ADAPTIVE CONTROL OF A SHUNT DC MOTOR WITH PERSISTENT EXCITATION

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Abstract: The adaptive input-output linearizing control has been successful in achieving the asymptotic output tracking stability for a shunt DC motor, however, persistent excitation (PE) of the regressor has not been investigated. The major difficulty is the ultimate behaviors of the estimated parameters and the field current are not predictable. Nevertheless, it is found that PE can be attained under some mild assumptions on the system and the reference trajectories. Simulation results confirming the assertion are given in the final.

Keywords: shunt DC motor; adaptive control; feedback linearization; persistent excitation.

1. INTRODUCTION

Shunt DC motors are widely used in various applications due to their capability of wide-range speed regulation and relatively high torque regarding their weight. Their dynamics can be adequately described by a three-state nonlinear model. The adaptive input-output linearizing control has been successful in accomplishing the tasks of trajectory tracking in the presence of parameter uncertainty (Chiasson & Bodson, 1991; Tafur-Sotelo & Vélez-Reyes, 2002). However, checkable conditions for the PE of the regressor are not available so far.

PE guarantees not only the exponential parametric stability but also improves robustness and transient performances (Narendra & Annaswamy, 1989). However, prior check of its fulfillment is not easy especially in a general nonlinear closed-loop system. It is even more difficult in this case because the estimated parameters and the unobservable field current entering the regressor are basically unpredictable. Nevertheless, under some checkable mild assumptions, it is found that the very desired property, i.e., persistent excitation, can actually be obtained eventually.

The remainder of the paper is organized as follows. The mathematical model and the properties achieved by the adaptive input-output linearizing control scheme are reviewed in Section 2. Sufficient conditions for the PE of the regressor are established in Section 3. A numerical example is given in Section 4 to illustrate the main idea of the assertion of this paper. Concluding remarks are finally made in Section 5.

2. PRELIMINARIES

A shunt DC motor, as depicted in Fig. 1, is a motor in which the field circuit is connected in parallel with the armature circuit. As a consequence,

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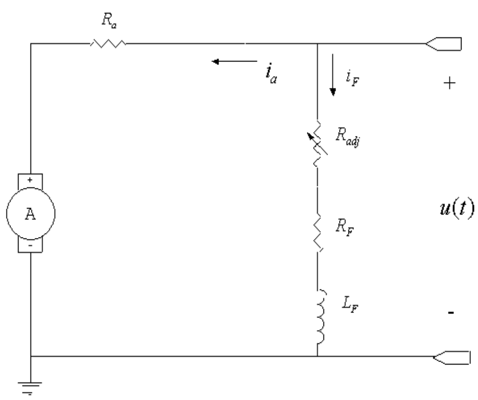

Fig. 1. Schematic diagram of a shunt DC motor

their dynamics are strongly coupled together. Its dynamical behavior can be described by

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\alpha^T\phi(x) + \beta x_3 u \\ \dot{x}_3 &= -\gamma_1 x_3 + \gamma_2 u \end{aligned} \tag{1}$$

where x_1 is the angular displacement of the motor shaft, x_2 is the corresponding angular velocity, x_3 is the field current, u is the control input, $\phi(x) = [x_2, x_2x_3^2, \tau_L]^T$, $\alpha = [B/J, K_mK_bK_F^2/(JR_a), 1/J]^T$, $\beta = K_mK_F/(JR_a)$, $\gamma = [(R_{\rm adj}+R_F)/L_F, 1/L_F]^T$. Physical meanings of the system parameters $m, J, B,\ K_m,\ K_b, K_F,\ R_a, R_F, R_{\rm adj},\ L_F$, and τ_L appeared above can be found in (Chiasson & Bodson, 1991), they are omitted here due to space limitation. By viewing x_1 as the output, the system (1) can be regarded as a linearizable system with relative degree two, where x_3 is the unobservable state (Chiasson & Bodson, 1991). Therefore, given a reference trajectory $x_1^d(t)$, the adaptive input-output linearizing control proposed by Sastry & Isidori (1989) can be applied to guarantee the asymptotic tracking stability. In this case, it can be written explicitly as

$$u = \frac{\hat{\alpha}^T\phi(x) + v}{\hat{\beta}x_3} \tag{2}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are the estimates for α and β respectively and the extra control input v is given by

$$v(t) = \dot{x}_2^d - k^Te, \tag{3}$$

with $k = [k_1, k_2]^T$ the control gain and $e = [e_1, e_2]^T = [x_1 - x_1^d, x_2 - \dot{x}_1^d]^T$ the tracking error vector. Apparently, full-state measurement and the knowledge of the load torque are required for implementing the control (2). Substituting (2) and (3) into (1), it yields

$$\begin{aligned}\dot{e} &= Ae + B(\tilde{\theta}^T\psi(t)) \\ \dot{x}_3 &= -\gamma_1 x_3 + \gamma_2 u \end{aligned} \tag{4}$$

where $\tilde{\theta} = [\hat{\alpha} - \alpha, \hat{\beta} - \beta]^T$ and

$$A = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$\psi(t) = [x_2, x_2x_3^2, \tau_L, -\frac{\hat{\alpha}^T\phi(x) + v(t)}{\hat{\beta}}]^T \tag{5}$$

The corresponding parameter update law is

$$\dot{\hat{\theta}}(t) = -c(e^TPB)\psi(t) \tag{6}$$

with $c > 0$ being the update gain and the symmetric positive-definite matrix P being the solution to the following Lyapunov equation

$$A^TP + PA = -Q, \quad Q > 0. \tag{7}$$

To avoid the control (2) from singularity and to ensure the boundedness of the state x_3, the following assumptions are needed.

A1) The parameter vector θ and its estimates $\hat{\theta}$ are confined by

$$0 < \theta_{im} \le \theta_i(,\hat{\theta}_i) \le \theta_{iM}, i = 1, \cdots, 4 \tag{8}$$

where the bounds θ_{im} and θ_{iM} are known *a priori*.

A2) The two criteria, namely, $|\ \hat{\theta}_4x_3\ | > \delta_0 > 0$ and $\gamma_1 - (\gamma_2\hat{\theta}_2/\hat{\theta}_4)x_2 > \delta_1 > 0$, hold for all time.

Remark 1. For fulfilling A1), certain parameter projection algorithms may be incorporated in real applications. Moreover, verification of A2) may not be easy if prior knowledge of γ_1 and γ_2 is not available. However, they are all assumed to sustain for simplicity.

Under A1)-A2), the control (2) ensures the following two properties (Chiasson & Bodson, 1991)

P1) All the signals in the closed-loop system are bounded.

P2) $e(t), d\hat{\theta}/dt \to 0$ as $t \to \infty$.

The properties P1)-P2) can be easily seen by selecting the Lyapunov function $V(e,\tilde{\theta}) = 1/2(e^TPe + \tilde{\theta}^T\Gamma^{-1}\tilde{\theta})$ and calculating its time derivatives, which results in

$$\begin{aligned}\dot{V}(e,\tilde{\theta}) &= e^TP\dot{e} + \tilde{\theta}^T\Gamma^{-1}\dot{\tilde{\theta}} \\ &\le -\lambda_{\min}(Q)\|e\|^2 \end{aligned} \tag{9}$$

where $\lambda_{\min}(Q)$ is the minimum eigenvalue of the matrix Q. The fact of $e \in L_2$ in (9), together with $\dot{e} \in L_\infty$ from (4), ensures the sustenance of P2) from Barbalat's lemma. Next, by multiplying both sides of (4) by x_3, it yields

$$\dot{q} = -2(\gamma_1 - \gamma_2\frac{\hat{\theta}_2}{\hat{\theta}_4}x_2)q + 2\frac{\gamma_2}{\hat{\theta}_4}(v + \hat{\theta}_1 x_2 + \hat{\theta}_3\tau_L) \quad (10)$$

where $q = x_3^2$. From A2) and the boundedness of the signals $e(t)$ and $\tilde{\theta}(t)$ guaranteed by (9), the system (10) can be regarded as an exponentially stable system with bounded input, and therefore P1) is ensured.

3. PERSISTENT EXCITATION

The obstacle for establishing the PE, as stated, lies in that $\hat{\theta}(t)$ and x_3 entering the regressor can not be predicted in advance. Nevertheless, by virtue of P1)-P2) and making some mild assumptions, the difficulty will be conquered in this section.

First, for ease of reference, the definition of PE is quoted here (Narendra & Annaswamy, 1989).

Definition 1 A bounded piecewise continuous signal vector $\Psi : R^+ \mapsto R^n$ is PE in R^n with a level of excitation ϵ_e if there exist constants $t_e, T_e > 0$ such that

$$\frac{1}{T_e}\int_{\xi}^{\xi+T_e} | \zeta^T\Psi(t) | \, dt \geq \epsilon_e, \quad \forall \xi \geq t_e \quad (11)$$

where ζ is any a unit vector in R^n.

In addition to the prior assumptions A1)-A2), the following assumptions are also included for attaining the goals.

A3) The load torque τ_L is a known constant.
A4) The reference trajectory $x_1^d(t)$ is smooth and T-periodic.
A5) The set of functions given below is linearly independent within $[0, T]$.

$$\ddot{x}_2^d x_2^d, (\dot{x}_2^d)^2, \dot{x}_2^d(x_2^d)^2, (x_2^d)\dot{x}_2^d, \dot{x}_2^d, (x_2^d)^3, (x_2^d)^2, 1 \quad (12)$$

Based on P1)-P2) and the above assumptions, it will be shown that the field current x_3 will stay in the vicinity of some T-periodic orbit after the time becomes sufficiently large. Let's proceed from defining two constants here

$$c_2 = \max_{t\geq 0} x_2^d(t),$$
$$\epsilon_a = \frac{\delta_1\theta_{4m}}{2\theta_{6M}}[\frac{c_2(\theta_{4m}+\theta_{2M})T}{\theta_{4m}} + \theta_{2M}]^{-1} \quad (13)$$

Since $e(t), \dot{\hat{\theta}}(t) \to 0$ as $t \to \infty$, by definition, there exists a $t_a > 0$ such that

$$\|e(t)\|, \|\dot{\hat{\theta}}(t)\| \leq \epsilon_a, \quad \forall t \geq t_a. \quad (14)$$

Moreover, it is easy to see that the following subset in the parameter space is well defined.

$$\Omega_\theta = \{\hat{\theta} \mid \hat{\theta}(t) \in R^4, t \geq t_a\} \quad (15)$$

Given a $t_b \geq t_a$, the vector $\bar{\theta} = \hat{\theta}(t_b)$ is apparently in Ω_θ. It follows that

$$\|\delta\hat{\theta}(t)\| \triangleq \|\hat{\theta}(t) - \bar{\theta}\| \leq \epsilon_a \cdot (t - t_b), \quad \forall t \geq t_b \quad (16)$$

Along that line, the lower bound for $(\gamma_1 - \gamma_2\frac{\bar{\theta}_2}{\bar{\theta}_4}x_2^d)$ can also be estimated as

$$\begin{aligned}
\gamma_1 - \gamma_2\frac{\bar{\theta}_2}{\bar{\theta}_4}x_2^d(t) &= (\gamma_1 - \gamma_2\frac{\hat{\theta}_2(t)}{\hat{\theta}_4(t)}x_2(t)) \\
&+\gamma_2\frac{\hat{\theta}_2(t)}{\hat{\theta}_4(t)}x_2(t) - \gamma_2\frac{\hat{\theta}_2(t)}{\hat{\theta}_4(t)}x_2(t) \\
&= (\gamma_1 - \gamma_2\frac{\hat{\theta}_2(t)}{\hat{\theta}_4(t)}x_2(t)) + \gamma_2(\frac{x_2^d}{\hat{\theta}_4(t)})\delta\hat{\theta}_2(t) \\
&-\gamma_2(\frac{\bar{\theta}_2 x_2^d}{\bar{\theta}_4\hat{\theta}_4(t)})\delta\hat{\theta}_4(t) + \gamma_2\frac{\hat{\theta}_2(t)}{\hat{\theta}_4(t)}e_2(t) \\
&\geq (\gamma_1 - \gamma_2\frac{\hat{\theta}_2(t)}{\hat{\theta}_4(t)}x_2(t)) - \theta_{6M}\frac{c_2(\theta_{4m}+\theta_{2M})}{\theta_{4m}^2}\cdot \\
&\quad | \delta\hat{\theta}(t) | -\theta_{6M}\frac{\theta_{2M}}{\theta_{4m}} | e_2(t) | \\
&\geq \delta_1 - \frac{\theta_{6M}}{\theta_{4m}}[\frac{c_2(\theta_{4m}+\theta_{2M})}{\theta_{4m}}(t - t_b) + \theta_{2M}]\epsilon_a \\
&> \delta_1/2 > 0, \quad \forall t \in [t_b, t_b + T]
\end{aligned} \quad (17)$$

By periodicity, the inequality (17) implies that

$$\gamma_1 - \gamma_2\frac{\bar{\theta}_2}{\bar{\theta}_4}x_2^d(t) > \delta_2 > 0, \forall t \geq 0, \bar{\theta} \in \Omega_\theta \quad (18)$$

where $\delta_2 > \delta_1/2$.

Consequently, the reference model

$$\dot{q}_m = -2(\gamma_1 - \gamma_2\frac{\bar{\theta}_2}{\bar{\theta}_4}x_2^d)q_m + u_p(t) \quad (19)$$

is an exponentially stable and periodically varying system with periodic input $u_p(t) = 2(\gamma_2/\bar{\theta}_4)(\dot{x}_2^d(t) + \bar{\theta}_1 x_2^d(t) + \bar{\theta}_3\tau_L)$. The state $q_m(t)$ then tends to the T-periodic trajectory $q_{\bar{\theta}}$ given by (Callier & Desoer, 1991)

$$q_{\bar{\theta}}(t) = \frac{h(t,0)}{1 - h(T,0)}\int_0^T h(T,\eta)u_p(\eta)d\eta$$

$$+\int_0^t h(t,\eta)u_p(\eta)d\eta, \quad t\in[0,T) \quad (20)$$

where $h(t,t_0)=\exp(-2\int_{t_0}^t[\gamma_1-\gamma_2\frac{\bar{\theta}_2}{\bar{\theta}_4}x_2^d(\tau)]d\tau)$ is the state transition matrix. Actually, given any a constant vector $\bar{\theta}\in\Omega_\theta$ in (19), there corresponds to a T-periodic trajectory $q_{\bar{\theta}}$, toward which the state $q_m(t)$ will converge. Denote Ω_q as the set of all the T-periodic trajectories $q_{\bar{\theta}}$ with respect to each $\bar{\theta}\in\Omega_\theta$. By subtracting (10) from (19), the error dynamics for $q(t)-q_m(t)$, denoted as $e_m(t)$, will be

$$\dot{e}_m(t)=-2[\gamma_1-\gamma_2\frac{\bar{\theta}_2}{\bar{\theta}_4}x_2^d(t)]e_m(t)+\Delta(t), \quad (21)$$

where

$$\begin{aligned}\Delta(t)=&2\gamma_2\{(\frac{x_2}{\bar{\theta}_4})\delta\hat{\theta}_1-(\frac{x_2^dq}{\bar{\theta}_4})\delta\hat{\theta}_2+\frac{1\tau_L}{\bar{\theta}_4}\delta\hat{\theta}_3\\&-\frac{1}{\hat{\theta}_4\bar{\theta}_4}(-\bar{\theta}_2x_2^dq+v+\hat{\theta}_1x_2+\hat{\theta}_3\tau_L)\delta\hat{\theta}_4\\&-\frac{k_1}{\bar{\theta}_4}e_1+\frac{1}{\bar{\theta}_4}(\bar{\theta}_1-k_2-\frac{\hat{\theta}_2\bar{\theta}_4}{\hat{\theta}_4}q)e_2\} \quad (22)\end{aligned}$$

On the other hand, the regressor $\psi(t)$ in (5) can be expressed as

$$\begin{aligned}\psi(t)&\triangleq\psi_d(t)+\delta\psi(t)\\&=[x_2^d,x_2^dq_{\bar{\theta}},\tau_L,\frac{\bar{\delta}_2^{\,T}\phi(x^d)+\dot{x}_2^d}{\bar{\beta}}]^T\\&\quad+[e_2,e_2q+x_2^de_q,0,f^T\delta\hat{\theta}+g^Te_t]^T \quad (23)\end{aligned}$$

where $e_q=q-q_{\bar{\theta}}$, $\delta\phi(x)=\phi(x)-\phi(x^d)$, $e_t=[e_1,e_2,e_q]^T$ and

$$\begin{aligned}f&=[f_1,f_2,f_3,f_4]^T\\&=[\frac{x_2^d}{\bar{\theta}_4},\frac{x_2q}{\bar{\theta}_4},\frac{\tau_L}{\bar{\theta}_4},-\frac{\hat{\theta}_1x_2+\hat{\theta}_2x_2q+\hat{\theta}_3\tau_L+v}{\bar{\theta}_4\hat{\theta}_4}]^T\\g&=[g_1,g_2,g_3]^T\\&=[-\frac{k_1}{\bar{\theta}_4},\frac{\hat{\theta}_1+\bar{\theta}_2q-k_2}{\bar{\theta}_4},\frac{\bar{\theta}_2x_2^d}{\bar{\theta}_4}]^T \quad (24)\end{aligned}$$

Consequently, the upper bound for $|\Delta(t)|$ in (22) can be written in a form of

$$|\Delta(t)|\le M_1\|\delta\hat{\theta}(t)\|+M_2\|e(t)\| \quad (25)$$

with

$$\begin{aligned}M_1=&2\gamma_2\max_t(|\frac{x_2}{\bar{\theta}_4}|,|\frac{x_2^dq}{\bar{\theta}_4}|,|\frac{\tau_L}{\bar{\theta}_4}|,|\frac{1}{\hat{\theta}_4\bar{\theta}_4}\\&\cdot\{-\bar{\theta}_2x_2^dq+v+\hat{\theta}_1x_2+\hat{\theta}_3\tau_L\}|)\\M_2=&2\gamma_2\max_{t\ge0}(|\frac{k_1}{\bar{\theta}_4}|+|\frac{1}{\bar{\theta}_4}(\bar{\theta}_1-k_2\\&-\frac{\hat{\theta}_2\bar{\theta}_4}{\hat{\theta}_4}q)|) \quad (26)\end{aligned}$$

Similarly, the upper bound for $|\zeta^T\delta\psi|$ can also be written as

$$\begin{aligned}|\zeta^T\delta\psi(t)|=&|\zeta_1e_2+\zeta_2(e_2q+x_2^de_q)+\zeta_4(f^T\delta\hat{\theta}\\&+g^Te_t)|\\\le&M_3\|\delta\hat{\theta}(t)\|+M_4\|e(t)\|\\&+M_5|e_q(t)| \quad (27)\end{aligned}$$

with $\zeta\in R^4$ an arbitrary unit vector and

$$\begin{aligned}M_3&=\max_{t\ge0}\|f(t)\|,\\M_4&=\max_{t\ge0}(|g_1(t)|,|g_2(t)|+|q(t)|+1)\\M_5&=\max_{t\ge0}(|x_2^d(t)|,|g_3(t)|) \quad (28)\end{aligned}$$

Apparently, the above constants $M_i, i=1,\cdots,5$ are all finite.

Now, by virtue of periodicity, PE of $\psi_d(t)$ is equivalent to the linear independence among its component functions within $[0,T]$ (Huang, 2004). However, since prior knowledge of $q_{\bar{\theta}}$ entering $\psi_d(t)$ is not available, verifiable conditions must be stated in terms of known functions. The remedy is stated here.

Lemma 2. Sustained A1)-A5), the function ψ_d in (23) is PE.

Proof. First, it will be shown that A5) implies that the component functions of $\psi_d(t)$ are linearly independent within $[0,T]$. Suppose they are not. Then, by definition, there exist constants $b_i, i=1,\cdots,4$, with at least one of them being nonzero, such that

$$b_1\dot{x}_2^d+b_2x_2^dq_{\bar{\theta}}+b_3x_2^d+b_4\tau_L=0 \quad (29)$$

When b_2 is zero, A5) will then be violated due to (29). Therefore, we only need to consider the case with $b_2\ne0$. Differentiating (29) with respect to time and re-arranging, it yields

$$\dot{q}_{\bar{\theta}}=-\frac{1}{b_2x_2^d}(b_1\ddot{x}_2^d+b_2\dot{x}_2^dq_{\bar{\theta}}+b_3\dot{x}_2^d) \quad (30)$$

Since $q_{\bar{\theta}}$ also satisfies (19), therefore, by equating it with (30) results in

$$\begin{aligned}&-\frac{1}{b_2x_2^d}(b_1\ddot{x}_2^d+b_2\dot{x}_2^dq_{\bar{\theta}}+b_3\dot{x}_2^d)\\&=-2(\gamma_1-\frac{\gamma_2\bar{\theta}_2}{\bar{\theta}_4}x_2^d)q_{\bar{\theta}}+2\frac{\gamma_2}{\bar{\theta}_4}(\dot{x}_2^d+\bar{\theta}_1x_2^d\\&+\gamma_2\tau_L) \quad (31)\end{aligned}$$

and therefore

$$q_{\bar{\theta}}=(2x_2^d(\gamma_1-\frac{\gamma_2\bar{\theta}_2}{\bar{\theta}_4}x_2^d)-\dot{x}_2^d)^{-1}\cdot\{2\frac{\gamma_2}{\bar{\theta}_4}x_2^d$$

$$(\dot{x}_2^d + \bar{\theta}_1 x_2^d + \gamma_2 \tau_L) + (b_1/b_2)\ddot{x}_2^d$$
$$+(b_3/b_2)\dot{x}_2^d\} \quad (32)$$

Finally, by substituting (32) into (29) and some straightforward manipulations, the following equation can be obtained

$$m_1 \ddot{x}_2^d x_2^d + m_2(\dot{x}_2^d)^2 + m_3 \dot{x}_2^d (x_2^d)^2 + m_4(x_2^d)$$
$$\cdot \dot{x}_2^d + m_5 \dot{x}_2^d + m_6 (x_2^d)^3 + m_7 (x_2^d)^2 = 0 \quad (33)$$

where $m_i, i = 1, \cdots, 7$ are certain constants, with at least one of them being nonzero. Hence, contradiction of A5) occurs.

The above linear independent property implies that (Huang, 2004)

$$\frac{1}{T}\int_0^T \mid \zeta^T \psi_d(t) \mid dt \geq \epsilon(\bar{\theta}), \quad \forall \bar{\theta} \in \Omega_\theta, \quad (34)$$

where $\zeta \in R^4$ is a unit vector, $\epsilon(\bar{\theta})$ is some positive number depending on $\bar{\theta}$. Since the set Ω_θ is bounded, the minimum of all those $\epsilon(\bar{\theta}), \forall \bar{\theta} \in \Omega_\theta$, denoted by ϵ_m, is well-defined, i.e.,

$$\frac{1}{T}\int_0^T \mid \zeta^T \psi_d(t) \mid dt \geq \epsilon_m > 0, \quad \forall \bar{\theta} \in \Omega_\theta \quad (35)$$

By the periodicity of the integrand in (35), it follows

$$\frac{1}{T}\int_\xi^{\xi+T} \mid \zeta^T \psi_d(t) \mid dt \geq \epsilon_m > 0,$$
$$\forall \xi \geq 0, \bar{\theta} \in \Omega_\theta \quad (36)$$

In other words, Lemma 1 implies the PE of all the possible functions ψ_d resulting from every possible $\bar{\theta} \in \Omega_\theta$. □

Since additive vanishing disturbances do not alter the PE of a signal, an immediate consequence of lemma 1 is that the regressor $\psi(t)$ will be persistently excited provided $\delta\psi(t) \to 0$ as $t \to \infty$. This will happen when $\hat{\theta}$ asymptotically converges to some constant vector $\bar{\theta} \in \Omega_\theta$. Unfortunately, the property of $\dot{\hat{\theta}} \to 0$ as $t \to \infty$ in P2) does not totally ensure its occurrence. Therefore, both $\Delta(t)$ and $\delta\psi(t)$ can not be viewed as vanishing disturbances in general. Nevertheless, if $\delta\psi$ in (23) can be shown to be less than ϵ_m for any a time period $[\xi, \xi + T]$ in (11), PE of the regressor can still be inferred. This will happen if we allow $\delta\psi$ be calculated with respect to each $q_{\bar{\theta}} \in \Omega_q$ closest to $q(t)$ within each time period $[\xi, \xi+T]$. It is actually the main idea behind the upcoming derivations.

Before the start, the following positive constants are defined.

$$c_t = \frac{1}{\delta_2} ln(\frac{k_c q_M M_5}{\epsilon_m}),$$
$$\epsilon_1 = \min(\epsilon_a, \frac{2\epsilon_m}{k_c}[M_3(2c_t + T) + M_4]^{-1},$$
$$\frac{\delta_2 \epsilon_m}{k_c M_5}[M_1(c_t + T) + M_2]^{-1}) \quad (37)$$

where k_c is a positive number at disposal and $q_M = \max \mid q(t) \mid, t \geq 0$.

It can be stated that

Theorem 1. The regressor ψ in the closed-loop system (4) will be persistently excited provided A1)-A5) above hold for all time.

Proof. From P2), there exists a positive $t_1 > 0$ with respect to ϵ_1, such that

$$\|e(\tau)\|, \|\dot{\hat{\theta}}(\tau)\| \leq \epsilon_1, \quad \forall \tau \geq t_1. \quad (38)$$

Define $t_p \triangleq t_1 + c_t$. The time instant $t_s = \xi - c_t$, for an arbitrarily given $\xi \geq t_p$, is well defined. Apparently, $\hat{\theta}(t_s) \in \Omega_\theta$. Let the constant vector $\bar{\theta}$ in (21) be equal to $\hat{\theta}(t_s)$. The $\delta\hat{\theta}$ there will be bounded by

$$\|\delta\hat{\theta}(\tau)\| = \|\hat{\theta}(\tau) - \bar{\theta}\| \leq \epsilon_1(\tau - t_s),$$
$$\forall \tau \geq t_s \quad (39)$$

and consequently the inequality (25) can be written as

$$\mid \Delta(\tau) \mid \leq [M_1(\tau - t_s) + M_2]\epsilon_1, \quad \forall \tau \geq t_s (40)$$

Based on (40), the deviation of $q(t)$ from $q_{\bar{\theta}}(t)$, denoted by $e_q(t)$, will be bounded by

$$\mid e_q(t) \mid \leq \mid e_m(t) \mid + \mid q_m(t) - q_{\bar{\theta}}(t) \mid$$
$$\leq \mid h(t, t_s) \mid \cdot (\mid e_m(t_s) \mid + \mid q_m(t_s)$$
$$-q_{\bar{\theta}}(t_s) \mid) + \int_{t_s}^t \mid h(t,\tau) \mid\mid \Delta(\tau) \mid d\tau$$
$$\leq e^{-\delta_2(t-t_s)}[\mid e_m(t_s) \mid + \mid q_m(t_s) - q_{\bar{\theta}}$$
$$(t_s) \mid] + \epsilon_1 \int_{t_s}^t \mid [M_1(\tau - t_s) + M_2] \cdot$$
$$e^{-\delta_2(t-\tau)} \mid d\tau$$
$$\leq 4q_M e^{-\delta_2(t-t_s)} + \frac{\epsilon_1}{\delta_2}[M_1(t - t_s) + M_2]$$
$$\leq 4q_M e^{-\delta_2 c_t} + \frac{\epsilon_1}{\delta_2}[M_1(c_t + T) + M_2]$$
$$\leq \frac{4\epsilon_m}{k_c M_5} + \frac{\epsilon_m}{k_c M_5}$$

$$\leq \frac{5\epsilon_m}{k_c M_5}, \quad \forall t \in [\xi, \xi + T], \xi \geq t_p \tag{41}$$

Accordingly, the lower bound for $| \zeta^T \psi(t) |$, by taking (27), (39) and (41) into account, can be estimated as

$$\begin{aligned} | \zeta^T \psi(t) | &\geq | \zeta^T \psi_d(t) | - M_3 \epsilon_1 (t - t_s) - M_4 \epsilon_1 \\ &\geq -M_5 \mid e_q \mid | \zeta^T \psi_d(t) | - [M_3 (t - t_s) \\ &\quad + M_4] \epsilon_1 - \frac{5\epsilon_m}{k_c} \end{aligned} \tag{42}$$

Let $T_e = T$ and $t_e = t_p$ in (11). The integral there can now be calculated as

$$\begin{aligned} &\frac{1}{T} \int_{\xi}^{\xi+T} | \zeta^T \psi(t) | \, dt \\ &\geq \frac{1}{T} \int_{\xi}^{\xi+T} | \zeta^T \psi_d(t) | \, dt - [\frac{1}{2T} M_3 (t - t_s)^2 |_{\xi}^{\xi+T} \\ &+ M_4] \cdot \epsilon_1 - \frac{5\epsilon_m}{k_c} \\ &\geq \frac{1}{T} \int_{\xi}^{\xi+T} | \zeta^T \psi_d(t) | \, dt - \frac{\epsilon_m}{k_c} - \frac{5\epsilon_m}{k_c} \\ &\geq (1 - \frac{6}{k_c}) \epsilon_m, \quad \forall \xi \geq t_p \end{aligned} \tag{43}$$

Therefore, by selecting any a $k_c > 6$, the PE of the regressor ψ can be concluded. □

4. SIMULATION

To demonstrate the validity of our assertion, a numerical example of the closed-loop system (4) is given in this section. To fulfill A4)-A5), the following reference trajectory is assigned

$$x_1^d(t) = \sin 0.8t + \sin t + \cos t \tag{44}$$

The adopted numerical values for the parameters in (1) are: $k_1 = 10.0$, $k_2 = 10.0$, $c = 2.0$,$\tau_L = 6.5$, $\theta = [0.1, 0.8, 1.0, 2.0]^T$, $\gamma = [0.8, 1.0]^T$. With respect to $x_1^d(t)$ in (44), it is not hard to conclude that A5) is fulfilled after some straightforward calculations. As can be expected, the estimation errors will converge to zero asymptotically as depicted in Fig. 2.

5. CONCLUSION

Sufficient conditions, listed in A1)-A5), for the PE of the regressor in an adaptive input-output linearizing tracking control of a shunt DC motor, are established. Once PE is ensured, as well known, it ensures not only the exponential stability of the parametric equilibrium, but also better transient performances. Therefore, the achievements here are appealing to corresponding control and identification designs.

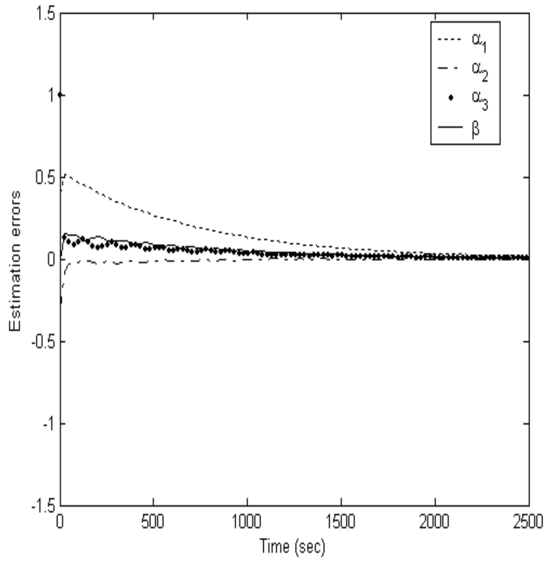

Fig. 2. Estimation errors vs. time

Extension of the results here to more general systems is interesting and under our investigation.

REFERENCES

Callier, F. M. & C. A. Desoer (1991). *Linear System Theory.* New York: Springer-Verlag.

Chiasson, J., & M. Bodson (1991). Nonlinear and adaptive control of a shunt DC motor. In *Proceedings of the IEEE International Conference on Systems Engineering,* Dayton, (pp. 73-76).

Huang, J. T. (2004). On parameter convergence of adaptive fully linearizble systems. In: *Proceedings of the 2004 American Control Conference.* Boston, (pp. 1929-1933).

Narendra, K. S. & A. M. Annaswamy (1989). *Stable Adaptive Systems.* Englewood Cliffs, NJ: Prentice-Hall.

Sastry, S. & A. Isidori (1989). Adaptive control of linearizable systems. *IEEE Transactions on Automatic Control*, **34**, 1123-1131.

Tafur-Sotelo, J. C., & M. Vélez-Reyes (2002). An adaptive feedback linearizing controller for a shunt DC motor. In *Proceedings of the IEEE Workshop on Computers in Power Electronics,* (pp 144-151).