddot{y}(t-r) + \dot{y}(t-r) + y(t-r)\right]$, where $a > 1$; when $r = 0$, the system is asymptotically stable while it is unstable for any $r > 0$ (Kolmanovskii and Myshkis, 1992).

In this paper, the effect of neutral delay on the stability of uncertain linear neutral systems will be investigated. Combining the descriptor system transformation (Niculescu, 2001*a*) and relaxation matrix method (He *et al.*, 2004; Yue and Han, 2004*a*), we will derive some new *neutral-delay-dependent* and discrete-delay dependent stability criteria through which one can get less conservative results. Examples will be given to show the effectiveness of the criteria.

Notation: $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, I is the identity matrix of appropriate dimensions, $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate. The notation $X > 0$ (respectively, $X \geq 0$), for $X \in \mathbf{R}^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, positive semi-definite). $\eta_i(A)$ denotes the ith eigenvalue of matrix A. For an arbitrarily matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. SYSTEM DESCRIPTION AND MAIN RESULT

Consider the following linear neutral system

$$\dot{y}(t) - F\dot{y}(t-r) = Hy(t) + Ky(t-\tau), t \geq t_0$$

$$y(t) = \phi(t), t \in [t_0 - \max\{\tau, r\}, t_0], \quad (1)$$

where $y(t) \in \mathbf{R}^n$. H, K and F are constant matrices of appropriate dimensions. If parameter uncertainties exist in the system matrices H and K and are of polytopic type, H and K can be expressed as

$$\left[\, H \;\; K \,\right] = \sum_{i=1}^{m} \lambda_i \left[\, H_i \;\; K_i \,\right], \quad (2)$$

where $\sum_{i=1}^{m} \lambda_i = 1$, $0 \leq \lambda_i \leq 1$. $r > 0$ and $\tau > 0$ are two constants which denote the neutral delay and discrete delay, respectively. In what follows, without loss of generality, we set $t_0 = 0$. For system (1), we need the following assumption.

Assumption 1. All the eigenvalues of matrix F are inside the open unit circle, i.e. $|\eta_i(F)| < 1$ $(i = 1, 2, ..., n)$.

Define (Niculescu, 2001*a*)

$$x_1(t) = y(t), x_2(t) = \dot{y}(t) - Hy(t). \quad (3)$$

Then, (1) can be transformed into an equivalent system

$$\dot{x}_1(t) = Hx_1(t) + x_2(t) \quad (4)$$

$$0 = -x_2(t) + Kx_1(t-\tau) + FHx_1(t-r) + Fx_2(t-r), \quad (5)$$

$$x_1(t) = \phi(t), x_2(t) = \dot{\phi}(t) - H\phi(t), \quad t \in [-\max\{\tau, r\}, 0]. \quad (6)$$

Letting $E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $A = \begin{bmatrix} H & I \\ 0 & -I \end{bmatrix}$, $A_1 = \begin{bmatrix} 0 & 0 \\ K & 0 \end{bmatrix}$ and $A_2 = \begin{bmatrix} 0 & 0 \\ FH & F \end{bmatrix}$, (4)-(6) can be rewritten as the following time-delay descriptor system

$$E\dot{x}(t) = Ax(t) + A_1x(t-\tau) + A_2x(t-r),$$
$$x_1(t) = \phi(t), x_2(t) = \dot{\phi}(t) - H\phi(t), \quad t \in [-\max\{\tau, r\}, 0], \quad (7)$$

where $x(t) = \left[\, x_1^T(t) \;\; x_2^T(t) \,\right]^T$. From (2), A, A_1 and A_2 can be expressed as

$$A = \sum_{i=1}^{m} \lambda_i A^i, A_1 = \sum_{i=1}^{m} \lambda_i A_1^i, A_2 = \sum_{i=1}^{m} \lambda_i A_2^i \quad (8)$$

where $A^i = \begin{bmatrix} H_i & I \\ 0 & -I \end{bmatrix}$, $A_1^i = \begin{bmatrix} 0 & 0 \\ K_i & 0 \end{bmatrix}$ and $A_2^i = \begin{bmatrix} 0 & 0 \\ FH_i & F \end{bmatrix}$.

To study the stability of (1), we first introduce two definitions.

Definition 1. The neutral system (1) is said to be exponentially stable, if there exist constants $\alpha > 0$ and $\beta > 0$ such that $\|y(t)\| \leq \alpha \sup_{-\max\{\tau,r\}\leq s\leq 0} \left\{ \|\phi(s)\|, \left\|\dot{\phi}(s)\right\| \right\} e^{-\beta t}$.

Definition 2. The descriptor system (7) is said to be $E-$ exponentially stable, if there exist constants $\alpha > 0$ and $\beta > 0$ such that $\|Ex(t)\| \leq \alpha \sup_{-\max\{\tau,r\}\leq s\leq 0} \left\{ \|\phi(s)\|, \left\|\dot{\phi}(s)\right\| \right\} e^{-\beta t}$.

Remark 1. It is clear to see that the exponential stability of (1) is equivalent to the $E-$ exponential stability of (7). Therefore, in the following we will employ system (7) to study the exponential stability of (1).

Now we state and establish the following result for the $E-$ exponential stability of (7).

Proposition 1. Under Assumption 1, for given scalars $\tau > 0$ and $r > 0$, system (7) is $E-$ exponentially stable if there exist matrices $P_1 > 0, P_2$, P_3, $W_i > 0$ $(i = 1, 2)$, $Q_j > 0$ $(j = 1, 2, 3)$, and matrices N_k, S_k, G_k and M_k $(k = 1, 2, 3, 4)$ of appropriate dimensions such that

$$\begin{bmatrix} \Omega & * & * & * \\ rN^T & -rQ_1 & * & * \\ \tau S^T & 0 & -\tau Q_2 & * \\ \delta(\tau,r)G^T & 0 & 0 & -\delta(\tau,r)Q_3 \end{bmatrix} < 0, \quad (9)$$

where

$$\Omega = \begin{bmatrix} \Gamma_{11} & * & * & * \\ \Gamma_{21} & \Gamma_{22} & * & * \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & * \\ \Gamma_{41} & \Gamma_{42} & \Gamma_{43} & \Gamma_{44} \end{bmatrix},$$

$$N = \left[\, N_1^T \; N_2^T \; N_3^T \; N_4^T \,\right]^T,$$

$$S = \left[\, S_1^T \; S_2^T \; S_3^T \; S_4^T \,\right]^T,$$

$$G = \left[\, G_1^T \; G_2^T \; G_3^T \; G_4^T \,\right]^T,$$

$$M = \left[\, M_1^T \; M_2^T \; M_3^T \; M_4^T \,\right]^T,$$

$$\delta(\tau,r) = \begin{cases} \tau - r, \; if \tau > r \\ r - \tau, \; if r > \tau \end{cases},$$

$$\begin{aligned} \Gamma_{11} &= W_1 + W_2 + N_1E + EN_1^T + S_1E \\ &\quad + ES_1^T - M_1A - A^TM_1^T, \\ \Gamma_{21} &= N_2E + S_2E - ES_1^T - EG_1^T \\ &\quad - M_2A - A_1^TM_1^T, \\ \Gamma_{22} &= -W_1 - S_2E - ES_2^T - G_2E - EG_2^T \\ &\quad - M_2A_1 - A_1^TM_2^T, \\ \Gamma_{31} &= S_3E + N_3E - EN_1^T + EG_1^T \\ &\quad - M_3A - A_2^TM_1^T, \\ \Gamma_{32} &= -EN_2^T - S_3E - G_3E - EG_2^T \\ &\quad - M_3A_1 - A_2^TM_2^T, \\ \Gamma_{33} &= -W_2 - N_3E - EN_3^T + G_3E + EG_3^T \\ &\quad - M_3A_2 - A_2^TM_3^T, \\ \Gamma_{41} &= P^T + N_4E + S_4E - M_4A + M_1^T, \\ \Gamma_{42} &= -S_4E - G_4E - M_4A_1 + M_2^T, \\ \Gamma_{43} &= -N_4E + G_4E + M_3^T - M_4A_2, \\ \Gamma_{44} &= rQ_1 + \tau Q_2 + \delta(\tau,r)Q_3 + M_4 + M_4^T, \end{aligned}$$

$$P = \begin{bmatrix} P_1 & P_2 \\ 0 & P_3 \end{bmatrix}.$$

Proof. See the full version (Yue and Han, 2004*b*) of the paper.

Considering the parameter uncertainties of type (8), we can obtain the following result.

Proposition 2. Under Assumption 1, for given scalars $\tau > 0$ and $r > 0$, the system (7) with parameter uncertainty described by (8) is $E-$ exponentially stable. if there exist matrices $P_1^l > 0, P_2^l$, P_3^l, $W_i^l > 0$ $(i = 1, 2)$, $Q_j^l > 0$ $(j = 1, 2, 3)$, and matrices N_k^l, S_k^l, G_k^l $(k = 1, 2, 3, 4; l = 1, 2, ..., m)$ and M_k $(k = 1, 2, 3, 4)$ of appropriate dimensions such that

$$\begin{bmatrix} \Omega^l & * & * & * \\ r\left(N^l\right)^T & -rQ_1^l & * & * \\ \tau\left(S^l\right)^T & 0 & -\tau Q_2^l & * \\ \delta(\tau,r)\left(G^l\right)^T & 0 & 0 & -\delta(\tau,r)Q_3^l \end{bmatrix} < 0, (10)$$

where

$$\Omega^l = \begin{bmatrix} \Gamma_{11}^l & * & * & * \\ \Gamma_{21}^l & \Gamma_{22}^l & * & * \\ \Gamma_{31}^l & \Gamma_{32}^l & \Gamma_{33}^l & * \\ \Gamma_{41}^l & \Gamma_{42}^l & \Gamma_{43}^l & \Gamma_{44}^l \end{bmatrix},$$

$$N^l = \left[\, \left(N_1^l\right)^T \; \left(N_2^l\right)^T \; \left(N_3^l\right)^T \; \left(N_4^l\right)^T \,\right]^T,$$

$$S^l = \left[\, \left(S_1^l\right)^T \; \left(S_2^l\right)^T \; \left(S_3^l\right)^T \; \left(S_4^l\right)^T \,\right]^T,$$

$$G^l = \left[\, \left(G_1^l\right)^T \; \left(G_2^l\right)^T \; \left(G_3^l\right)^T \; \left(G_4^l\right)^T \,\right]^T,$$

$$M = \left[\, M_1^T \; M_2^T \; M_3^T \; M_4^T \,\right]^T,$$

$$\delta(\tau,r) = \begin{cases} \tau - r, \; if \tau > r \\ r - \tau, \; if r > \tau \end{cases},$$

Γ_{ij}^l $(i, j = 1, 2, 3, 4)$ are the same as Γ_{ij} in Proposition 1 by replacing A, A_1, A_2, P, $P_1 > 0, P_2$, P_3, $W_i > 0$ $(i = 1, 2)$, $Q_j > 0$ $(j = 1, 2, 3)$, and matrices N_k, S_k, G_k $(k = 1, 2, 3, 4)$ with A^l, A_1^l, A_2^l, P^l, $P_1^l > 0, P_2^l$, P_3^l, $W_i^l > 0$ $(i = 1, 2)$, $Q_j^l > 0$ $(j = 1, 2, 3)$, and matrices N_k^l, S_k^l, G_k^l $(k = 1, 2, 3, 4; l = 1, 2, ..., m)$,respectively, where $P^l = \begin{bmatrix} P_1^l & P_2^l \\ 0 & P_3^l \end{bmatrix}$.

Remark 2. Proposition 1 provides a **neutral-delay-dependent** and discrete-delay-dependent criterion for system (7). Similar to the proof of Proposition 1, see (Yue and Han, 2004*b*), we can obtain a **neutral-delay-independent** and discrete-delay-dependent condition for system (7). More specifically, we can conclude that under Assumption 1, for a given scalar $\tau > 0$, the $E-$ exponential stability of system (7) is **neutral-delay-independent** and discrete-delay-dependent if there exist matrices $P_1 > 0, P_2$, P_3, $W_i > 0$ $(i = 1, 2)$, $Q_2 > 0$, and matrices S_k, M_k $(k = 1, 2, 3, 4)$ of appropriate dimensions such that

$$\begin{bmatrix} \breve{\Omega} & * \\ \tau S^T & -\tau Q_2 \end{bmatrix} < 0, \tag{11}$$

where

$$\breve{\Omega} = \begin{bmatrix} \breve{\Gamma}_{11} & * & * & * \\ \breve{\Gamma}_{21} & \breve{\Gamma}_{22} & * & * \\ \breve{\Gamma}_{31} & \breve{\Gamma}_{32} & \breve{\Gamma}_{33} & * \\ \breve{\Gamma}_{41} & \breve{\Gamma}_{42} & \breve{\Gamma}_{43} & \breve{\Gamma}_{44} \end{bmatrix},$$
$$S = \left[S_1^T \; S_2^T \; S_3^T \; S_4^T \right]^T,$$
$$M = \left[M_1^T \; M_2^T \; M_3^T \; M_4^T \right]^T,$$

and $\breve{\Gamma}_{ij}$ $(i, j = 1, 2, 3, 4)$ are the same as Γ_{ij} by setting $N_i = 0$, $G_i = 0$ $(i = 1, 2, 3, 4)$ and $Q_1 = Q_3 = 0$. As for the system (7) with parameter uncertainty of type (8), we can also obtain a similar **neutral-delay-independent** and discrete-delay-dependent condition.

In above two propositions, we consider the E-exponential stability of system (7) for the case where $\tau > r$ or $r > \tau$. When $\tau = r$, (7) reduces to the following system

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + \tilde{A}_1 x(t - \tau), \\ x_1(t) &= \phi(t), x_2(t) = \dot{\phi}(t) - L\phi(t), \\ & t \in [-\tau, 0], \end{aligned} \tag{12}$$

where $A = \begin{bmatrix} H & I \\ 0 & -I \end{bmatrix}$, $\tilde{A}_1 = \begin{bmatrix} 0 & 0 \\ K + FH & F \end{bmatrix}$. Similarly, when parameter uncertainties exist in the matrices M and L and are of polytopic type,

$$A = \sum_{i=1}^{m} \lambda_i A^i, \tilde{A}_1 = \sum_{i=1}^{m} \lambda_i \tilde{A}_1^i \tag{13}$$

where $A^i = \begin{bmatrix} H_i & I \\ 0 & -I \end{bmatrix}$ and $\tilde{A}_1^i = \begin{bmatrix} 0 & 0 \\ K_i + FH_i & F \end{bmatrix}$. Then, similar to Propositions 1 and 2, we can obtain the following results for the case of $\tau = r$.

Proposition 3. Under Assumption 1, for a given scalar $\tau > 0$, system (12) is $E-$ exponentially stable if there exist matrices $P_1 > 0$, P_2, P_3, $W > 0$, $Q > 0$, N_i and M_i $(i = 1, 2, 3)$ of appropriate dimensions such that

$$\begin{bmatrix} \tilde{\Gamma}_{11} & * & * & * \\ \tilde{\Gamma}_{21} & \tilde{\Gamma}_{22} & * & * \\ \tilde{\Gamma}_{31} & \tilde{\Gamma}_{32} & \tilde{\Gamma}_{33} & * \\ \tau N_1^T & \tau N_2^T & \tau N_3^T & -\tau Q \end{bmatrix} < 0, \tag{14}$$

where

$$\begin{aligned} \tilde{\Gamma}_{11} &= W + N_1 E + E N_1^T - M_1 A - A^T M_1^T, \\ \tilde{\Gamma}_{21} &= -E N_1^T + N_2 E - \tilde{A}_1^T M_1^T - M_2 A, \\ \tilde{\Gamma}_{22} &= -W - N_2 E - E N_2^T - M_2 \tilde{A}_1 - \tilde{A}_1^T M_2^T, \\ \tilde{\Gamma}_{31} &= P^T + M_1^T + N_3 E - M_3 A, \\ \tilde{\Gamma}_{32} &= M_2^T - N_3 E - M_3 \tilde{A}_1, \\ \tilde{\Gamma}_{33} &= \tau Q + M_3 + M_3^T, \end{aligned}$$
$$P = \begin{bmatrix} P_1 & P_2 \\ 0 & P_3 \end{bmatrix}.$$

Proposition 4. Under Assumption 1, for given scalars $\tau > 0$, the system (12) with parameter uncertainty described by (13) is $E-$ exponentially stable if there exist matrices $P_1^l > 0$, P_2^l, P_3^l, $W^l > 0$, $Q^l > 0$, N_i^l $(l = 1, 2, ..., m)$ and M_i $(i = 1, 2, 3)$ of appropriate dimensions such that

$$\begin{bmatrix} \tilde{\Gamma}_{11}^l & * & * & * \\ \tilde{\Gamma}_{21}^l & \tilde{\Gamma}_{22}^l & * & * \\ \tilde{\Gamma}_{31}^l & \tilde{\Gamma}_{32}^l & \tilde{\Gamma}_{33}^l & * \\ \tau \left(N_1^l\right)^T & \tau \left(N_2^l\right)^T & \tau \left(N_3^l\right)^T & -\tau Q^l \end{bmatrix} < 0, \tag{15}$$

where $\tilde{\Gamma}_{ik}^l$ $(i, k = 1, 2, 3)$ are the same as $\tilde{\Gamma}_{ik}$ in Corollary 1 by replacing A, $\tilde{A}_1$, P, $P_1 > 0$, P_2, P_3, $W > 0$, $Q > 0$, and N_i with A^l, $\tilde{A}_1^l$, P^l, $P_1^l > 0$, P_2^l, P_3^l, $W^l > 0$, $Q^l > 0$, and N_i^l, respectively, where $P^l = \begin{bmatrix} P_1^l & P_2^l \\ 0 & P_3^l \end{bmatrix}$.

3. NUMERICAL EXAMPLES

Example 1. Consider the linear neutral system (1) with

$$H = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, K = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix},$$
$$F = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}.$$

Two cases are considered for this example.

Case I: $\tau = r$

Using criteria in (Han, 2004*a*; He *et al.*, 2004; Lien and Chen, 2003) and this paper, the results on the maximum allowed time-delay for stability are compared in Table 1. It can be seen that the result using the method in this paper indeed improves the ones derived using the mentioned existing methods. Other results surveyed in (He *et al.*, 2004) are even more conservative.

Table 1. Comparison of $\tau_{\max}$ using different methods.

Methods	$\tau_{\max}$
(Lien and Chen, 2003)	0.8844
(Han, 2004*a*)	1.6014
(He *et al.*, 2004)	1.6527
This paper	1.7884

Case II: $\tau \neq r$

Applying the **neutral-delay-independent** and discrete-delay-dependent stability condition mentioned in Remark 2, the maximum discrete delay $\tau_{\max}$ for asymptotic stability is computed as 1.7124 while the result in (He *et al.*, 2004) was reported as 1.6527.

The effect of the neutral delay r on the maximum discrete delay $\tau_{\max}$ is shown in Table 2 employing the methods in (He *et al.*, 2004) and this paper. It is clear to see that the method is this paper can give better results than that in (He *et al.*, 2004).

Table 2. Bound $\tau_{\max}$ calculated for various r.

r	0.01	0.05	0.1	0.2
(He *et al.*, 2004)	—	—	1.7100	1.6987
This paper	1.7594	1.7511	1.7447	1.7351
r	0.7	0.8	0.9	1.0
(He *et al.*, 2004)	1.6624	1.6591	1.6564	1.6543
This paper	1.7166	1.7173	1.7194	1.7213
r	1.5	1.6	1.6527	1.7884
(He *et al.*, 2004)	1.6527	1.6527	1.6527	1.6527
This paper	1.7464	1.7545	1.7619	1.7884
r	0.3	0.4	0.5	0.6
(He *et al.*, 2004)	1.6883	1.6792	1.6718	1.6664
This paper	1.7246	1.7211	1.7200	1.7161
r	1.1	1.2	1.3	1.4
(He *et al.*, 2004)	1.6531	1.6527	1.6527	1.6527
This paper	1.7263	1.7297	1.7352	1.7402
r	1.8	1.9	2.0	1000
(He *et al.*, 2004)	—	—	—	1.6527
This paper	1.7720	1.7509	1.7377	1.7125

Table 3 shows that when $|\tau - r| \to 0^+$, the maximum discrete delay $\tau_{\max}$ approaches the result for the case of $\tau = r$.

Table 3. Bound $\tau_{\max}$ calculated for different r around τ.

r	1.7584	1.7684	1.7784	1.7884
$\tau_{\max}$	1.7780	1.7783	1.7814	1.7884
r	1.7984	1.8084	1.8184	
$\tau_{\max}$	1.7760	1.7711	1.7701	

Example 2. Consider the uncertain linear neutral system (1) with

$$H = \begin{bmatrix} -2+\delta_1 & 0 \\ 0 & -1+\delta_2 \end{bmatrix},$$

$$K = \begin{bmatrix} -1+\gamma_1 & 0 \\ -1 & -1+\gamma_2 \end{bmatrix},$$

$$F = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

where δ_i and γ_i ($i = 1, 2$) denote the parameter uncertainties satisfying $|\delta_1| \leq 1.6$, $|\delta_2| \leq 0.05$, $|\gamma_1| \leq 0.1$, $|\gamma_2| \leq 0.3$.

When $\tau = r$, it is reported in (Yue and Han, 2004*a*) that the maximum allowed value of $\tau_{\max}$ is 1.54. Now we consider the case of $\tau \neq r$. Table 4 lists the numerical results for different r. From this table one can see again that $|\tau - r| \to 0^+$, the maximum discrete delay $\tau_{\max}$ approaches the result for the case of $\tau = r$.

Table 4. Comparison of $\tau_{\max}$ using different methods.

r	0.1	0.5	1.0	1.1	1.2	1.3
$\tau_{\max}$	1.42	1.40	1.43	1.44	1.45	1.47
r	1.4	1.54	1.6	1.7	1.8	1.9
$\tau_{\max}$	1.49	1.54	1.49	1.45	1.43	1.41

4. CONCLUSION

Some new neutral-delay-dependent and discrete-delay dependent stability criteria have been obtained for a class of linear neutral systems. It has been shown through numerical examples that the criteria in this paper can provide less conservative results than some existing method. We have also concluded that when neutral delay is the same as discrete delay, the maximum allowed delay bound for stability can be achieved.

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