# REFERENCE MODEL AND LYAPUNOV ANALYSIS APPROACH : APPLICATION TO AN INDUCTION MOTOR

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Abstract: This paper deals with a robust $H_2/H_\infty$ synthesis and an analysis technique applied to an induction motor. In order to robustify a linearizing decoupled feedback against parametric variations, $H_2/H_\infty$ controllers with a reference model are added. The main interests are performances and real decoupling between the rotor flux and the speed in spite of parameter variations. A stability analysis method using parameter-dependent Lyapunov function is also proposed to verify the global stability. Real time implementations are carried out.


Keywords: Robust control, Induction motor, $H_2/H_\infty$, Parameter dependent Lyapunov function.

## 1. INTRODUCTION

The industrial development of power electronics and real time controller board allows everyday to experiment new algorithms of control more sophisticated. These algorithms counteract some nonlinearities and minimize the number of sensors while increasing performances. Techniques like field oriented control developed by Blaschke (Blaschke, 1972) can be improved by achieving exact input-output decoupling and linearization via a nonlinear state feedback as shown in (Marino *et al.*, 1993).

The behavior of these nonlinear feedbacks of linearization are deteriorated by the presence of parametric variations. Nonlinearities and couplings are reintroduced by these variations. A model of the effects of the variations of parameter is made up, in order to carry out a linear robust synthesis of controller. This one would maintain faculties of decoupling of the initial nonlinear feedback.

A model of reference (Morari and Zafiriou, 1989) is added to the loop of linearization in order to improve the synthesis of the robust controller.

Consequently, we propose to use a mixed optimization of $H_2/H_\infty$ by using linear matrix inequalities which tools as those introduced by (Chilali and Gahinet, 1996; Gahinet *et al.*, 1995). This approach synthesize a robust controller who allows good performance. An analysis tool is performed to analyze the robustness of the synthesized controller by adding the neglected nonlinearities and the polytopic description.

The proposed approach consider an optimization of the polytopic parameter variation of a linear system in order to find a Lyapunov function which include a maximized $H_2$ norm bounded nonlinearity. The originality of the method is in the combination of reference model and $H_2/H_\infty$ design coupled with an analysis tool which takes into account nonlinearities and parameter variation neglected in the design step.

## 2. PROBLEM FORMULATION

### *2.1 Preliminaries*

In this paper, multi-inputs and multi-outputs nonlinear systems with parametric uncertainties are considered, they are defined by the following equations :

$$\dot{\xi} = A\left(\tilde{\theta}\right)\xi + B\left(\tilde{\theta}\right)v + Hd(\tilde{\theta}, \xi(t)) \qquad (1)$$

$$y = C_n.\xi \qquad (2)$$

where

$$A\left(\tilde{\theta}\right) = \left(A_n + \sum_{p=1}^{q} A_p.\tilde{\theta}_p\right) \qquad (3)$$

$$B\left(\tilde{\theta}\right) = \left(B_n + \sum_{p=1}^{q} B_p.\tilde{\theta}_p\right) \qquad (4)$$

$$Hd(\tilde{\theta}, \xi(t)) = \begin{bmatrix} R_1\left(\xi, \tilde{\theta}\right) \\ \vdots \\ R_n\left(\xi, \tilde{\theta}\right) \end{bmatrix} \qquad (5)$$

index $n$ corresponds to the nominal system and $p$ to the perturbed system. Where $\xi \in \mathbb{R}^n$ denotes vector of state variables, $v \in \mathbb{R}^m$ denotes the input vectors, $\theta \in \mathbb{R}^q$ denotes the vector of the uncertain parameters which are assumed to be time-unvarying and $y \in \mathbb{R}^m$ denotes the output vector.
The different matrices $A_i$ and $B_i$ are constant.

## 3. ROBUST CONTROL SYNTHESIS

In order to synthesize the output feedback controller, the variations are not considered. This condition is taken into account to have an optimum controller which can be synthesized with L.M.I. tools. Differently, the solution must be found with the bilinear tools (B.M.I., iterative L.M.I. ), but it is not sure that the solution is the optimal one.

So, the feedback controller is designed on the nominal linear part of the model (1).

### *3.1 Reference model*

In order to counteract the deviations of the perturbed linearized model, a tracking model based on a reference model is used, see (Morari and Zafiriou, 1989). The reference model control objective is to design a compensator so that the input-output operator of the closed-loop system matches that of a specified reference model.

For this study, the reference model is taken equal to the nominal linearized model $\left(i.e.\ \tilde{\theta} = 0\right)$ :

In fact, the reference model acts to attenuate the contribution of the nonlinear part $Hd(\tilde{\theta}, \xi(t))$ of the model (1) in presence of parameter deviation.

### *3.2 Performance specifications*

A mixed $H_2/H_\infty$ linear synthesis is designed in order to take into account some performances. The control problem can be formulated as an $H_\infty$ optimization problem on the nominal case in (Doyle *et al.*, 1989; Safonov *et al.*, 1993) and for a treatment of $H_2/H_\infty$ output-feedback problem with an L.M.I. formulation in (Chilali and Gahinet, 1996). The $H_\infty$ specifications of closed loop system can be integrated by shaping the sensitivity function S and the complementary sensitivity function T (or KS).

The $H_2$ optimization will concerns the error outputs between the reference model and the linearized system. So, a shaping filter $W_e(s)$ can be used to integrate these specifications.

## 4. ROBUST STABILITY ANALYSIS

In order to prove the global stability for the closed loop of the designed controller, a stability analysis is presented integrating the nonlinear perturbation of the equation (1) and the parameter variation which have been put aside for the synthesis.

Integrating the reference model and the controller, let the final closed loop system be described as:

$$\dot{X} = A_{f_{cl}}\left(\tilde{\theta}\right)X + H_{f_{cl}}.d(\tilde{\theta}, X(t)) \qquad (6)$$

Considering this uncertain model, a sufficient LMI condition for the system to prove its stability is proposed. This condition involves Lyapunov functions that depend on the polytopic structure of the uncertainty in order to reduce the conservatism of the method. This work refers to the results of (Geromel *et al.*, 1998). The affine structure of the previous model which depends of the parameter variation (equation 6:$A_g\left(\tilde{\theta}\right)$ and $B_g\left(\tilde{\theta}\right)$) can be used to perform a polytopic description of the structure.

### 4.1 Problem statement

Considering a polytope $\mathcal{A}$ (which corresponds to $A_{f_{cl}}\left(\tilde{\theta}\right)$) precisely defined by its known vertices, it is aimed to find the greatest nonlinear uncertainty $d(\tilde{\theta}, X(t))$ structured by $H_f$, in the sense of a norm, so that stability is ensured in the presence of both uncertainties. More precisely, a bound $\rho$ is derived so that if:

$$\frac{||d(\tilde{\theta}, X(t))||_2}{||X(t)||_2} < \rho \quad , \forall X(t) \in \mathbb{R}^{n*} \qquad (7)$$

The system (6) is stable whatever the value of $\alpha \in \Delta$.

This problem is closed to the derivation of the so-called "robust stability bounds".

Many investigations have been led on this topic since Patel and Toda have proposed a first result. A robust stability bound is in fact the inverse of an upper bound of $\mu$.
Most of those works consider only one uncertainty case: unstructured one or parametric structured one. An original point in this work is the presence of both the polytopic uncertainty and the nonlinear one.

### 4.2 LMI condition

In this part, a LMI condition for robust stability of model (6) is proposed.

*Theorem 1.* : Let a system be described by (6) where $A$ is some matrix belonging to a polytope $\mathcal{A}$, where $H_f \in \mathbb{R}^{n\times q}$ is a known matrix and where $d(.)$ is a norm bounded function from $\mathbb{R} \times \mathbb{R}^n$ to $\mathbb{R}^q$. This system is robustly stable **if**

$$\frac{||d(\tilde{\theta}, X(t))||_2}{||X(t)||_2} < \rho = \nu^{\frac{1}{2}} \quad \forall X(t) \in \mathbb{R}^{n*} \qquad (8)$$

**and if** there exist $F \in \mathbb{R}^{n\times n}$, $G \in \mathbb{R}^{n\times n}$ and $N$ symmetric matrices $P_j = P_j^* \in \mathbb{R}^{n\times n}$, $j = 1, ..., N$ such that, $\forall j \in \{1, ..., N\}$, the following LMIs hold:

$$\begin{cases} M_j = \\ \begin{bmatrix} A_j'F' + FA_j + \nu I_n & P_j - F + A_j'G & P_jH_f \\ P_j - F' + G'A_j & -(G + G') & \mathbb{O} \\ H_f'P_j & \mathbb{O} & -I_q \end{bmatrix} < 0 \\ \\ P_j > 0 \end{cases} \qquad (9)$$

**Proof:** The reasoning uses some arguments borrowed from (Peaucelle *et al.*, 2000) but because of the presence of the additive terms relevant to the nonlinearity the proof has to be detailed.
Assume that there exist some matrices $F$, $G$, $P_j$, $j = 1, ..., N$ and a scalar number $\nu$ solution of (9). Define the following parameter-dependent matrices $M(\alpha)$ and $P(\alpha)$ ($\forall\alpha \in \Delta$)are obviously convex combinations of respectively $M_j$ and $P_j$, $j = 1, ..., N$. Hence, for any instance $\alpha \in \Delta$, there exists a triple of $\mathbb{R}^{n\times n}$-matrices, $\{F, G, P(\alpha)\}$. Thus, inequality (9) holds for the vertices of polytope $\mathcal{A}$ and also for each point inside $\mathcal{A}$. The notation ".$(\alpha)$" and "$\forall\alpha \in \Delta$" are omitted to make the proof shorter. If $M$ is negative definite whatever $\alpha \in \Delta$ is, then the following inequality stands $\forall X(t) \in \mathbb{R}^{n*}$:

$$\left[\, X'(t) \;\; X'(t)A' \;\; d'(\tilde{\theta}, X(t)) \,\right] M \begin{bmatrix} X(t) \\ AX(t) \\ d(\tilde{\theta}, X(t)) \end{bmatrix} < 0 \qquad (10)$$

$$\Leftrightarrow X'(t)(A'P + PA)X(t) + \nu X'(t)X(t) + d'(\tilde{\theta}, X(t))H_f'PX(t) + X'(t)PH_f.d(\tilde{\theta}, X(t)) - d'(\tilde{\theta}, X(t))d(\tilde{\theta}, X(t)) < 0 \, \forall X(t) \in \mathbb{R}^{n*} \qquad (11)$$

If inequality (8) holds, then it comes:

$$d'(\tilde{\theta}, X(t))d(\tilde{\theta}, X(t)) < \nu X'(t)X(t) \qquad (12)$$

Taking (11) and (12) into account yields:

$$X'(t)(A'P + PA)X(t) + d'(\tilde{\theta}, X(t))H_f'.PX(t) + X'(t)PH_f.d(\tilde{\theta}, X(t)) < 0 \quad \forall X(t) \in \mathbb{R}^{n*} \qquad (13)$$

Inequality (13) can be written $\dot{V}(X(t)) < 0$ with $V(X(t))$ being a parameter-dependent Lyapunov. Hence, system (6) is stable. □

Condition given in theorem 1 is tractable from a computational point of view. Maximizing $\nu$ while LMI system (9) holds for some $F$, $G$ and $P_j$ can be achieved owing to the function `fminu` of the LMI Toolbox of Matlab. If the LMIs are found feasible, the optimal value of $\nu$ is such that for any nonlinear uncertainty $d(\tilde{\theta}, X(t))$ whose spectral norm is less than $\rho = \nu^{\frac{1}{2}}$, system (6) is stable whatever $\alpha \in \Delta$ is.

## 5. SYNTHESIS AND ANALYSIS APPROACH APPLIED TO A NONLINEAR EXPERIMENTAL SET-UP: AN INDUCTION MOTOR

The proposed methodology has been applied for a 1.1kW squirrel induction motor, whose data are listed in the appendix. Considering the previously described model such that $\theta_1 = \bar{\theta}_1\left(1 + \tilde{\theta}_1\right)$

and $\theta_2 = \bar{\theta}_2\left(1+\tilde{\theta}_2\right)$ (where $\bar{\theta}_i$ corresponds to the medium value of $\theta_i$)), the functioning changes in induction motor resistances $\tilde{\theta}_1$ range in $[-0.5, +0.5]$ and inductances change $\tilde{\theta}_2$ range in $[-0.2, 0.2]$.

### *5.1 Modelization approach*

The induction motor model is nonlinear, therefore a linearizing input-output control feedback is used like thus described in section 2.1 (see (Marino *et al.*, 1993),(Cauet *et al.*, 2002)) concerning induction motor linearization). When a parameter deviation occurs, the closed-loop behavior can be described by a two parts model, a linear part which is composed of an affine dependency of the parameter variation and an additive nonlinear part. Here is the linearized system, see (Cauet *et al.*, 2002) for detailed calculus :

$$\dot{\xi} = \left(A_n + \sum_{p=1}^{2} A_p.\tilde{\theta}_p\right)\xi + \left(B_n + B_2.\tilde{\theta}_2\right)v + R\left(\xi, \tilde{\theta}\right) \tag{14}$$

Where:

$$\xi = \left[x_1, \dot{x}_1, h_2 = (x_2^2 + x_3^2), L_{f_n} h_2\right]' \tag{15}$$

And

$$A_n = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -b_{10} & b_{11} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -b_{20} & b_{21} \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -a_4 - a_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & A_{1_{43}} & -a_4 - 3a_2 \end{bmatrix}$$

$$A_{1_{43}} = 2a_2\left(-a_4 - a_2 + a_2 a_3 M\right)$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ b_{10} & -b_{11} & 0 & 0 \\ 0 & 0 & 2a_2 & 0 \\ 0 & 0 & A_{2_{43}} & -2a_2 - b_{21} \end{bmatrix}$$

$$A_{2_{43}} = b_{20} - 2a_2^2 a_3 M - 4a_2^2$$

$$B_n = \begin{bmatrix} 0 & 0 \\ b_{11} & 0 \\ 0 & 0 \\ 0 & b_{21} \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ -b_{11} & 0 \\ 0 & 0 \\ 0 & -b_{21} \end{bmatrix}$$

$$\text{and where } R\left(\xi, \tilde{\theta}\right) = \begin{bmatrix} 0 \\ R_1\left(\xi, \tilde{\theta}\right) \\ 0 \\ R_2(x) \end{bmatrix} \tag{16}$$

(17)

With

$$R_1\left(\xi, \tilde{\theta}\right) = -\tilde{\theta}_2 \frac{a_1 p}{M}\left(\frac{x_1 x_4}{2a_2} + x_1 x_3\right)$$

$$R_2\left(\xi, \tilde{\theta}\right) = \tilde{\theta}_2\left(\frac{2pa_2 M}{a_1} x_1 x_2\right) + \left(\tilde{\theta}_1 + \tilde{\theta}_2\right) . \left(\frac{2\alpha_n^2 M^2}{x_3}\left[\left(\frac{x_4 + 2a_2 x_3}{2a_2 M}\right)^2 + \frac{x_2^2}{a_1^2}\right]\right)$$

Subscript $n$ refers to the nominal parameter values. Where the state vector of the original nonlinear model, $x$, is given by: $x = [x_1, x_2, x_3, x_4, x_5]'$. $x_1$ is the induction machine speed, $(x_2, x_3)$ denote the $(\alpha, \beta)$ rotor flux, $(x_4, x_5)$ denote the $(\alpha, \beta)$ stator current. The input vector $u = [u_1, u_2]'$ represents the $(\alpha, \beta)$ components of the stator voltage inputs.

The other parameters are listed in the appendix, $a_1 = \frac{pM}{JL_r}$, $a_2 = \frac{R_r}{L_r}$ , $a_3 = \frac{M}{\sigma L_s L_r}$, $a_4 = \frac{M^2 Rr}{\sigma L_s L_r^2} + \frac{R_s}{\sigma L_s}$ and $a_5 = \frac{1}{\sigma L_s}$ be a reparametrization of the induction motor model (in function of the real parameters described in the appendix). The closed loop poles of the nominal linearized model are represented by $b_{10} = 2790$, $b_{11} = -201$, $b_{20} = 96100$, $b_{21} = -620$.

### *5.2 Design of weighting functions*

In this study, the controlled output are $\xi_1(1) = x_1$ (the rotor speed) and $\xi_2(1) = x_2^2 + x_3^2$ (the square of the flux norm).

The design of the weighting functions $W_S$ ,$W_{KS}$ and $W_e$ associated to the requirements is performed as follows :

$$W_S = diag(w_{S1}, w_{S2}) \text{ with } w_{Si}$$
$$\text{of the form } \frac{\frac{s}{M_s} + w_{bsi}}{s + \varepsilon_s . w_{bsi}}$$
$$W_{KS} = diag(w_{KS1}, w_{KS2}) \text{ with } w_{KSi}$$
$$\text{of the form } \frac{s + \frac{w_{bksi}}{M_u}}{\varepsilon_{ks}.s + w_{bksi}}$$
$$W_e = diag(w_{e1}, w_{e2}) \text{ with } w_{ei} = 1$$

The weighting filters $w_{S1}$ $(w_{S2})$ and $w_{KS1}$ $(w_{KS2})$ are respectively the weighting function of S and KS of the dynamics of $x_1$ $\left(\left(x_2^2 + x_3^2\right)\right)$ where $x_1$ represents the rotor speed and $\left(x_2^2 + x_3^2\right)$ represents the square modulus of the rotor flux.

Each parameter of $W_S(s)$, $W_{KS}(s)$ and $W_e(s)$ has a physical meaning.

In our case, we choose $\varepsilon_s \ll 1$ , $M_s \simeq 2$,$w_{bs1} = 60$ $rad/s$ and $w_{bs2} = 120$ $rad/s$.
While the weighting filters of KS are designed to

non-saturate the actuators, so $M_u = 3$ $w_{bks1} = 300\ rad/s$, $w_{bks2} = 20000\ rad/s$ and $\varepsilon_{ks} = 2$.

Concerning the filter $W_e$ , $M_e \simeq 3$ , $w_{e_1} = 60\ rad/s$ $w_{e_2} = 120\ rad/s$ and $\varepsilon_e = 2$.

For this choice of the weighting filters, the algorithm with $H_2/H_\infty$ optimization via LMI approach has been used to obtain a $\gamma$ achieved for the value 1.13 ($H_\infty$ bound) and $\mu = 0.05$ ($H_2$ bound). The value of $\gamma$ is more than 1 but already allows to have the specified performances.

The controller which has been designed is composed of two diagonal sub-controller.

In order to implement the designed controller, a reduction using (Etien *et al.*, 2000) has been applied. The following reduced controller $K(s)$ has been obtained:

$$K = \begin{bmatrix} k_1(s) & 0 & 0 & 0 \\ 0 & k_2(s) & 0 & 0 \\ 0 & 0 & \dfrac{5.933s + 1714}{s + 363.4} & 0 \\ 0 & 0 & 0 & \dfrac{6.08s + 3355}{s + 1155} \end{bmatrix}$$

$$\text{with } k_1(s) = \frac{2.013s^2 + 3904s + 5.775\ 10^4}{s^2 + 1292s + 386.7}$$

$$k_2(s) = \frac{0.2061s + 94.15}{s + 1.199}$$

### *5.3 Analysis numerical results.*

The nonlinear term $R(x)$ in (16) can be redefined to have the proposed structure of the term $H_{f_{cl}}.d$ in (6) owing to:

$$H_{f_{cl}} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}^T \tag{18}$$

Considering all the nonlinearities that can occur, in practice, while the induction motor is functioning, it was calculated that the worst case leads to a bound norm $\rho_{max} = ||d(\tilde{\theta}, X(t))||_{max} = 115$. In order to demonstrate the robust stability with respect to both polytopic and nonlinear uncertainties, it then matters that our analytical tool leads to a value of $\rho$ that is greater than $\rho_{max}$. We have used, on one hand, some classical tools relevant to quadratic stability, *i.e.* involving a Fixed Lyapunov Function (FLF) and, on the other hand, our condition involving Parameter-Dependent Lyapunov Functions (PDLF) with either $H$ given in (18) or with H equals to $I_n$. The performed computation is summarized in the table 1.

Table 1 highlights the importance of introducing $H$ to avoid pessimistic computation and it emphasizes the fact that the framework involving PDLF enables to prove robust stability for this induction motor model whereas quadratic stability is more pessimistic.

| | FLF | PDLF | FLF;$H = I_n$ | PDLF;$H = I_n$ |
|---|---|---|---|---|
| $\rho$ | 159.72 | 170.9 | 0.03 | 0.04 |

Table 1. Comparison between quadratic stability and parameter-dependent Lyapunov stability

### *5.4 Implementation on experimental set-up*

The solution to test the robustness of the controller consists to vary the parameters in the control loop.

In our case, it acts to make evolve the parameters in the linearization loop.

Let us underline that a nonlinear observer proposed in (Verghese and Sanders, 1988) is used with regards to real time requirement. An encoder transfer the rotor position information to the controller.

Experimental results have been brought on these figures for the reference trajectories of figure (1). We have superposed the results for the different couple of resistances and inductances.

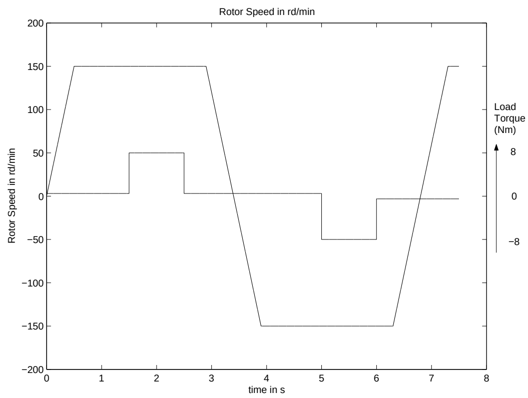


Fig. 1. Benchmark for Speed and Load torque

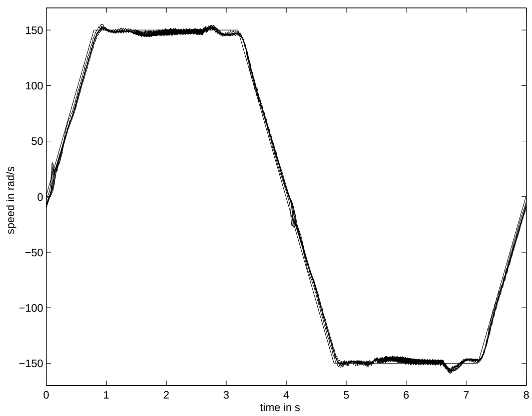


Fig. 2. Speed response

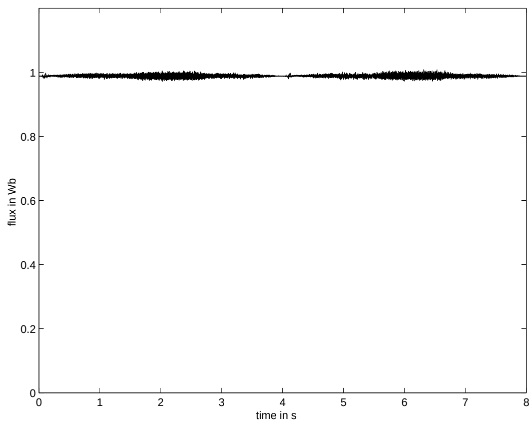


Fig. 3. Rotor flux modulus

We can notice that the flux rotor modulus is really decoupled of the rotor speed but this rotor flux is the estimated one. The odd scaling of the figure (3) is used for clarity. As a matter of fact, the results will be fewer impressive. The various control loops act in complementarity to have an identical behavior (decoupling and performance), and this, in spite of the variations of parameters.

## 6. CONCLUSION

It must be noticed that the synthesis method makes a possible way to maintain a real decoupling between the outputs of the nonlinear model of the induction motor in spite of the resistance variations and inductance variations. The analysis tool can perform a verification of the stability of the closed loop before the implementation.

In final, these results have been confirmed by experimental tests on the real plant.

## 7. APPENDIX A

Motor parameters:

$P = 1.1kW$ motor power, $p = 2$ pair number of poles, $Ls = 471.8mH$ stator inductance, $M = 447.5mH$ mutual inductance, $Lr = 471.8mH$ rotor inductance, $Rs = 9.65\Omega$ stator resistance, $Rr = 4.3047\Omega$ rotor resistance,$\sigma = 1 - \frac{M^2}{L_s L_r}$ Blondel coefficient, $J = 29.3.10^{-3}\ Kg.m^2$ rotor inertia, $\mathrm{T}_{\max} = 17.5N.m$ maximum motor torque.

## REFERENCES


Blaschke, F. (1972). The principle of field orientation applied to the new transvector close-loop control system for rotating field machines. *Siemens-Rev* **39**, 217–220.

Cauet, S., L. Rambault, O. Bachelier and D. Mehdi (2002). Parameter-dependent lyapunov functions applied to analysis of induction motor stability. *Control Engineering Practice* **10**(3), 337–345.

Chilali, M. and P. Gahinet (1996). $H_\infty$ Design with Pole Placement Constraints : An LMI Approach. *I.E.E.E Trans. on Aut. Contr.* **41**(3), 358–367.

Doyle, J. C., K. Glover, P. P. Khargonekar and B. A. Francis (1989). State space solutions to $H_2$ and $H_\infty$ control problems. *I.E.E.E transactions on Automatic Control* **34**(8), 831–847.

Etien, E., J.D. Gabano, L. Rambault and J.C. Trigeassou (2000). Frequential moments :application to controller reduction. In: *American Cont. Conf.*. Chicago.

Gahinet, P., A. Nemirovski, A.J. Laub and M. Chilali (1995). LMI Control Toolbox.

Geromel, J. C., M.C. de Oliveira and L. Hsu (1998). LMI characterization of structural and robust stability. *Linear Algebra and its Applications* **285**, 69–80.

Marino, R., S. Peresada and P. Valigi (1993). Adaptive input-output linearizing control of inductions motors. *I.E.E.E Transactions on Automatic Control* **38**(2), 208–221.

Morari, M. and E. Zafiriou (1989). *Robust Process Control.* Prentice-Hall. New Jersey.

Peaucelle, D., D. Arzelier, O. Bachelier and J. Bernussou (2000). A new robust D-stability condition for real convex polytopic uncertainty. **40**(1), 21–30.

Safonov, M.G., J. Doyle, K. Glover, A. Packard and R. Smith (1993). *Robust Control Toolbox for Use with Matlab.* The Math Works Company.

Verghese, G.C and S. R. Sanders (1988). Observers for flux estimation in induction machines. *I.E.E.E Transactions Indust. Elec.* **35**(1), 85–94.