DYNAMIC SURFACE CONTROL APPROACH TO ADAPTIVE ROBUST CONTROL OF NONLINEAR SYSTEMS IN SEMI-STRICT FEEDBACK FORM

Zi-Jiang Yang * **Toshimasa Nagai** *
Shunshoku Kanae * **Kiyoshi Wada** *

* *Department of Electrical and Electronic Systems Engineering,*
Kyushu University, Hakozaki, Fukuoka 812-8581, Japan
TEL: (+81)92-642-3904, FAX: (+81)92-642-3939
E-mail: yoh@ees.kyushu-u.ac.jp

Abstract: This paper consider the adaptive robust control of a class of single-input-single-output (SISO) nonlinear systems in semi-strict feedback form, where nonlinearities can exist in the input channel of each subsystem. To overcome the problem of "explosion of terms", the recently developed dynamic surface control technique is generalized to the nonlinear systems under study. At each step of design, a feedback controller strengthened by nonlinear damping terms to counteract modelling errors is designed to guarantee input-to-state practical stability of the corresponding subsystem, and then parameter adaptions are introduced to reduce the ultimate error bound. Finally, simulational examples are included to verify the results of theoretical analysis.

Keywords: Adaptive control, Robust control, Dynamic surface control, Nonlinear damping term, Input-to-state practical stability, Semi-strict feedback form.

1. INTRODUCTION

A drawback of the backstepping design procedure is the "explosion of terms" caused by the repeated differentiations of the virtual inputs. Recently, the dynamic surface control (DSC) technique has been proposed to avoid this problem by introducing a first-order low-pass filter at each step of the conventional backstepping design procedure (Yip & Hedrick 1998; Swaroop, Hedrick, Yip & Gerdes, 2000; Wang & Huang 2001).

In this paper, motivated by the pioneering works of the DSC technique reported in the literature, the theory and methodology are generalized to a class of SISO nonlinear systems in semi-strict feedback form, where nonlinearities can exist in the input channel of each subsystem. The design procedure is performed in a step by step manner. At each step of design, a feedback controller strengthed by nonlinear damping terms to counteract modelling errors is degined to guarantee input-to-state practical stability (ISpS which is an extension of the concept of input-to-state stability (ISS), see Jiang & Praly (1998) for definition) of the corresponding subsystem, and then parameter adaptions are introduced to reduce the ultimate error bound. Simulational examples are included to verify the results of theoretical analysis.

2. STATEMENT OF THE PROBLEM

Consider the following SISO nonlinear system in semi-strict feedback form:

$$\begin{aligned}\dot{x}_i &= f_i(\overline{x}_i) + g_i(\overline{x}_i)x_{i+1} + \Delta_i(\overline{x}_n, t),\ i = 1, \cdots, n-1\\ \dot{x}_n &= f_n(\overline{x}_n) + g_n(\overline{x}_n)u + \Delta_n(\overline{x}_n, t)\\ y &= x_1\end{aligned} \tag{1}$$

where, $\overline{x}_i = [x_1, \cdots, x_i]^T (i = 1, \cdots, n)$ is the state vector up to the ith subsystem, y is the system output; $f_i(\overline{x}_i), g_i(\overline{x}_i)$ are C^1 functions; $\Delta_i(\overline{x}_n, t)$ is the lumped nonrepeatable nonlinearities or external disturbances which may be nonlipschitz, but is continuous in its arguments.

The nonlinearities are modelled as follows.

$$\begin{aligned}
&f_i(\overline{x}_i) = f_{0i}(\overline{x}_i) + \Delta_{fi}(\overline{x}_i, \boldsymbol{a}_i) + \eta_{fi}(\overline{x}_i)\\
&g_i(\overline{x}_i) = g_{0i}(\overline{x}_i) + \Delta_{gi}(\overline{x}_i, \boldsymbol{b}_i) + \eta_{gi}(\overline{x}_i)\\
&\Delta_{fi}(\overline{x}_i, \boldsymbol{a}_i) = \boldsymbol{\phi}_i^T(\overline{x}_i)\boldsymbol{a}_i, \ \ \Delta_{gi}(\overline{x}_i, \boldsymbol{b}_i) = \boldsymbol{\psi}_i^T(\overline{x}_i)\boldsymbol{b}_i\\
&\boldsymbol{\phi}_i^T(\overline{x}_i) = [\phi_{i,1}(\overline{x}_i), \cdots, \phi_{i,M_{fi}}(\overline{x}_i)]\\
&\boldsymbol{\psi}_i^T(\overline{x}_i) = [\psi_{i,1}(\overline{x}_i), \cdots, \psi_{i,M_{gi}}(\overline{x}_i)]\\
&\boldsymbol{a}_i^T = [a_{i,1}, \cdots, a_{i,M_{fi}}], \ \ \boldsymbol{b}_i^T = [b_{i,1}, \cdots, b_{i,M_{gi}}]
\end{aligned} \tag{2}$$

where, $f_{0i}(\overline{x}_i), g_{0i}(\overline{x}_i)$ are known nominal nonlinear functions; $\Delta_{fi}(\overline{x}_i, \boldsymbol{a}_i), \ \Delta_{gi}(\overline{x}_i, \boldsymbol{b}_i)$ are linearly parameterized uncertain functions with known regressors $\boldsymbol{\phi}_i^T(\overline{x}_i), \ \boldsymbol{\psi}_i^T(\overline{x}_i)$ and unknown parameter vectors $\boldsymbol{a}_i, \ \boldsymbol{b}_i$; $\eta_{fi}(\overline{x}_i), \ \eta_{gi}(\overline{x}_i)$ are unmodelled uncertainties (approximation errors) of $f_i(\overline{x}_i), \ g_i(\overline{x}_i)$ respectively. When the parameters are adjusted by some suitable adaptive laws, the estimates of $f_i(\overline{x}_i), g_i(\overline{x}_i)$ are given by

$$\begin{aligned}
\widehat{f}_i(\overline{x}_i) &= f_{0i}(\overline{x}_i) + \widehat{\Delta}_{fi}\left(\overline{x}_i, \widehat{\boldsymbol{a}}_{it}\right)\\
\widehat{g}_i(\overline{x}_i) &= g_{0i}(\overline{x}_i) + \widehat{\Delta}_{gi}\left(\overline{x}_i, \widehat{\boldsymbol{b}}_{it}\right)
\end{aligned} \tag{3}$$

where $\widehat{\boldsymbol{a}}_{it}, \widehat{\boldsymbol{b}}_{it}$ are adaptively updated parameter vectors at time instant t.

Assumption 1: Define the desired domain of operation (a compact set) as

$$\Omega_x = \left\{\overline{x}_n \ \middle| |\overline{x}_n| \le d, {}^{\exists}d > 0\right\} \subset \mathbb{R}^n \tag{4}$$

$\Delta_{fi}(\overline{x}_i, \boldsymbol{a}_i), \Delta_{gi}(\overline{x}_i, \boldsymbol{b}_i)$ are sufficiently complex such that the modelling errors of $f_i(\overline{x}_i), g_i(\overline{x}_i)$ are sufficiently small on Ω_x for ${}^{\exists}\varepsilon_{fi}, {}^{\exists}\varepsilon_{gi} > 0$:

$$\sup_{\overline{x}_n \in \Omega_x} \left|\eta_{fi}(\overline{x}_i)\right| \le \varepsilon_{fi}, \quad \sup_{\overline{x}_n \in \Omega_x} |\eta_{gi}(\overline{x}_i)| \le \varepsilon_{gi} \tag{5}$$

Assumption 2: The lower and upper bounds of the parameter vectors are known *a priori*, i.e.,

$$\underline{\boldsymbol{a}_i} \le \boldsymbol{a}_i \le \overline{\boldsymbol{a}_i}, \quad \underline{\boldsymbol{b}_i} \le \boldsymbol{b}_i \le \overline{\boldsymbol{b}_i} \tag{6}$$

where $\le$ holds element-wisely for the vectors.

Notice that the standard adaptive laws

$$\begin{aligned}
\dot{\widehat{a^s_{i,m}}}_t &= \gamma_{ai} \ \phi_{i,m}(\overline{x}_i) S_i, \quad m = 1, 2, \cdots, M_{fi}\\
\dot{\widehat{a^s_{i,m}}}_t &= \gamma_{bi} \ \frac{\alpha_{i0}\psi_{i,m}(\overline{x}_i)}{\widehat{g}_i(\overline{x}_i)} S_i, \quad m = 1, 2, \cdots, M_{gi}
\end{aligned} \tag{7}$$

do not ensure that the estimated parameters stay in a prescribed range. Here, $\gamma_{ai}, \gamma_{bi} \ge 0$ are adaptive gains, S_i and α_{i0} will be defined later.

To let the esimated parameters stay in a prescribed range and has continuous first derivative, we adopt the following smooth projection technique (Yao & Tomizuka, 1997). Let $\boldsymbol{\theta} = \boldsymbol{a}_i$ or $\boldsymbol{\theta} = \boldsymbol{b}_i$. Then we have:

$$\widehat{\theta_m}_t = \pi\left(\widehat{\theta^s_m}_t\right) = \begin{cases}
\overline{\theta_m} + \varepsilon\left[1 - \exp\left(-(\widehat{\theta^s_m}_t - \overline{\theta_m})/\varepsilon\right)\right] & \text{for } \widehat{\theta^s_m}_t > \overline{\theta_m}\\
\widehat{\theta^s_m}_t & \text{for } \widehat{\theta^s_m}_t \in \left[\underline{\theta_m}, \ \overline{\theta_m}\right]\\
\underline{\theta_m} - \varepsilon\left[1 - \exp\left(\widehat{\theta^s_m}_t - \underline{\theta_m})/\varepsilon\right)\right] & \text{for } \widehat{\theta^s_m}_t < \underline{\theta_m}
\end{cases} \tag{8}$$

for $m = 1, \cdots, M$, where $M = M_{fi}$ or $M = M_{gi}$.

The adaptive laws with smooth projection ensure

$$\widehat{a_{i,m}}_t \in \left[\underline{a_{i,m}} - \varepsilon, \ \overline{a_{i,m}} + \varepsilon\right], \quad \widehat{b_{i,m}}_t \in \left[\underline{b_{i,m}} - \varepsilon, \ \overline{b_{i,m}} + \varepsilon\right] \tag{9}$$

where ε is a small positive number. Furthermore, define $\widetilde{\boldsymbol{\theta}^s_t} = \widehat{\boldsymbol{\theta}^s_t} - \boldsymbol{\theta}$, and let $\gamma = \gamma_{ai}$ or $\gamma = \gamma_{bi}$. Then we have

$$V(\widetilde{\boldsymbol{\theta}^s_t}, \boldsymbol{\theta}) = \sum_{m=1}^{M} \frac{1}{\gamma} \int_0^{\widetilde{\theta^s_m}_t} \left[\pi\left(\nu_m + \theta_m\right) - \theta_m\right] d\nu_m, \quad \gamma > 0 \tag{10}$$

It can be verified that $V(\widetilde{\boldsymbol{\theta}^s_t}, \boldsymbol{\theta})$ is positive definite with respect to $\widetilde{\boldsymbol{\theta}^s_t}$, and (Yao & Tomizuka, 1997)

$$\frac{\partial}{\partial \widetilde{\boldsymbol{\theta}^s_t}} V(\widetilde{\boldsymbol{\theta}^s_t}, \boldsymbol{\theta}) = \frac{\widetilde{\boldsymbol{\theta}}_t^T}{\gamma}, \quad \widetilde{\boldsymbol{\theta}}_t = \widehat{\boldsymbol{\theta}}_t - \boldsymbol{\theta} \tag{11}$$

The results are used for analysis of the performance of the adaptive laws.

Assumption 3: The input gain $g_i(\overline{x}_i)$ of the ith subsystem is bounded away from zero with known sign. Thus, without loss of generality, we assume $g_i(\overline{x}_i) > 0$ for $\overline{x}_n \in \Omega_x$.

Assumption 4: The parameter bounds are chosen such that

$$\begin{gathered}
\widehat{g}_i(\overline{x}_i) = g_{0i}(\overline{x}_i) + \widehat{\Delta}_{gi}\left(\overline{x}_i, \widehat{\boldsymbol{b}}_{it}\right) > 0\\
\frac{g_i(\overline{x}_i)}{\widehat{g}_i(\overline{x}_i)} > {}^{\exists}C_{gi} > 0
\end{gathered} \tag{12}$$

for any $\underline{b_{i,m}} - \epsilon_{bi} \le \widehat{b_{m,i}}_t \le \overline{b_{i,m}} + \epsilon_{bi}$ $(m = 1, \cdots, M_{gi})$.

Assumption 5: There exist finite positive constants $M_{\Delta fi}, M_{\Delta i}, M_{\Delta gi} < \infty$ and known smooth functions $d_{\Delta i}(\overline{x}_i, t), d_{\Delta fi}(\overline{x}_i)$ such that the following inequalities hold for $\overline{x}_n \in \Omega_x$:

$$\begin{gathered}
\left|\frac{\Delta_i(\overline{x}_n, t)}{\sqrt{d_{\Delta i}(\overline{x}_i, t)^2 + 1}}\right| \le M_{\Delta i}, \quad \left|\frac{\widetilde{f}_i(\overline{x}_i)}{\sqrt{d_{\Delta fi}(\overline{x}_i)^2 + 1}}\right| \le M_{\Delta fi}\\
\left|\frac{\widetilde{g}_i(\overline{x}_i)}{g_i(\overline{x}_i)}\right| \le M_{\Delta gi}
\end{gathered} \tag{13}$$

where

$$\begin{aligned}
\widetilde{f}_i(\overline{x}_i) &= f_i(\overline{x}_i) - \widehat{f}_i(\overline{x}_i) = -\boldsymbol{\phi}_i^T(\overline{x}_i)\widetilde{\boldsymbol{a}}_{it} + \eta_{fi}(\overline{x}_i)\\
\widetilde{g}_i(\overline{x}_i) &= g_i(\overline{x}_i) - \widehat{g}_i(\overline{x}_i) = -\boldsymbol{\psi}_i^T(\overline{x}_i)\widetilde{\boldsymbol{b}}_{it} + \eta_{gi}(\overline{x}_i)
\end{aligned} \tag{14}$$

Notice that $d_{\Delta i}(\overline{x}_i, t), \ d_{\Delta fi}(\overline{x}_i)$ will be used for nonlinear damping terms.

Assumption 6: All of the following known nonlinear functions

$$f_{0i}(\overline{x}_i), g_{0i}(\overline{x}_i), \boldsymbol{\phi}_i^T(\overline{x}_i), \boldsymbol{\psi}_i^T(\overline{x}_i), d_{\Delta i}(\overline{x}_i, t), d_{\Delta fi}(\overline{x}_i)$$

which will be used in the designed controller are C^1 functions.

Assumption 7: The reference trajectory $y_r(t)$ is appropriately chosen as a sufficiently smooth function such that

$$\begin{aligned}
\Omega_{y_r} = \Big\{ y_r, \dot{y}_r, \ddot{y}_r \Big| \ & |y_r| \le \overline{y}_r, |\dot{y}_r| \le \overline{\dot{y}}_r,\\
& |\ddot{y}_r| \le \overline{\ddot{y}}_r, {}^{\exists}\overline{y}_r, {}^{\exists}\overline{\dot{y}}_r, {}^{\exists}\overline{\ddot{y}}_r > 0 \Big\} \subset \mathbb{R}^3
\end{aligned} \tag{15}$$

3. DESIGN OF THE CONTROLLER

In this section, we show the design procedure of the proposed adaptive robust nonlinear controller.

Step 1:

Define the output tracking error as

$$S_1 = x_1 - y_r \tag{16}$$

Then, we have the following dynamics of the first subsystem:

$$\begin{aligned}\dot{S}_1 = &\widehat{f}_1(\overline{x}_1) + \widetilde{f}_1(\overline{x}_1) + \widehat{g}_1(\overline{x}_1)\left(1 + \frac{\widetilde{g}_1(\overline{x}_1)}{\widehat{g}_1(\overline{x}_1)}\right)x_2 \\ &+\Delta_1(\overline{x}_n, t) - \dot{y}_r\end{aligned} \tag{17}$$

To stabilize the subsystem, we design the virtual input $\overline{\xi}_2$ as the followng:

$$\begin{aligned}\overline{\xi}_2 &= \frac{\alpha_{10} - \alpha_{11} - \alpha_{12} - \alpha_{13} - \alpha_{14}}{\widehat{g}_1(\overline{x}_1)} \\ \alpha_{10} &= -c_1 S_1 - f_{01}(\overline{x}_1) - \boldsymbol{\phi}_1^T(\overline{x}_1)\widehat{\boldsymbol{a}}_{1t} + \dot{y}_r \\ \alpha_{11} &= \kappa_{11}\sqrt{d_{\Delta 1}(\overline{x}_1)^2 + 1}S_1 \\ \alpha_{12} &= \kappa_{12}\sqrt{\alpha_{10}^2 + \nu}S_1 \\ \alpha_{13} &= \kappa_{13}\sqrt{d_{\Delta f1}(\overline{x}_1)^2 + 1}S_1 \\ \alpha_{14} &= \kappa_{14}\widehat{g}_1(\overline{x}_1)S_1\end{aligned} \tag{18}$$

where $c_1, \kappa_{11}, \kappa_{12}, \kappa_{13} > 0$. α_{10} is a feedback controller with model compensation, $\alpha_{11}, \alpha_{12}, \alpha_{13}$ are nonlinear damping terms to respectively counteract $\Delta_1(\overline{x}_n, t), \widetilde{g}_1(\overline{x}_1), \widetilde{f}_1(\overline{x}_1)$, and α_{14} with $\kappa_{14} > 1$ is a nonlinear damping term to counteract error signal due to S_2 which will be defined later.

Motivated by the DSC technique (Yip & Hedrick 1998; Swaroop, Hedrick, Yip & Gerdes, 2000; Wang & Huang 2001), a first-order low-pass filter with a small positive time-constant τ_2 is introduced here to avoid involving calculation of $\dot{\overline{\xi}}_2$. Letting $\overline{\xi}_2$ pass through the low-pass filter and defining a new signal ξ_{2d}, we have

$$\tau_2\dot{\xi}_{2d} + \xi_{2d} = \overline{\xi}_2, \quad \xi_{2d}(0) = \overline{\xi}_2(0) \tag{19}$$

Define the error signal $S_2 = x_2 - \xi_{2d}$ and the error signal between $\overline{\xi}_2$ and its filtered version ξ_{2d}

$$y_2 = \xi_{2d} - \overline{\xi}_2 = -\tau_2\dot{\xi}_{2d} \tag{20}$$

It follows that

$$x_2 = S_2 + y_2 + \overline{\xi}_2 = B_{x2}\left(S_2, \boldsymbol{S}_{1a}, c_1, \boldsymbol{\kappa}_1, \widehat{\boldsymbol{\theta}}_{1t}, y_r, \dot{y}_r\right) \tag{21}$$

where B_{x2} is an appropriate continuous function of its arguments, and $\boldsymbol{S}_{1a} = [S_1, y_2]^T, \boldsymbol{\kappa}_1 = [\kappa_{11}, \kappa_{12}, \kappa_{13}, \kappa_{14}]^T, \widehat{\boldsymbol{\theta}}_{1t} = \left[\widehat{\boldsymbol{a}}_{1t}^T, \widehat{\boldsymbol{b}}_{1t}^T\right]^T$.

Our task here is to establish the ISpS of the combined subsystem $\mathcal{C}1$:

$$\mathcal{C}1 : \begin{cases}\dot{S}_1 = -c_1 S_1 - D_1 S_1 + \widetilde{f}_1(\overline{x}_1) + \dfrac{\widetilde{g}_1(\overline{x}_1)}{\widehat{g}_1(\overline{x}_1)}\alpha_{10} \\ \qquad +\Delta_1(\overline{x}_n, t) + g_1(\overline{x}_1)(S_2 + y_2) \\ \dot{y}_2 = -\dfrac{y_2}{\tau_2} + B_{y2}\end{cases} \tag{22}$$

where D_1 and B_{y2} are appropriate continuous functions of their arguments:

$$\begin{aligned}D_1 &= \Big(\kappa_{11}\sqrt{d_{\Delta 1}(\overline{x}_1)^2 + 1} + \kappa_{12}\sqrt{\alpha_{10}^2 + 1} \\ &\quad +\kappa_{13}\sqrt{d_{\Delta f1}(\overline{x}_1)^2 + 1} + \kappa_{14}\widehat{g}_1(\overline{x}_1)\Big)\frac{g_1(\overline{x}_1)}{\widehat{g}_1(\overline{x}_1)} \\ &= D_1\left(S_1, c_1, \boldsymbol{\kappa}_1, \widehat{\boldsymbol{\theta}}_{1t}, y_r, \dot{y}_r\right) \\ B_{y2} &= B_{y2}\Big(S_2, \boldsymbol{S}_{1a}, c_1, \boldsymbol{\kappa}_1, \widehat{\boldsymbol{\theta}}_{1t}(\boldsymbol{\gamma}_1), \widehat{\boldsymbol{\theta}}_{1t}(\overline{\boldsymbol{\theta}_1}, \underline{\boldsymbol{\theta}_1}), \\ &\quad \Delta_1(\overline{x}_n, t), y_r, \dot{y}_r, \ddot{y}_r\Big)\end{aligned} \tag{23}$$

and $\boldsymbol{\gamma}_1 = [\gamma_{a1}, \gamma_{b1}]^T, \overline{\boldsymbol{\theta}_1} = \left[\overline{\boldsymbol{a}_1}^T, \overline{\boldsymbol{b}_1}^T\right]^T, \underline{\boldsymbol{\theta}_1} = \left[\underline{\boldsymbol{a}_1}^T, \underline{\boldsymbol{b}_1}^T\right]^T$.

To this purpose, we make assumption 8:

Assumption 8: The states of the combined error system stay in a compact set $\Omega_{S_{1a}}^{large} \times \Omega_{S_2}^{large} \subset \mathbb{R}^3$

$$\begin{aligned}\Omega_{S_{1a}}^{large} &= \left\{\boldsymbol{S}_{1a} \,\middle|\, |\boldsymbol{S}_{1a}| \le \overline{\boldsymbol{S}}_{1a}^{large}, \exists\, \overline{\boldsymbol{S}}_{1a}^{large} > 0\right\} \subset \mathbb{R}^2 \\ \Omega_{S_2}^{large} &= \left\{S_2 \,\middle|\, |S_2| \le \overline{S}_2^{large}, \exists\, \overline{S}_2^{large} > 0\right\} \subset \mathbb{R}^1\end{aligned} \tag{24}$$

According to assumptions 7 and 8, $|D_1|$ has a maximum $\overline{D}_1$ on $\Omega_{S_{1a}}^{large} \times \Omega_{y_r}$, and $|B_{y2}|$ has a maximum $\overline{B}_{y2}$ on $\Omega_{S_{1a}}^{large} \times \Omega_{y_r} \times \Omega_{S_2}^{large}$. Let the time-constant of the low-pass filter satisfy

$$\frac{1}{\tau_2} \ge \frac{1}{4}\overline{g_1(\overline{x}_1)} + c_1 + \overline{D}_1 + \frac{\overline{B}_{y2}^2}{2\epsilon} \tag{25}$$

where $\overline{g_1}$ is the maximum of g_1 on $\Omega_x \subset \mathbb{R}^n$, and ϵ is an arbitrary positive number. Notice that (25) has transparent physical meaning, i.e., stronger control efforts, larger adaptive gains and faster changing reference trajectory require a smaller filter time-constant τ_2.

Then the ISpS property of the combined system (22) can be shown in the following lemma (the proof is omitted due to limit of paper length):

Lemma 1. Let assumptions 1∼8 hold. If S_2 is made uniformly bounded at the next step, then there exists a set of $\tau_2, c_1, \boldsymbol{\kappa}_1, \boldsymbol{\gamma}_1, \overline{\boldsymbol{\theta}}_1, \underline{\boldsymbol{\theta}}_1$ for an appropriate set of initial conditions such that the combined system (22) is ISpS:

$$|\boldsymbol{S}_{1a}| \le |\boldsymbol{S}_{1a}(0)|e^{-c_1 t/2} + \sqrt{\frac{\epsilon}{c_1}} + \sup_{0\le\tau\le t}\mu_1(\tau)$$

where $\mu_1(t)$ is a uniformly bounded signal:

$$\begin{aligned}\mu_1(t) &= \mu_{11}(t) + \mu_{12}(t)|S_2| \\ \mu_{11}(t) &= \frac{\left|\widetilde{f}_1(\overline{x}_1) + \dfrac{\widetilde{g}_1(\overline{x}_1)}{\widehat{g}_1(\overline{x}_1)}\alpha_{10} + \Delta_1(\overline{x}_1, t)\right|}{\dfrac{c_1}{2} + D_1 - g_1(\overline{x}_1)} \\ \mu_{12}(t) &= \frac{|g_1(\overline{x}_1)|}{\dfrac{c_1}{2} + D_1 - g_1(\overline{x}_1)}\end{aligned}$$

The lemma implies that we can make $\boldsymbol{S}_{1a}$ stay in a compact subset $\Omega_{S_{1a}} \subset \Omega_{S_{1a}}^{large}$. Also, assumption 8 holds in the sense that we can find a compact set $\mathcal{D}_{S_{1a}} \subset \Omega_{S_{1a}} \subset \Omega_{S_{1a}}^{large}$ such that $\boldsymbol{S}_{1a} \in \Omega_{S_{1a}} \subset \Omega_{S_{1a}}^{large}$ for all $\boldsymbol{S}_{1a}(0) \in \mathcal{D}_{S_{1a}}$. Finally, notice the nonlinear damping terms appear in the denominator of μ_1 so that the modelling errors that appear in the numerator are suppressed.

Furthermore, to analyze the ultimate error bound achieved by the adaptive law, we define the following Lyapunov function:

$$V_1 = \frac{\boldsymbol{S}_{1a}^T \boldsymbol{S}_{1a}}{2} + V(\widetilde{\boldsymbol{a}}_{1t}^s, \boldsymbol{a}_1) + V(\widetilde{\boldsymbol{b}}_{1t}^s, \boldsymbol{b}_1) \tag{26}$$

Then we have the following results:

Lemma 2. Let the conditions and results of lemma 1 hold. If the adaptive law (8) where $i = 1$ is used, and S_2 is made uniformly ultimately bounded with ultimate bound $\overline{S}_2^u$ at the next step, then

$$|\boldsymbol{S}_{1a}(t)| \leq \frac{C_{\Delta 1} M_{\Delta 1}}{\kappa_{11}} + \frac{C_{g1}\varepsilon_{g1}}{\kappa_{12}} + \frac{C_{f1}\varepsilon_{f1}}{\kappa_{13}} + \frac{\overline{S}_2^u}{\kappa_{14} - 1} + \sqrt{\frac{\epsilon}{c_1}} \quad \text{as } t \geq {}^\exists T_1 > 0$$

with ${}^\exists C_{f1}, {}^\exists C_{g1}, {}^\exists C_{\Delta 1} > 0$.

Steps $2 \leq i \leq n-1$:

The dynamics of the ith subsystem is obtained as

$$\dot{S}_i = \dot{x}_i - \dot{\xi}_{id} = f_i(\overline{x}_i) + g_i(\overline{x}_i)x_{i+1} + \Delta_i(\overline{x}_n, t) - \dot{\xi}_{id} \tag{27}$$

Then we design the following controller as we did in step 1:

$$\begin{aligned} \overline{\xi}_{i+1} &= \frac{\alpha_{i0} - \alpha_{i1} - \alpha_{i2} - \alpha_{i3} - \alpha_{i4}}{\widehat{g}_i(\overline{x}_i)} \\ \alpha_{i0} &= -c_i S_i - f_{0i}(\overline{x}_i) - \boldsymbol{\phi}_i^T(\overline{x}_i)\widehat{\boldsymbol{a}}_{it} + \dot{\xi}_{id} \\ \alpha_{i1} &= \kappa_{i1}\sqrt{d_{\Delta i}(\overline{x}_i)^2 + 1}S_i \\ \alpha_{i2} &= \kappa_{i2}\sqrt{\alpha_{i0}^2 + 1}S_i \\ \alpha_{i3} &= \kappa_{i3}\sqrt{d_{\Delta fi}(\overline{x}_i)^2 + 1}S_i \\ \alpha_{i4} &= \kappa_{i4}\widehat{g}_i(\overline{x}_i)S_i \end{aligned} \tag{28}$$

where $c_i, \kappa_{i1}, \kappa_{i2}, \kappa_{i3} > 0, \kappa_{i4} > 1$.

Letting $\overline{\xi}_i$ pass through the low-pass filter and defining a new signal ξ_{id}, we have

$$\tau_{i+1}\dot{\xi}_{(i+1)d} + \xi_{(i+1)d} = \overline{\xi}_{i+1}, \quad \xi_{(i+1)d}(0) = \overline{\xi}_{i+1}(0) \tag{29}$$

Define the error signals

$$S_{i+1} = x_{i+1} - \xi_{(i+1)d}, \quad y_{i+1} = \xi_{(i+1)d} - \overline{\xi}_{i+1} \tag{30}$$

It follows that

$$\begin{aligned} x_{i+1} = B_{x(i+1)}\big(&S_{i+1}, \tau_2 \cdots, \tau_i, \boldsymbol{S}_{1a}, c_1, \boldsymbol{\kappa}_1, \widehat{\boldsymbol{\theta}}_{1t}, \cdots, \\ &\boldsymbol{S}_{ia}, c_i, \boldsymbol{\kappa}_i, \widehat{\boldsymbol{\theta}}_{it}, y_r, \dot{y}_r\big) \end{aligned} \tag{31}$$

where $B_{x(i+1)}$ is an appropriate continuous function of its arguments, and $\boldsymbol{S}_{ia} = [S_i, y_{i+1}]^T, \boldsymbol{\kappa}_i = [\kappa_{i1}, \kappa_{i2}, \kappa_{i3}, \kappa_{i4}]^T, \widehat{\boldsymbol{\theta}}_{it} = \big[\widehat{\boldsymbol{a}}_{it}^T, \widehat{\boldsymbol{b}}_{it}^T\big]^T$.

Our task here is to establish the ISpS of the combined subsystem $\mathcal{C}i$:

$$\mathcal{C}i : \begin{cases} \dot{S}_i = -c_i S_i - D_i S_i + \widetilde{f}_i(\overline{x}_i) + \dfrac{\widetilde{g}_i(\overline{x}_i)}{\widehat{g}_i(\overline{x}_i)}\alpha_{i0} \\ \qquad + \Delta_i(\overline{x}_n, t) + g_i(\overline{x}_i)(S_{i+1} + y_{i+1}) \\ \dot{y}_{i+1} = -\dfrac{y_{i+1}}{\tau_{i+1}} + B_{y(i+1)} \end{cases} \tag{32}$$

where D_i and $B_{y(i+1)}$ are appropriate continuous functions of their arguments:

$$\begin{aligned} D_i &= \Big(\kappa_{i1}\sqrt{d_{\Delta i}(\overline{x}_i)^2 + 1} + \kappa_{i2}\sqrt{\alpha_{i0}^2 + 1} \\ &\quad + \kappa_{i3}\sqrt{d_{\Delta fi}(\overline{x}_i)^2 + 1} + \kappa_{i4}\widehat{g}_i(\overline{x}_i)\Big)\frac{g_i(\overline{x}_i)}{\widehat{g}_i(\overline{x}_i)} \\ &= D_i\Big(S_i, \boldsymbol{S}_{1a}, \cdots, \boldsymbol{S}_{(i-1)a}, \tau_2, \cdots, \tau_i, \\ &\quad c_1, \boldsymbol{\kappa}_1, \widehat{\boldsymbol{\theta}}_{1t}, \cdots, c_i, \boldsymbol{\kappa}_i, \widehat{\boldsymbol{\theta}}_{it}, y_r, \dot{y}_r\Big) \end{aligned} \tag{33}$$
$$\begin{aligned} B_{y(i+1)} &= B_{y(i+1)}\Big(S_{i+1}, \tau_2, \cdots, \tau_{i+1}, \\ &\quad \boldsymbol{S}_{1a}, c_1, \boldsymbol{\kappa}_1, \dot{\widehat{\boldsymbol{\theta}}}_{1t}(\boldsymbol{\gamma}_1), \widehat{\boldsymbol{\theta}}_{1t}(\overline{\boldsymbol{\theta}_1}, \underline{\boldsymbol{\theta}_1}), \Delta_1(\overline{x}_n, t), \cdots, \\ &\quad \boldsymbol{S}_{ia}, c_i, \boldsymbol{\kappa}_i, \dot{\widehat{\boldsymbol{\theta}}}_{it}(\boldsymbol{\gamma}_i), \widehat{\boldsymbol{\theta}}_{it}(\overline{\boldsymbol{\theta}_i}, \underline{\boldsymbol{\theta}_i}), \Delta_i(\overline{x}_n, t), y_r, \dot{y}_r, \ddot{y}_r\Big) \end{aligned}$$

and $\boldsymbol{\gamma}_i = [\gamma_{ai}, \gamma_{bi}]^T, \overline{\boldsymbol{\theta}_i} = \big[\overline{\boldsymbol{a}_i}^T, \overline{\boldsymbol{b}_i}^T\big]^T, \underline{\boldsymbol{\theta}_i} = \big[\underline{\boldsymbol{a}_i}^T, \underline{\boldsymbol{b}_i}^T\big]^T$.

To this purpose, we make assumption 9:

Assumption 9: The states of the combined error system stay in a compact set $\Omega_{S_{ia}}^{large} \times \Omega_{S_{i+1}}^{large} \subset \mathbb{R}^3$

$$\begin{aligned} \Omega_{S_{ia}}^{large} &= \Big\{\boldsymbol{S}_{ia}\Big| \, |\boldsymbol{S}_{ia}| \leq \overline{\boldsymbol{S}}_{ia}^{large}, {}^\exists\overline{\boldsymbol{S}}_{ia}^{large} > 0\Big\} \subset \mathbb{R}^2 \\ \Omega_{S_{i+1}}^{large} &= \Big\{S_{i+1} \;\Big|\; |S_{i+1}| \leq \overline{S}_{i+1}^{large}, {}^\exists\,\overline{S}_{i+1}^{large} > 0\Big\} \subset \mathbb{R}^1 \end{aligned} \tag{34}$$

According to assumptions 7∼9, $|D_i|$ has a maximum $\overline{D}_i$ on $\Omega_{S_{1a}}^{large} \times \cdots \times \Omega_{S_{ia}}^{large} \times \Omega_{y_r}$, and $|B_{y(i+1)}|$ has a maximum $\overline{B}_{y(i+1)}$ on $\Omega_{S_{1a}}^{large} \times \cdots \times \Omega_{S_{ia}}^{large} \times \Omega_{y_r} \times \Omega_{S_{i+1}}^{large}$. Let

$$\frac{1}{\tau_{i+1}} \geq \frac{1}{4}\overline{g_i(\overline{x}_i)} + c_i + \overline{D}_i + \frac{\overline{B}_{y(i+1)}^2}{2\epsilon} \tag{35}$$

where $\overline{g_i}$ is the maximum of g_i on $\Omega_x \subset \mathbb{R}^n$, and ϵ is an arbitrary positive number.

Then we have the following results:

Lemma 3. Let assumptions 1∼9 hold. If S_{i+1} is made uniformly bounded at the next step, then there exists a set of $\tau_2, \cdots, \tau_{i+1}, c_1, \boldsymbol{\kappa}_1, \boldsymbol{\gamma}_1, \overline{\boldsymbol{\theta}}_1, \underline{\boldsymbol{\theta}}_1, \cdots, c_i, \boldsymbol{\kappa}_i, \boldsymbol{\gamma}_i, \overline{\boldsymbol{\theta}}_i, \underline{\boldsymbol{\theta}}_i$ for an appropriate set of initial conditions such that the combined system (32) is ISpS:

$$|\boldsymbol{S}_{ia}| \leq |\boldsymbol{S}_{ia}(0)|e^{-c_i t/2} + \sqrt{\frac{\epsilon}{c_i}} + \sup_{0 \leq \tau \leq t} \mu_i(\tau)$$

where $\mu_i(t)$ is a uniformly bounded signal:

$$\begin{aligned} \mu_i(t) &= \mu_{i1}(t) + \mu_{i2}(t)|S_{i+1}| \\ \mu_{i1}(t) &= \frac{\left|\widetilde{f}_i(\overline{x}_i) + \dfrac{\widetilde{g}_i(\overline{x}_i)}{\widehat{g}_i(\overline{x}_i)}\alpha_{i0} + \Delta_i(\overline{x}_1, t)\right|}{\dfrac{c_i}{2} + D_i - g_i(\overline{x}_i)} \\ \mu_{i2}(t) &= \frac{|g_i(\overline{x}_i)|}{\dfrac{c_i}{2} + D_i - g_i(\overline{x}_i)} \end{aligned}$$

Lemma 4. Let the conditions and results of lemma 3 hold. If the adaptive law (8) for each i is used and S_{i+1} is made uniformly ultimately bounded with ultimate bound $\overline{S}_{i+1}^u$ at the next step, then

$$|\boldsymbol{S}_{ia}(t)| \leq \frac{C_{\Delta i} M_{\Delta i}}{\kappa_{i1}} + \frac{C_{gi}\varepsilon_{gi}}{\kappa_{i2}} + \frac{C_{fi}\varepsilon_{fi}}{\kappa_{i3}} + \frac{\overline{S}_{i+1}^u}{\kappa_{i4} - 1} + \sqrt{\frac{\epsilon}{c_i}} \quad \text{as } t \geq {}^\exists T_i > 0$$

with ${}^\exists C_{fi}, {}^\exists C_{gi}, {}^\exists C_{\Delta i} > 0$.

Step n:

The dynamics of the nth subsystem is obtained as

$$\dot{S}_n = f_n(\overline{x}_n) + g_n(\overline{x}_n)u + \Delta_n(\overline{x}_n, t) - \dot{\xi}_{nd} \quad (36)$$

Then to stabilize the final subsystem we design the following controller:

$$\begin{aligned} u &= \frac{\alpha_{n0} - \alpha_{n1} - \alpha_{n2} - \alpha_{n3}}{\widehat{g}_n(\overline{x}_n)} \\ \alpha_{n0} &= -c_n S_n - f_{0n}(\overline{x}_n) - \boldsymbol{\phi}_n^T(\overline{x}_n)\widehat{\boldsymbol{a}_{n}}_t + \dot{\xi}_{nd} \\ \alpha_{n1} &= \kappa_{n1}\sqrt{d_{\Delta n}(\overline{x}_n)^2 + 1}S_n \\ \alpha_{n2} &= \kappa_{n2}\sqrt{\alpha_{n0}^2 + 1}S_n \\ \alpha_{n3} &= \kappa_{n3}\sqrt{d_{\Delta fn}(\overline{x}_n)^2 + 1}S_n \end{aligned} \quad (37)$$

Substituting u into the subsystem, we have

$$\begin{aligned} \dot{S}_n &= -c_n S_n - D_n S_n - \boldsymbol{\phi}_n^T(\overline{x}_n)\widetilde{\boldsymbol{a}}_{nt} + \eta_{fn}(\overline{x}_n) \\ &\quad + \frac{-\boldsymbol{\psi}_n^T(\overline{x}_n)\widetilde{\boldsymbol{b}}_{nt} + \eta_{gi}(\overline{x}_n)}{\widehat{g}_n(\overline{x}_n)}\alpha_{n0} + \Delta_n(\overline{x}_n, t) \end{aligned} \quad (38)$$

where

$$\begin{aligned} D_n &= \Big(\kappa_{n1}\sqrt{d_{\Delta n}(\overline{x}_n)^2 + 1} + \kappa_{n2}\sqrt{\alpha_{n0}^2 + 1} \\ &\quad + \kappa_{n3}\sqrt{d_{\Delta fn}(\overline{x}_n)^2 + 1}\Big)\frac{g_n(\overline{x}_n)}{\widehat{g}_n(\overline{x}_n)} \end{aligned} \quad (39)$$

Similar to the previous steps, we have the following results:

Lemma 5. Let assumptions 1∼9 hold. There exists a set of $\tau_2, \cdots, \tau_n, c_1, \boldsymbol{\kappa}_1, \boldsymbol{\gamma}_1, \overline{\boldsymbol{\theta}}_1, \underline{\boldsymbol{\theta}}_1, \cdots, c_n, \boldsymbol{\kappa}_n, \boldsymbol{\gamma}_n, \overline{\boldsymbol{\theta}}_n, \underline{\boldsymbol{\theta}}_n$ for an appropriate set of initial conditions such that subsystem (38) is ISS:

$$|S_n| \le |S_n(0)|e^{-c_n t/2} + \sup_{0\le\tau\le t} \mu_{n1}(\tau)$$

where $\mu_{n1}(t)$ is a uniformly bounded signal:

$$\mu_{n1}(t) = \frac{\left|\widetilde{f}_n(\overline{x}_n) + \frac{\widetilde{g}_n(\overline{x}_n)}{\widehat{g}_n(\overline{x}_n)}\alpha_{n0} + \Delta_n(\overline{x}_n, t)\right|}{\frac{c_n}{2} + D_n}$$

Lemma 6. Let the conditions and results of lemma 5 hold. If the adaptive law (8) where $i = n$ is used, then

$$|S_n(t)| \le \frac{C_{\Delta n}M_{\Delta i}}{\kappa_{n1}} + \frac{C_{gn}\varepsilon_{gn}}{\kappa_{n2}} + \frac{C_{fn}\varepsilon_{fn}}{\kappa_{n3}}, \text{ as } t \ge {}^{\exists}T_n > 0$$

with ${}^{\exists}C_{fn}, {}^{\exists}C_{gn}, {}^{\exists}C_{\Delta n} > 0$.

4. ISPS STABILITY AND ULITIMATE BOUND OF THE OVERALL ERROR SYSTEM

Recall the results of lemmas 1, 3 and 5. We can find that the overall error system is a cascade of the subsystems. Along the same lines of the proof of lemma C.4 in Krstic, Kanellakopoulos & Kokotovic (1995), we have the ISpS property of the overall error system:

$$|\boldsymbol{S}(t)| \le \sqrt{2}\lambda_n e^{-\rho_n t/2}|\boldsymbol{S}(0)| + \mathcal{E}_{\varepsilon(n)} + \mathcal{E}_{\mu(n)}(0, t) \quad (40)$$

where $\boldsymbol{S} = [\boldsymbol{S}_{1a}^T, \cdots, \boldsymbol{S}_{(n-1)a}^T, S_n]^T$,

$$\begin{aligned} &\mathcal{E}_{\varepsilon 1} = \sqrt{\frac{\epsilon}{c_1}},\ \lambda_1 = 1,\ \rho_1 = c_1,\ \mathcal{M}_1 = \overline{\mu}_{12} = \|\mu_{12}\|_\infty, \\ &\mathcal{E}_{\mu 1}(0, t) = \left[\sup_{0\le\tau\le t} \mu_{11}(\tau)\right] \end{aligned} \quad (41)$$

$$\begin{aligned} \lambda_j &= \lambda_{j-1}^2 + \Big((\lambda_{j-1}+1)\mathcal{M}_{j-1} + 1\Big) \\ \rho_j &= \min(\rho_{j-1}/2,\ c_j/2) \\ \overline{\mu}_{j2} &= \|\mu_{j2}\|_\infty \\ \mathcal{M}_j &= \Big((\lambda_{j-1}+1)\mathcal{M}_{j-1} + 1\Big)\overline{\mu}_{j2} \\ \mathcal{E}_{\varepsilon j} &= (\lambda_{j-1}+1)\mathcal{E}_{\varepsilon(j-1)} \\ &\quad + \Big((\lambda_{j-1}+1)\mathcal{M}_{j-1} + 1\Big)\sqrt{\frac{\epsilon}{c_j}} \\ \mathcal{E}_{\mu j}(0, t) &= (\lambda_{j-1}+1)\mathcal{E}_{\mu(j-1)}(0, t) \\ &\quad + \Big((\lambda_{j-1}+1)\mathcal{M}_{j-1} + 1\Big)\left[\sup_{0\le\tau\le t} \mu_{j1}(\tau)\right] \end{aligned} \quad (42)$$

for $j = 2, \cdots, n-1$, and

$$\begin{aligned} \lambda_n &= \lambda_{n-1}^2 + \Big((\lambda_{n-1}+1)\mathcal{M}_{n-1} + 1\Big) \\ \rho_n &= \min(\rho_{n-1}/2,\ c_n/2) \\ \mathcal{E}_{\varepsilon n} &= (\lambda_{n-1}+1)\mathcal{E}_{\varepsilon(n-1)} \\ \mathcal{E}_{\mu n}(0, t) &= (\lambda_{n-1}+1)\mathcal{E}_{\mu(n-1)}(0, t) \\ &\quad + \Big((\lambda_{n-1}+1)\mathcal{M}_{n-1} + 1\Big)\left[\sup_{0\le\tau\le t} \mu_{n1}(\tau)\right] \end{aligned} \quad (43)$$

From lemmas 2, 4 and 6, we have the ultimate error bound as

$$\begin{aligned} |\boldsymbol{S}(t)| \le &\sum_{i=1}^{n} \alpha_{\Delta i}M_{\Delta i} + \sum_{i=1}^{n} \alpha_{\varepsilon_g i}\ \varepsilon_{gi} + \sum_{i=1}^{n} \alpha_{\varepsilon_f i}\ \varepsilon_{fi} \\ &+ \sum_{i=1}^{n-1} \alpha_{\epsilon i}\sqrt{\epsilon} \qquad \text{as} \quad t \ge {}^{\exists}T_u > 0 \end{aligned} \quad (44)$$

where ${}^{\exists}\alpha_{\varepsilon_f i}, {}^{\exists}\alpha_{\varepsilon_g i}{}^{\exists}\alpha_{\Delta i}, {}^{\exists}\alpha_{\epsilon i} > 0$. Notice that these constants can be made small by the control gains $\kappa_{i1}, \kappa_{i2}, \kappa_{i3}, \kappa_{i4}$ and c_i.

Finally, the theoretical results are summarized in the following theorem (detailed analysis is omitted due to the limit of paper length):

Theorem 1. Let the assumptions and results of lemmas 1∼6 hold. If the reference trajectory, the initial error signals and the design parameters are chosen appropriately, the following results hold.

(1) There exists a compact set $\mathcal{D}_x \subset \Omega_x$ of the initial states such that $\overline{x}_n \in \Omega_x$ for all $\overline{x}_n(0) \in \mathcal{D}_x$.
(2) The overall error system is ISpS as characterized in (40).
(3) The ultimate bound of $|\boldsymbol{S}(t)|$ can be made sufficiently small as shown in (44).

Remark: Our practical purpose however, is to let the output $y = x_1$ track the reference trajectory y_r. Therefore, it is not necessary to make all of the error signals very small by paying great efforts of damping control and network adaption. Therefore we can set weak control gains and discard adaptive networks for the subsystems with large index i. The trajectory tracking task can be achieved by adopting relatively strong control gains and adaptive networks for the first subsystem (17). See the results of lemmas 1 and 2. This policy is called "partial adaption".

5. SIMULATIONAL EXAMPLES AND CONCLUSIONS

Consider the following nonlinear system:

$$\begin{aligned} \dot{x}_1 &= f_1(\overline{x}_1) + g_1(\overline{x}_1)x_2 + \Delta_1(\overline{x}_3, t) \\ \dot{x}_2 &= f_2(\overline{x}_2) + g_2(\overline{x}_2)x_3 + \Delta_2(\overline{x}_3, t) \\ \dot{x}_3 &= f_3(\overline{x}_3) + g_3(\overline{x}_3)u + \Delta_3(\overline{x}_3, t) \end{aligned} \tag{45}$$

where

$$\begin{aligned} f_1(\overline{x}_1) &= 2x_1^2 \sin x_1 \\ f_2(\overline{x}_2) &= x_1^2 + x_1 x_2 + x_2 \cos x_1 \\ f_3(\overline{x}_3) &= x_1 x_3 + x_1^2 \sin x_3 + x_2^2 + x_3^2 + x_3 \sin x_2 \\ g_1(\overline{x}_1) &= 1 + 0.2x_1^2 \\ g_2(\overline{x}_2) &= 2 + (x_2 \cos x_1)^2 \\ g_3(\overline{x}_3) &= 3 + 2e^{\sin(x_1+x_2+x_3)} + \frac{x_1x_2 + x_2x_3 + x_1x_3}{{x_1}^2 + {x_2}^2 + {x_3}^2 + 1} \\ \Delta_1(\overline{x}_3, t) &= 0.3\left(x_1^2\right)^{1/3} \sin x_2 \\ \Delta_2(\overline{x}_3, t) &= 0.3x_1 x_2 \sin x_3 \\ \Delta_3(\overline{x}_3, t) &= 0.3x_3 \sin(10t) \end{aligned} \tag{46}$$

The *a priori* known nominal functions are:

$$\begin{aligned} &f_{01}(\overline{x}_1) = 0,\ f_{02}(\overline{x}_2) = 0,\ f_{03}(\overline{x}_3) = 0 \\ &g_{01}(\overline{x}_1) = 1,\ g_{02}(\overline{x}_2) = 2,\ g_{03}(\overline{x}_3) = 3 \end{aligned} \tag{47}$$

The following known smooth functions are used for nonlinear damping terms:

$$\begin{aligned} &d_{\Delta_1}(\overline{x}_1) = x_1, && d_{\Delta_{f1}}(\overline{x}_1) = x_1^2 \\ &d_{\Delta_2}(\overline{x}_2) = x_1 x_2, && d_{\Delta_{f2}}(\overline{x}_2) = x_1^2 + x_2^2 \\ &d_{\Delta_3}(\overline{x}_3) = x_3, && d_{\Delta_{f3}}(\overline{x}_3) = x_1^2 + x_2^2 + x_3^2 \end{aligned} \tag{48}$$

Design parameters are given as follows.

$$\begin{aligned} &c_1 = c_2 = c_3 = 5,\ \tau_2 = 0.015,\ \tau_3 = 0.01 \\ &\kappa_{11} = \kappa_{14} = 4,\ \kappa_{12} = \kappa_{13} = 2 \\ &\kappa_{21} = \kappa_{22} = \kappa_{23} = 1,\ \kappa_{14} = 2 \\ &\kappa_{31} = \kappa_{32} = \kappa_{33} = 1 \end{aligned} \tag{49}$$

Two controllers are implemented. The first one is a fixed robust controller without parameter adaption, whose results are shown in figure 1. It can be verified that all the internal signals are bounded.

The second one is a partially adaptive robust controller where only the first subsystem's nonlinear functions are updated adaptively. The nonlinear functions are approximated by RBF netoworks where the numbers of the basis functions are chosen as $M_{f1} = M_{g1} = 15$. The adaptive gains are chosen as $\gamma_{a1} = 80$ $\gamma_{b1} = 2$. It can be found in figure 2 that the tracking error S_1 is significantly reduced with satisfactory transient behaviour.

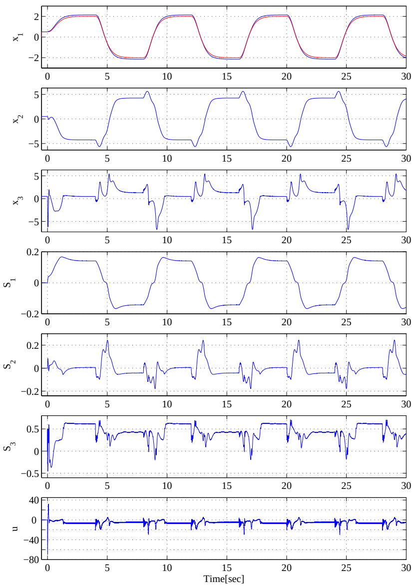

Fig. 1. Results of a fixed robust controller without parameter adaption.

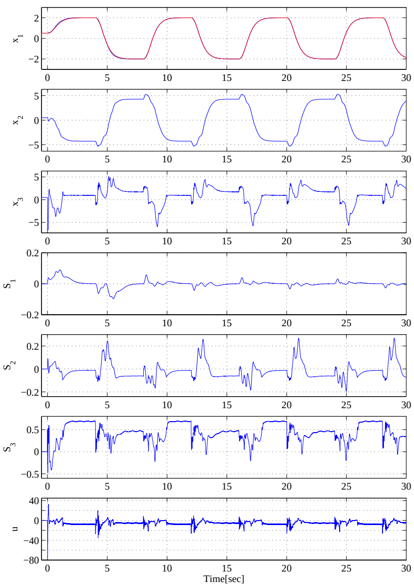

Fig. 2. Results of a partially adaptive robust controller where only the first subsystem's nonlinear functions are updated adaptively.

REFERENCES

Jiang, Z. P. & Praly, L. (1998). Design of robust adaptive controllers for nonlinear systems with dynamic uncertainties. *Automatica, 34*, 825-840.

Krstic, M., Kanellakopoulos, L. & Kokotovic, P. (1995). *Nonlinear and adaptive control Design*, John Wiley & Sons.

Swaroop, D. , Hedrick, J. K., Yip, P. P. & Gerdes, J. C. (2000). Dynamic surface control for a class of nonlinear systems. *IEEE Trans. Automatic Control, 45*, 1893-1899.

Wang, D. & Huang, J. (2001). Neural network based adaptive dynamic surface control for nonlinear systems in strick-feedback form. *40th IEEE Conference on Decision and Control, Orlando*

Yao, B. & Tomizuka, M. (1997). Adaptive robust control of SISO nonlinear systems in a semi-strict feedback form. *Automatica, 33*, 893-900.

Yip, P. P. & Hedrick, J. K. (1998). Adaptive dynami surface control: a simplified algorithm for adsaptive backstepping control of nonlinear systems. *International Journal of Control, 71*, 959-979.