AC INDUCTION MACHINE SPEED OBSERVER WITH ROTOR RESISTANCE ADAPTATION

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Abstract: AC induction motors become very popular for motion control applications due to their simple and reliable construction. Control of drives based on AC induction motors is a quite complex task. Provided the vector control algorithm is used we need to know not only the rotor speed but also the position of the magnetic flux inside the motor during the control process. In most applications the magnetic flux phasor position has to be calculated. But there are also applications in which even speed sensors should be omitted. In such a situation, we have to solve the task of state reconstruction only from voltage and currents measurements. In the current paper, we present a method based on deterministic evaluation of measurement using the state observer based on Lyapunov function. The rotor resistance estimation is also discussed as it is one of main problems in AC induction machine speed estimation.

Keywords: Induction motors, Observers, Control, Parameter estimation

1. INTRODUCTION

Induction motors become more and more popular due to their reliable construction. If we intend to use AC induction motor in a low cost application (e.g. mass produced washing machine) it is necessary to optimize production costs. Many applications require precise speed control. The speed sensor is quite expensive device comparing to other parts of the drive. That is why we are trying to develop a reliable control system which estimates the rotor speed from electrical quantities instead of using a speed sensor.

The idea of using a state observer for evaluation of the signals needed for AC induction motor control is known (Holtz, 2002; Rajashekara *et al.*, 1996). In most cases, such applications exploit the Kalman filter algorithm to estimate the values of states that cannot be measured directly. The Kalman filter provides a unified method for the state observer design and that is why it is relatively easy to use (Shi *et al.*, 2002). Another possibility is to find a state observer fitted exactly for the drive. It is possible to use a simple structure similar to Luenberger's observer with many advantages - low computational demands and results similar to Kalman filter algorithm (Vaclavek *et al.*, 2002).

The algorithms need an accurate model of the controlled drive to compute good estimate of unknown state variables. The parameters, especially resistances, can change dramatically during the drive operation because of working temperature changes. It would be very useful if the observer algorithm could also estimate some of AC induction drive parameters.

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One of the biggest problems in AC induction machine rotor speed estimation is inaccurate knowledge of the rotor resistance. Rotor resistance can vary rapidly due to machine temperature changes. In our concept we are trying to get some additional speed information by rotor slot harmonics analysis, which allows us to make on-line rotor resistance estimation.

1.1 AC Induction Motor Model

There are many models of AC induction motors. We use the so-called t-model structure known also as Kovacs model.

$$\frac{d\mathbf{\Psi}_s}{dt} = \mathbf{u}_s - R_s \mathbf{i}_s \tag{1}$$

$$\frac{d\mathbf{\Psi}_r}{dt} = j\, z_p \omega \mathbf{\Psi}_r - R_r \mathbf{i}_r \tag{2}$$

$$\frac{d\omega}{dt} = \frac{1}{J}\left(T - T_{load}\right) \tag{3}$$

$$\mathbf{\Psi}_s = L_s \mathbf{i}_s + L_m \mathbf{i}_r \tag{4}$$

$$\mathbf{\Psi}_r = L_m \mathbf{i}_s + L_r \mathbf{i}_r \tag{5}$$

$$T = 1.5 z_p \Im\left(\overline{\mathbf{\Psi}}_s \mathbf{i}_s\right) \tag{6}$$

where $\mathbf{\Psi}_s = \Psi_{s\alpha} + j\Psi_{s\beta}$, $\mathbf{\Psi}_r = \Psi_{r\alpha} + j\Psi_{r\beta}$ are stator and rotor magnetic flux phasors in $\alpha\beta$ coordinates, $\mathbf{u}_s = u_{s\alpha} + j\, u_{s\beta}$ is stator voltage phasor, $\mathbf{i}_s = i_{s\alpha} + j\, i_{s\beta}$, $\mathbf{i}_r = i_{r\alpha} + j\, i_{r\beta}$ are stator and rotor current phasors, ω is rotor angular speed, T, T_{load} are driving and load torques, R_s, R_r are stator and rotor resistances, z_p is the number of pole-pairs, J is the rotor inertia and L_s, L_r, L_m are inductances.

As the load torque and the rotor inertia are not usually known it is not possible to use equations (3) and (6). The remaining equations can be after some computations transferred into more suitable form

$$\frac{d\mathbf{i}'_s}{dt} = \mathbf{u}_s - \xi_1 \mathbf{i}'_s + \mathbf{\Psi}'_r\left(\xi_2 - j\,\omega z_p\right) \tag{7}$$

$$\frac{d\mathbf{\Psi}'_r}{dt} = \xi_3 \mathbf{i}'_s - \mathbf{\Psi}'_r\left(\xi_2 - j\,\omega z_p\right) \tag{8}$$

where

$$\mathbf{i}'_s = \mathbf{i}_s \frac{L_s L_r - L_m^2}{L_r} \tag{9}$$

$$\mathbf{\Psi}'_r = \mathbf{\Psi}_r \frac{L_m}{L_r} \tag{10}$$

$$\xi_1 = \frac{R_s L_r^2 + R_r L_m^2}{L_s L_r^2 - L_r L_m^2} \tag{11}$$

$$\xi_2 = \frac{R_r}{L_r} \tag{12}$$

$$\xi_3 = R_r \frac{L_m^2}{L_s L_r^2 - L_r L_m^2} \tag{13}$$

Equations (7) and (8) represent some kind of model normalized according to the motors inductances. The model has only three independent parameters ξ_1, ξ_2, ξ_3. It is clear, that also inductances have to be known to use (9)-(13), but it is not usually problem. Inductances can be measured off-line (Seok and Sul, 2001) and we can suppose that they do not change significantly during normal drive operation.

2. STATE OBSERVER

The used control structure is shown on Fig. 1. The state observer can use measurement of stator voltage and modified current $\mathbf{u}_s, \mathbf{i}'_s$. It should estimate modified rotor magnetic flux $\mathbf{\Psi}'_r$, rotor angular speed ω, modified stator current $\mathbf{i}'_s$ (filtering purposes), and, if possible, also parameters ξ_1, ξ_2, ξ_3.

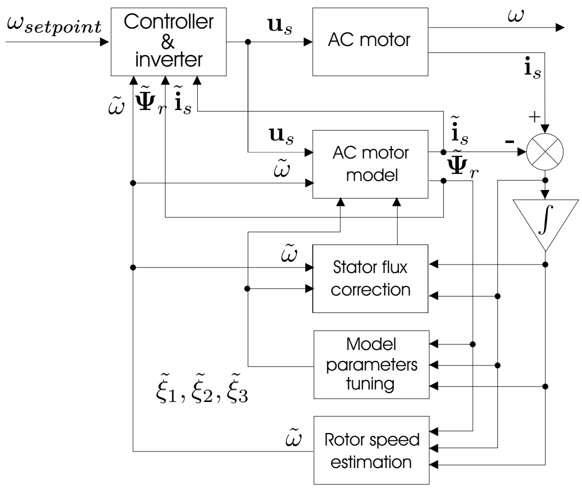

Fig. 1. Speed and flux observer structure

The observer algorithm design is based on Lyapunov stability analysis (Vaclavek and Blaha, 2003). The complete algorithm can be summarized into following equations:

$$\Delta\mathbf{i}'_s = \tilde{\mathbf{i}}'_s - \mathbf{i}'_s \tag{14}$$

$$\frac{dx'}{dt} = \Delta\mathbf{i}'_s \tag{15}$$

$$y = \Delta\mathbf{i}'_s + k_1 x' \tag{16}$$

$$\delta = \left(\tilde{\xi}_1 + \tilde{\xi}_2 - k_1 - k_2 - j\, z_p \tilde{\omega}\right)\Delta\mathbf{i}'_s - \tag{17}$$
$$- \left(1 + k_1 k_2\right) x'$$

$$\frac{d\tilde{\mathbf{i}}'_s}{dt} = \mathbf{u}_s - \tilde{\xi}_1 \tilde{\mathbf{i}}'_s + \tilde{\mathbf{\Psi}}'_r\left(\tilde{\xi}_2 - j\,\tilde{\omega} z_p\right) + \delta \tag{18}$$

$$\frac{d\mathbf{\Psi}'_r}{dt} = \tilde{\xi}_3 \tilde{\mathbf{i}}'_s - \tilde{\mathbf{\Psi}}'_r\left(\tilde{\xi}_2 - j\,\tilde{\omega} z_p\right) \tag{19}$$

$$\frac{d\tilde{\omega}}{dt} = -k_\omega \Im\left(\overline{\left(y + \Delta\mathbf{i}'_s\right)}\left(\tilde{\mathbf{\Psi}}'_r + \Delta\mathbf{i}'_s\right)\right) \tag{20}$$

$$\frac{d\tilde{\xi}_1}{dt} = k_{\xi_1} \Re\left(y \overline{\tilde{\mathbf{i}}'_s}\right) \tag{21}$$

$$\frac{d\tilde{\xi}_2}{dt} = -k_{\xi_2} \Re\left(\overline{\left(y + \Delta\mathbf{i}'_s\right)}\left(\tilde{\mathbf{\Psi}}'_r + \Delta\mathbf{i}'_s\right)\right) \tag{22}$$

$$\frac{d\tilde{\xi}_3}{dt} = k_{\xi_3}\Re\left(\Delta\mathbf{i}'_s\overline{\mathbf{i}'_s}\right) \tag{23}$$

with respect to (9)-(13). Behavior of the algorithm can be tuned by set of selectable positive parameters $\{k_1, k_2, k_\omega, k_{\xi_1}, k_{\xi_2}, k_{\xi_3}\}$.

2.1 Parameters Estimation

The observer algorithm allows not only computation of the rotor magnetic flux phasor position and rotor angular speed but also parameters estimation using equations (21)-(23). If the motor inductances are known it should be possible to compute rotor resistance using (12) or (13) and stator resistance from (11). Simulation experiments proved that it is really possible to estimate stator resistance R_s, but not the rotor resistance R_r. The reason is very simple and can be shown using the original AC induction motor model equation (2). The magnitude of the rotor magnetic flux phasor is held constant during normal drive operation. In consequence the rotor current phasor $\mathbf{i}_r$ is orthogonal to the rotor magnetic flux phasor $\mathbf{\Psi}_r$. The equation (2) can be then rewritten into form

$$\frac{d\mathbf{\Psi}_r}{dt} = j\mathbf{\Psi}_r\left(z_p\omega + R_r\frac{|\mathbf{i}_r|}{|\mathbf{\Psi}_r|}\right) \tag{24}$$

It is clear, that if any difference between measured and estimated stator current is detected it is not possible to determine whether the difference is caused by a change in ω or in R_r. Only value of the complete term $z_p\omega + R_r\frac{|\mathbf{i}_r|}{|\mathbf{\Psi}_r|}$ can be estimated. We have to know either rotor angular speed or rotor resistance and then it is possible to compute the other variable. Although there is a possibility off addition of some variations to the rotor magnetic flux (e.g. sinusoidal or square wave changes to rotor flux reference value) in such way that phasors $\mathbf{i}_r$ and $\mathbf{\Psi}_r$ will not be orthogonal, this method is practically unusable in many cases due to limitations in maximum available stator voltage. That is why it is necessary to find other methods for obtaining the value of the rotor resistance. It leads to practical result that observer parameters k_{ξ_2}, k_{ξ_3} should be set $k_{\xi_2} = k_{\xi_3} = 0$.

In the steady state when the rotor speed and stator frequency is constant only rotor speed will be affected by the error in the rotor resistance. This is very important as we can suppose that observer provides correct magnetic flux phasor position even in the case of significant error in the rotor resistance. Rotor current can be also estimated using (5). That is why we can compute rotor resistance easily from (24) in steady state if actual rotor speed is known. The rotor resistance is then

$$R_r = (\omega_s - z_p\omega)\frac{|\mathbf{\Psi}_r|}{|\mathbf{i}_r|} \tag{25}$$

where ω_s is synchronous magnetic flux rotational speed, which is known.

Rotor speed ω in (25) can be determined by slot harmonics analysis. This method can provide actual rotor speed estimate independent on machine electrical parameters. The problem of this method is that it can be compared to using a very low resolution encoder (about 15-30 pulses per revolution). It is clear that in such case we can obtain good speed estimate in steady state with poor dynamic performance. That is why we use speed estimation based on stator voltage and current measurement as was described before and this auxiliary rotor speed information obtained using rotor slot harmonics analysis can be used for rotor resistance adaptation.

2.2 Rotor Slot Harmonics Analysis

Three phase induction machine is normally supplied by a system of symmetrical three phase harmonic voltages. The magnetic flux will contain not only fundamental but also higher harmonics. Some of them are caused by nonlinearities in magnetizing characteristics and by asymmetry in the machine parameters or geometry. But there are also harmonics produced by the reluctance variation caused by stator slots and moving rotor slots. Frequency of these harmonics is determined be the rotor rotational speed.

It can be shown (Nandi *et al.*, 2001; Vas, 1993), that the fundamental rotor slot harmonic can be expressed as

$$f_{sh} = \frac{N_r}{z_p}f_r - f_s \tag{26}$$

where f_{sh} is fundamental rotor slot harmonic frequency, N_r is number of rotor slots, z_p is number of pole pairs, $f_r = \frac{\omega}{2\pi}$ is rotor rotational frequency and f_s is synchronous stator frequency.

The flux variations caused by rotor slots generate rotor slot harmonics in the stator current. Unfortunately it can be quite difficult to detect these harmonics in the stator current due to their very low amplitude comparing to the fundamental

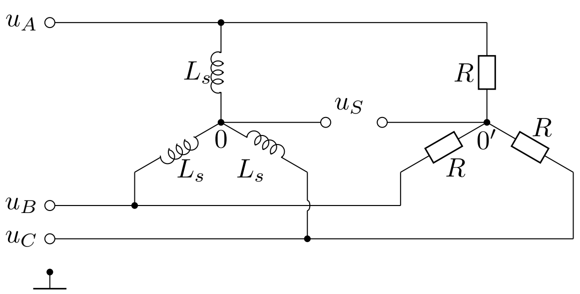

Fig. 2. Signal measurement for rotor slot harmonics analysis

stator current harmonic. That is why we use the other possibility. It is possible monitor voltage u_s between the neutral point 0 of the stator windings and the neutral point $0'$ constructed by the three phase resistive network as shown on Fig. 2. It is possible to show, that the amplitude of the harmonic at stator frequency f_s and also its multiplies Nf_s is significantly reduced in the voltage u_s (it is not zero because of machine parameters and geometry asymmetry).

Actual rotor speed can be computed by following algorithm:

(1) Signal u_s is filtered by low pass filter to remove high frequency high-voltage component introduced by PWM.
(2) Signal spectra is computed using FFT algorithm.
(3) Stator frequency f_s is determined.
(4) Frequency range $\left\langle \frac{N_r}{z_p}\frac{\omega_{min}}{2\pi} - f_s; \frac{N_r}{z_p}\frac{\omega_{max}}{2\pi} - f_s \right\rangle$ is searched for the harmonic f_{sh} with the highest amplitude. The harmonics with frequency Nf_s where $N = 1, 2, 3, \ldots$ are excluded from the search. Drive is supposed to work at rotor speed in the interval $\langle \omega_{min}; \omega_{max} \rangle$. The speed bounds are determined from stator frequency and maximal possible slip value.
(5) Rotor speed is computed using

$$\omega = (f_{sh} + f_s)\frac{2\pi z_p}{N_r} \tag{27}$$

Some of the rotor slot harmonics can be attenuated because of the machine geometrical construction. If it is possible, the machine construction should be selected to provide easy rotor slot harmonics detection at reasonable frequencies (Nandi *et al.*, 2003). In many cases we do not know the rotor slots number N_r. Then we have to make experiment when the rotor speed is being measured by a speed sensor and the rotor slots number is computed from (26) as

$$N_r = \frac{f_{sh} + f_s}{f_r} z_p \tag{28}$$

3. ALGORITHM IMPLEMENTATION

The observer algorithm was tested on the real AC induction machine using our experimental system. A small 250 W induction machine with 2 pole pairs was used with approximate parameters values $R_s = 32\Omega$, $R_r = 22\Omega$, $L_s = 0.85H$, $L_r = 0.85H$, $L_m = 0.7H$. The load torque was produced by DC permanent magnet motor mechanically connected to the induction machine. The observer was implemented on the Freescale 56F805 Hybrid Controller evaluation board together with the complete control system based on the rotor flux oriented vector control algorithm (Lepka, 2003). The Freescale 56F805 hybrid controller chip is equipped with peripherals needed to control electrical drives and also the evaluation board is suitable for motor control application testing. The Freescale 3-Phase AC high-voltage brushless DC power stage has been used to supply control signals to the machine.

At this time the electronics used in the experiment does not provide functionality needed for capturing of the signal needed for rotor slot harmonics analysis. That is why the data for this part of the experiment has been captured using HP 89410A vector signal analyzer and then processed in Matlab environment. We plan to make slight modifications in the electronics to be able to process rotor slot harmonics analysis directly on Freescale Hybrid Controller.

4. RESULTS

4.1 Rotor speed estimation using stator quantities measurement

In the first experiment the ability of speed changes tracking was studied. The motor was running without any load. Observer parameters were tuned experimentally to values $k_1 = 2$, $k_2 = 300$, $k_\omega = 8000$, $k_{\xi_1} = 2000$, $k_{\xi_2} = 0$, $k_{\xi_3} = 0$. The algorithm was able to track the rotor speed changes without significant error even in case of rapid speed change. The comparison of real and estimated rotor speed is shown on Fig. 3.

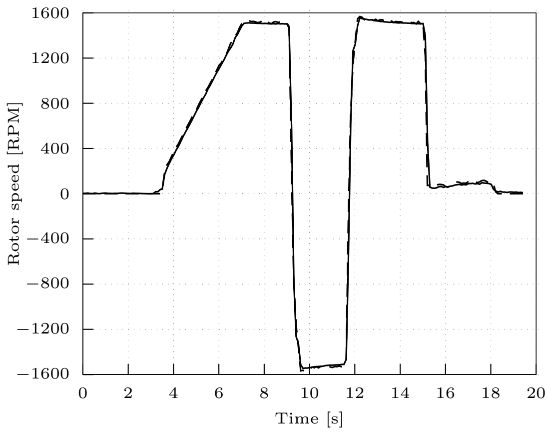

Fig. 3. Comparison of real rotor speed (dashed) and estimated value (solid)

The relative speed estimation error $\delta_\omega = \frac{\omega - \tilde{\omega}}{\omega}100$ is shown on Fig. 4. Complete speed range has been evaluated for machine without external load torque (only friction torque is present), while experiment with the load torque fixed at $0.5Nm$ has been made only for limited speed range. At this time we are not able to control constant load torque at rotor speed under 500 RPM and we cannot run the induction machine at speed over 1600 RPM producing higher torque because of power limitation. We can see that the algorithm can achieve speed estimation error lower than 2% at speed higher than 1000 RPM. This precision is satisfactory for common AC induction machine applications. Both curves o the Fig. 4 have the shape of a hyperbolic function. The main reason of the speed estimation steady state error is the difference between the real rotor resistance and its value used in the observer calculation. It can be shown that if we keep the load torque constant, the rotor current $\mathbf{i}_r$ is also constant. From (24) it is clear, that if the speed estimation error is caused by inaccurate rotor resistance knowledge then the absolute value of the speed estimate error is constant and the relative error will be hyperbolic function of the rotor speed. The relative error is positive for the machine running without load as the machine temperature is relatively low and the real rotor resistance is lower than the fixed rotor resistance value used in the observer calculation. On the other hand the relative speed error for the machine running under high load is negative as the machine temperature become high resulting in the higher real rotor resistance. Based on the Fig. 4 we can make conclusion, that the main reason for the speed estimate error in the proposed algorithm is the rotor resistance error.

4.2 Rotor slots harmonics analysis

In the first step we had to identify number of rotor slots as it was not known for the machine used in the experiment. In fact the number N_r is not necessarily equal to the rotor slots number as it can be influenced by the rotor eccentricity (Nandi *et al.*, 2001). We have measured the rotor speed $\omega = 16.67 rads^{-1}$ while the stator frequency was $f_s = 47Hz$. The rotor slot harmonic is present at frequency $f_{sh} = 254Hz$ as can be seen from Fig. 5. Then we can compute N_r from (27) as

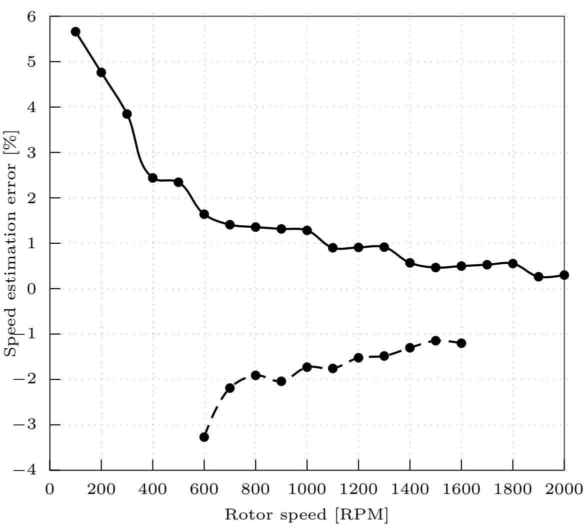

Fig. 4. Estimated speed relative error for no load (solid) and load torque 0.5 Nm (dashed)

$$N_r = (f_{sh} + f_s)\frac{2\pi z_p}{\omega} \tag{29}$$

and we will get $N_r = 36$.

The process of searching for the rotor slot harmonics can be described using figure 5. There are several significant spectral lines. The first one at point A is at the frequency of the stator voltage fundamental harmonic. In the ideal case there will be no harmonic at this frequency and this harmonic component occurs only due to machine stator windings parameters asymmetry. The spectral lines at points C and E stand for the third and ninth stator voltage harmonics and are excluded from the search. As we know the breakdown slip of our machine to be $s_b = 50\%$ we can compute the minimal possible rotor speed

$$\begin{aligned}\omega_{min} &= f_s(1-s)\frac{2/pi}{z_p} = \\ &= 47(1-0.5)\frac{2/pi}{2} = 73.8 rad\, s^{-1}\end{aligned} \tag{30}$$

and minimal possible rotor slot harmonics frequency

$$\begin{aligned}f_{sh_{min}} &= \frac{N_r}{z_p}\frac{\omega_{min}}{2\pi} - f_s = \frac{36}{2}\frac{73.8}{2\pi} - 47 = \\ &= 164.4Hz\end{aligned} \tag{31}$$

Because of this minimal possible rotor slot harmonic frequency also the spectral line B is excluded. If we search for the harmonic component with the highest amplitude omitting excluded spectral lines we find the spectral line D at frequency $f_{sh} = 254Hz$ and we can compute rotor speed using (27)

$$\begin{aligned}\omega &= (f_{sh} + f_s)\frac{2\pi z_p}{N_r} = (254 + 47)\frac{4\pi}{36} = \\ &= 105 rad\, s^{-1} = 1003 RPM\end{aligned} \tag{32}$$

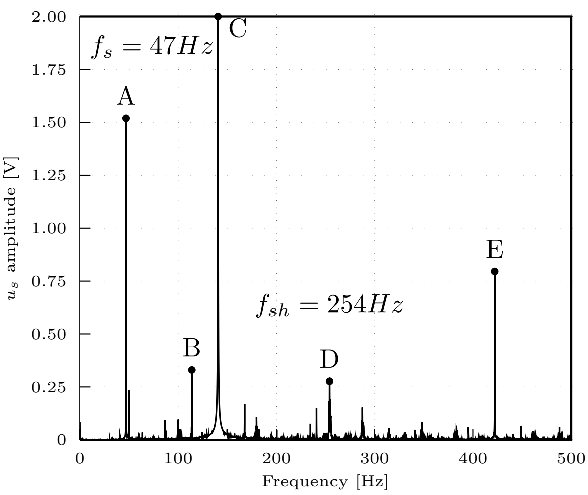

Fig. 5. Neutral point voltage spectra for rotor speed 1000 RPM

Now we are able to estimate rotor speed only from knowledge of stator frequency and rotor slot harmonic frequency using (27). The comparison

Table 1. Comparison of real and estimated speed

f_s [Hz]	f_{sh} [Hz]	real ω [RPM]	estimated ω [RPM]	estimate error [%]
31	149	600	600	0
31	209	800	800	0
47	254	1000	1003	0.3
47	344	1300	1304	0.3
62	326	1300	1294	0.5
62	417	1600	1597	0.2

of real and estimated rotor speed can be seen in Tab. 1. The error in the speed estimate is very low and is independent on the machine load and temperature conditions and that is why it is possible to use this auxiliary speed information for rotor resistance adaptation as was mentioned earlier.

The possibility of rotor slot harmonics detection in the stator current was also studied. The rotor slot harmonics amplitude is very low comparing to the fundamental harmonic of the stator current. It is very difficult to filter out the stator current fundamental harmonic effectively as its frequency is too close to frequency of rotor slot harmonic. That is why it is necessary to process the signal including this high current component. It leads to problem that A/D converter with high dynamic range and high resolution would be necessary. As the sensorless AC induction machine control should be aimed especially to low cost applications we should suppose, that our hardware will not be equipped with such A/D converters. This leads to conclusion, that the processing of the windings neutral point voltage can be more suitable in many applications.

5. CONCLUSIONS AND FUTURE WORKS

The presented speed observer algorithm has been proved partially in testing on real AC induction machine and in simulation using Matlab environment. The algorithm has been implemented on Freescale 56F805 Hybrid Controller chip that is suitable for motor control applications. As the algorithm does not depend on the control scheme it is possible to use it with classical vector control scheme (Lepka, 2003) as well as with other AC induction machine control algorithms like DTC (Vas, 1998) or PDSFC (Blaha and Vaclavek, 2003). It was shown, that the correct knowledge of the rotor resistance value is essential for accurate rotor speed estimation. It has been also shown, that it is possible to estimate rotor speed using the rotor slot harmonics analysis. The accuracy of the estimate is sufficient to be used for the rotor resistance adaptation.

Future research will concentrate especially on practical implementation of rotor slot harmonics analysis on Freescale Hybrid Controller. As consequence we will be able to improve our speed and flux estimation algorithm by adding on-line rotor resistance adaptation ability.

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