OPTIMAL DECENTRALIZED CONTROL OF DYNAMICAL SYSTEMS UNDER UNCERTAINTIES

N.M. Dmitruk [*,1] **O.V. Ridchenko** [**]

[*] *Institute of Mathematics, National Academy of Science of Belarus*
[**] *Belorussian State University*

Abstract: In this paper an optimal control problem for a group of interconnected linear time-varying systems under bounded disturbances and incomplete information about its states is considered. A constructive method of on-line decentralized control, when control functions are distributed between subsystems so that they conduct self-control with some view on actions of the others, is presented.

Keywords: optimal control, decentralized control, large-scale systems, uncertainty, feedback control methods.

1. INTRODUCTION

Numerical methods for solving optimal control problems, in the theory, allow the dimensions of state and control variables to be arbitrary. But when implementing these methods for solving real problems system order may play a crucial role. High dimension is a main characteristics of large-scale systems and is a basic obstacle for application of common numerical methods for solution of such problems.

For large-scale systems decentralized control is often the only method of control since employment of a single centralized controller may be economically or computationally unfeasible or even result in the "curse of dimensionality". Many significant contributions has been made to the development of decentralized control theory for large-scale dynamic systems since the 70s (see, (Sandell *et al.*, 1978; Siljak, 1991; Lunze, 1992) and references therein). Recently, main attention has been devoted to the stabilization of linear and nonlinear large-scale systems by both state and output feedback controls (see, e.g. Yan *et al.*, 1998, 2003; Labibi *et al.*, 2002).

In this paper the linear time-varying system model is in input-output decentralized form (Lunze, 1992), i.e. the interconnections between subsystems are only due to the self dynamics of the group. Each subsystem operates under parametric bounded disturbances and its states are not fully available, instead some linear combinations of them are measured with bounded errors. All subsystems have a collective goal to steer the group to a target set in fixed time providing maximum value to a linear performance index. The aim of the paper is to describe a method of on-line decentralized feedback construction implemented for each subsystem via an estimator processing the measurements of subsystem states and a controller generating current control functions for each sampling period. Interconnections between subsystems allow the delayed exchange of information about the optimal behaviour selected in the past.

The paper follows the constructive approach in (Gabasov *et al.*, 2000a,b; 2002; Balashevich *et al.*, 2002) for optimal control and observation of linear and nonlinear dynamical systems in real-time and develops the results on decentralized control of large-scale determined system with fully available states studied in (Gabasov *et al.*, 2005).

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2. DYNAMICAL SYSTEMS UNDER UNCERTAINTIES

Consider a group of q control subsystems, where the i-th subsystem on the interval $T = [t_*, t^*]$ is modeled by a linear differential equation of the form

$$\dot{x}_i = A_i(t)x_i + \sum_{j=1, j\neq i}^{q} A_{ij}(t)x_j + B_i(t)u_i + M_i(t)w_i. \qquad (1)$$

Here $A_{ij}(t) \in R^{n_i \times n_j}$, $(A_{ii}(t) = A_i(t))$, $B_i(t) \in R^{n_i \times r_i}, M_i(t) \in R^{n_i \times p_i}, t \in T$, $i,j \in I = \{1, 2, \ldots, q\}$, are piecewise continuous functions, $x_i = x_i(t) \in R^{n_i}$ is a state of the i-th subsystem at the instant t; $u_i = u_i(t) \in U_i \subset R^{r_i}$ is a value of a discrete control of the i-th subsystem: $u_i(t) \equiv u_i(\tau), t \in [\tau, \tau + h[, \tau \in T_h = \{t_*, t_* + h, \ldots, t^* - h\}$, $(h = (t^* - t_*)/N$ is a sampling period, $N > 0)$, $U_i = \{u_i \in R^{r_i} : \; u_{*i} \leq u_i \leq u_i^*\}$ is a bounded set of admissible values of the i-th control; $w_i = w_i(t) \in R^{p_i}$, $t \in T$, if a finite parametric disturbance: $w_i(t) = \Lambda_i(t)v_i$, where $\Lambda_i(t) \in R^{p_i \times l_i}, t \in T$, is known piecewise continuous function, $v_i \in R^{l_i}$ is an unknown parameter of the disturbance with values from a bounded set $V_i = \{v_i \in R^{l_i} : w_{*i} \leq v_i \leq w_i^*\}$.

Let the initial states $x_i(t_*) \in R^{n_i}$, $i \in I$, of the subsystems be unknown and represented in the form

$$x_i(t_*) = x_{0i} + G_i z_i,$$

where $x_{0i} \in R^{n_i}$, $G_i \in R^{n_i \times k_i}$ are known vector and matrix; $z_i \in Z_i \subset R^{k_i}$ is an unknown parameter from a bounded set $Z_i = \{z_i \in R^{k_i} : d_{*i} \leq z_i \leq d_i^*\}$.

Sets Z_i, V_i, $\Gamma_i = \{\gamma_i = (z_i, v_i) : z_i \in Z_i, v_i \in V_i\}$, $i \in I$; $\Gamma = \{\gamma = (\gamma_i, i \in I) : \gamma_i \in \Gamma_i, i \in I\}$ are called *a priory* distributions of parameters of the initial state, the disturbance, i-th subsystem (1) and the group, respectively.

Suppose that in the course of the control process the exact states of the subsystems are not available, the information about them is given by signals

$$y_i(\theta) = C_i(\theta)x_i(\theta) + \xi_i(\theta), \;\; \theta \in T_h, \qquad (2)$$

of the measurer. Here $C_i(\theta) \in R^{m_i \times n_i}$, are values of a piecewise continuous function at discrete instants $\theta \in T_h$; $\xi_i(\theta) \in R^{m_i}$, $\theta \in T_h$, are errors of the measurer satisfying the inequalities $\xi_{*i} \leq \xi_i(\theta) \leq \xi_i^*, \theta \in T_h; \quad 0 < \|\xi_i^* - \xi_{*i}\|$.

Assume that a centralized control of the group, when one controller generates all control functions $u_i(t)$, $t \in T$, $i \in I$, as described in (Gabasov *et al.*, 2000a), is impossible due to some reasons, e.g. high expenses. The purpose of this paper is to present a decentralized control scheme, when every i-th subsystem at every instant $\tau \in T_h$ calculates its own control, based on signals of the measurer and some information about the behaviour of other subsystems. In this paper it is supposed that interconnections between the subsystems allow the exchange of prospective behaviour constructed at the previous instant $\tau - h$.

3. CLASSICAL FEEDBACK CONTROL PRINCIPLE

Following the classical closed-loop control principle, discrete decentralized control of the group is performed as follows. At the instant $t = t_*$ the controller of the i-th subsystem receives the output signal $y_i(t_*)$ from its measurer, and control signals $u_j(t_* - 0), j \in I_i = I \setminus i$, from the other controllers about their supposed behaviour, formed before the control process starts. Using this information, the i-th controller chooses value $u_i(t_*) = u_i(t_*, y_i(t_*); u_j(t_* - 0), j \in I_i) \in U_i$ according to some prescribed in advance rules, and feeds the control function $u_i(t) = u_i(t_*), t \in [t_*, t_* + h[$, to the input of its subsystems. Under these controls and disturbances $w_i(t)$, $t \in [t_*, t_* + h[$, $i \in I$, the group at the instant $t_* + 2h$ is driven into a new state $x(t_* + h) = (x_i(t_* + h), i \in I)$, and measurers (2), $i \in I$, calculate signals $y_i(t_* + h), i \in I$. At an arbitrary instant $\tau \in T_h$ the i-th controller basing on output signals $y_i(\tau)$ and control signals $u_j(\tau - h)$, $j \in I_i$, chooses $u_i(\tau) = u_i(\tau, y_{i\tau}(\cdot); u_j(\tau - h), j \in I_i) \in U_i$, where $y_{i\tau}(\cdot) = (y_i(\theta), \theta \in T_h(\tau)), T_h(\tau) = \{t_*, \ldots, \tau\}$. The control function $u_i(t) = u_i(\tau)$, $t \in [\tau, \tau + h[$, is fed to the input of the i-th subsystem.

As a result of described operations functions $u_i(\tau) = u_i(\tau, y_{i\tau}(\cdot); u_j(\tau - h), j \in I_i)$, $\tau \in T_h$, $i \in I$, depending on signals $y_{i\tau}(\cdot)$ are obtained.

Let $Y_i(\tau)$ be a totality of all signals $y_{i\tau}(\cdot)$ of the i-th measurer (2), which can be obtained by τ.

Definition. A function

$$u_i = u_i(\tau, y_{i\tau}(\cdot); u_j(\tau - h), j \in I_i), \qquad (3)$$
$$y_{i\tau}(\cdot) \in Y_i(\tau), u_j(\tau - h) \in U_j, \; j \in I_i, \; \tau \in T_h,$$

is said to be a (discrete) decentralized feedback of the i-th subsystem, a totality $u = (u_i, i \in I)$ is called a decentralized feedback of the group.

Let $X^* = \{(x_i \in R^{n_i}, i \in I) : g_* \leq \sum_{i \in I} H_i x_i \leq g^*\}$ be a prescribed target set with $H_i \in R^{m \times n_i}$, $g_* < g^*$; and $X^*(u)$ be a set of all terminal states $x(t^*) = (x_i(t^*), i \in I)$ of the group closed by feedback u, which are consistent with signals $y_{it^*}(\cdot)$, $i \in I$.

Definition. A decentralized feedback of the group is said to be admissible if $X^*(u) \subset X^*$.

Let the quality of an admissible decentralized feedback of the group be described by a functional

$$J(u) = \min_{X^*(u)} \sum_{i \in I} c_i' x_i,$$

where $c_i \in R^{n_i}$, $i \in I$, are given vectors.

Definition. An admissible decentralized feedback u^0 is said to be optimal if $J(u^0) = \max_u J(u)$, where maximum is calculated over all admissible feedbacks (3), $i \in I$.

The optimal decentralized feedback provides a guaranteeng result, being the best control under the worse conditions.

According to the definitions the classical closed-loop control principle implies the calculation of the optimal decentralized feedback before the control process starts. If it is constructed, the group under disturbances can be controlled in real-time with no additional calculation costs. Unfortunately, the synthesis problem for subsystems under uncertainty is intractable if not unsolvable even for low dimensions. Therefore the idea of construction the optimal decentralized feedback in classical sense as described in this section is abandoned and real-time control is employed. The real-time control means that the control function $u(t)$, $t \in [\tau, \tau + h[$, is calculated in the course of the control process for each sampling instant $\tau \in T_h$ after information about the realized signal $y(\tau)$ becomes available. Describing on-line optimal feedback construction we follow the ideas of (Gabasov *et al.*, 1995), where centralized controls are considered. On-line calculation of current control actions is also a core of the popular model predictive control methodology (Mayne *et al.*, 2000).

4. ON-LINE DECENTRALIZED FEEDBACK CONTROL PRINCIPLE

Before the control process starts an optimal open-loop control $u^0(t) = (u_i^0(t), i \in I)$, $t \in T$, for the group is constructed (see below the problem which is to be solved). Information $u_j^0(\cdot) = (u_j^0(t), t \in T)$, $j \in I$, Q_b^i is transmitted to the i-th subsystem.

For an arbitrary subsystem $i \in I$ describe the concrete control process, where parameters z^*, v^* and errors $\xi^*(\theta)$, $\theta \in T_h$, have realized. At the start of the process the optimal open-loop control $u_i^0(t)$, $t \geq t_*$, is fed to the input of the i-th subsystem and it obtains from its measurer the signal $y_i^*(t_*)$. Using this signal and previously received information $u_j^0(\cdot), j \in I_i$, the controller of the i-th subsystem calculates the optimal open-loop control $u_i^d(t|t_*, y_i^*(t_*), u_j^0(\cdot), j \in I_i)$, $t \in [t_*, t^*]$ (see below the problem which is to be solved). Both optimal open-loop control and optimal support is communicated to the other subsystems of the group and the control function fed to the input of the i-th subsystem is switched to $u_i^*(t) = u_i^d(t|t_*, y_i^*(t_*), u_j^0(\cdot), j \in I_i)$, $t \geq t_* + s_i(t_*)$, where $s_i(t_*) < h$ is time spent to calculate the optimal open-loop control.

Let the control process is carried out till the instant $\tau \in T_h$, which means that: 1) the control $u_i^*(t)$, $t \in [t_*, \tau[$, has been fed to the input of the i-th subsystem before τ, 2) the i-th controller has received the optimal open-loop controls $u_j^{d\tau-h}(\cdot) = (u_j^d(t|\tau - h, y_{j\tau-h}^*(\cdot), u_k^{d\tau-2h}(\cdot), k \in I_j), t \in [\tau - h, t^*]), j \in I_i$, of other subsystems, constructed at the previous moment $\tau - h$; 3) it also knows its own optimal open-loop control $u_i^d(t|\tau-h, y_{i\tau-h}^*(\cdot), u_j^{d\tau-2h}(\cdot), j \in I_i)$, $t \in [\tau-h, t^*]$ from the previous moment; 4) at the instant τ the measured signal $y_i^*(\tau)$ is obtained.

Using information 1)-4) the optimal open-loop control $u_i^d(t|\tau, y_{i\tau}^*(\cdot), u_j^{d\tau-h}(\cdot), j \in I_j)$, $t \in [\tau, t^*]$, for a current moment τ are to be constructed. The following analysis is empoyed to elaborate the algorithm for constructing the functions $u_i^d(t|\tau, y_{i\tau}^*(\cdot), u_j^{d\tau-h}(\cdot), j \in I_j)$, $t \in [\tau, t^*]$.

Firstly, note that the signal $y_i^*(\tau)$ can be represented in the form $y_i^*(\tau) = C_i(\tau)(x_i^*(\tau) + x_{0i}(\tau)) + \xi_i(\tau)$, where $x_0(t) = (x_{0i}(t), i \in I)$, $t \in [t_*, \tau]$, is a trajectory of a determined control group

$$\dot{x}_i = A_i(t)x_i + B_i(t)u_i + \sum_{j \in I_i} A_{ij}(t)x_j, \quad (4)$$
$$x_i(t_*) = x_{i0}, \;\; u_i(t) \equiv u_i^*(t), t \in [t_*, \tau[, i \in I;$$

and $x^*(t) = (x_i^*(t), i \in I)$, $t \in [t_*, \tau]$, is a trajectory of a nondeterministic group without control

$$\dot{x}_i = A_i(t)x_i + \sum_{j \in I_i} A_{ij}(t)x_j + M_i(t)w_i, \quad (5)$$
$$x_i(t_*) = G_i z_i, \;\; w_i(t) = \Lambda_i(t)v_i, t \in [t_*, \tau], i \in I,$$

for the parameters $z_i = z_i^*$, $v_i = v_i^*$, $i \in I$.

Using known control functions $u_j^*(t)$, $t \in [\tau - h, \tau]$, $j \in I_i$, calculate a state $x_{0i}(\tau)$ and subtract this known term from the signal $y_i^*(\tau)$:

$$y_{i0}^*(\tau) = y_i^*(\tau) - C_i(\tau)x_{0i}(\tau).$$

Since this operation has been performed at every moment $t_*, t_* + h, \ldots, \tau - h$, by the instant τ the signals $y_{i0\tau}^*(\cdot) = (y_{i0}^*(t), t \in T_h(\tau))$ are at hand. These signals coincide with the output measurer (2) would have produce for group (5) in the observation process with realized parameters $z_i = z_i^*$, $v_i = v_i^*$, $i \in I$.

Secondly, introduce sets $\hat{\Gamma}(\tau)$ and $\hat{\Gamma}^i(\tau)$.

Definition. A set $\hat{\Gamma}(\tau)$ is called a current distribution of the group parameter $\gamma = (\gamma_i, i \in I)$, $i \in I$. It consists of vectors $\gamma \in \Gamma$, consistent with the signal $y^*_{0\tau}(\cdot) = (y^*_{i0\tau}(\cdot), i \in I)$, i.e. such that there exist an initial state $x_i(t_*) = G_i z_i, i \in I$, $z_i \in Z_i$, and a disturbance $w_i(t) = \Lambda_i(t)v_i$, $t \in [t_*, \tau[, i \in I$, $v_i \in V_i$, together with possible measurement errors $\xi_i(t)$, $t \in T_h(\tau)$, $i \in I$, able to generate the signal $y^*_{0\tau}(\cdot) = (y^*_{i0\tau}(\cdot), i \in I)$.

Definition. A set $\hat{\Gamma}^i(\tau)$ is said to be the i-th approximation of $\hat{\Gamma}(\tau)$. It consists of vectors $\gamma \in \Gamma$, able to produce the signal $y^*_{i0\tau}(\cdot)$.

Obviously, $\hat{\Gamma}(\tau) = \bigcap_{i \in I} \hat{\Gamma}^i(\tau)$.

Let $\hat{X}^{*i}(\tau)$ be the i-th approximation of the current distrubution of the terminal state of the group, which is a set of all terminal states of group (5) with parameters from $\hat{\Gamma}^i(\tau)$.

Thirdly, in what follows estimates of the set $\hat{X}^{*i}(\tau)$ are used. These estimates are the projections of $\hat{X}^{*i}(\tau)$ on the system of directions given by the rows $h_{(k)}$, $k \in K = \{1, 2, \ldots, m\}$ of the matrix $H = (H_i, i \in I)$:

$$\alpha^i_k(\tau) = \max h'_{(k)}x,\ x \in \hat{X}^{*i}(\tau),\ k \in K, \quad (6)$$
$$\beta^i_k(\tau) = \min h'_{(k)}x,\ x \in \hat{X}^{*i}(\tau),\ k \in K.$$

Problems (6) are called current optimal observation problems of the i-th subsystem.

Suppose that by the instant τ the i-th subsystem obtained the estimates $\alpha^j_k(\tau - h), \beta^j_k(\tau - h), k \in K$, $j \in I_i$, calculated by all other subsystems for the previous instant $\tau - h$. Values $\bar{\alpha}^i(\tau) = (\bar{\alpha}^i_k(\tau), k = \overline{1, m}), \bar{\beta}^i(\tau) = (\bar{\beta}^i_k(\tau), k \in K)$:

$$\bar{\alpha}^i_k(\tau) = \min\{\alpha^i_k(\tau), \alpha^j_k(\tau - h), j \in I_i\},\ k \in K,$$

$$\bar{\beta}^i_k(\tau) = \max\{\beta^i_k(\tau), \beta^j_k(\tau - h), j \in I_i\},\ k \in K,$$

are said to be the i-th coherent estimates of the distribution $\hat{X}^*(\tau) = \bigcap_{i \in I} \hat{X}^{*i}(\tau)$ of the terminal state of the group.

Finally, the optimal current program is defined.

Definition. A discrete control $u_i(t)$, $t \in [\tau, t^*]$, is said to be a current open-loop control (current program) if at the instant t^* it steers the group

$$\dot{x}_i = A_i(t)x_i + \sum_{j \in I_i} A_{ij}(t)x_j + B_i(t)u_i,$$
$$x_i(\tau) = x_{0i}(\tau),$$
$$\dot{x}_j = A_j(t)x_j + \sum_{k \in I_i} A_{jk}(t)x_k$$
$$+B_j(t)u^d_j(t|\tau - h, y^*_{i\tau}(\cdot), u^{d\tau-2h}_k(\cdot), k \in I_j),$$
$$x_j(\tau) = x_{0j}(\tau),\ j \in I_i,$$

to the set

$$\bar{X}^{*i}(\tau) = \{(x_j \in R^{n_j}, j \in I) : g_* - \bar{\beta}^i(\tau) \le \sum_{j \in I} H_j x_j \le \bar{g}^* - \bar{\alpha}^i(\tau)\}.$$

Quality of the current program $u_i(t)$, $t \in [\tau, t^*]$, is given by a value $\tilde{I}(u_i) = \min \sum_{j \in I} c'_j x_j(t^*)$, where minimum is calculated over all $\gamma \in \hat{\Gamma}^i(\tau)$.

Definition. An optimal current program $u^d_i(t|\tau, y^*_{i\tau}(\cdot), u^{d\tau-h}_j(\cdot), j \in I_i)$, $t \in [\tau, t^*]$, is defined by the equality $\tilde{I}(u^d_i) = \max \tilde{I}(u_i)$, where maximum is calculated over all current programs.

The optimal current program is a solution to the following problem

$$\sum_{j \in I} c'_j x_j(t^*) \to \max, \quad (7)$$
$$\dot{x}_i = A_i(t)x_i + \sum_{j \in I_i} A_{ij}(t)x_j + B_i(t)u_i,$$
$$x_i(\tau) = x_{0i}(\tau),$$
$$\dot{x}_j = A_j(t)x_j + \sum_{k \in I_i} A_{jk}(t)x_k$$
$$+B_j(t)u^d_j(t|\tau - h, y^*_{i\tau}(\cdot), u^{d\tau-2h}_k(\cdot), k \in I_j),$$
$$x_j(\tau) = x_{0j}(\tau),\ j \in I_i,$$
$$g_* - \bar{\beta}^i(\tau) \le \sum_{j \in J} H_j x_j(t^*) \le g^* - \bar{\alpha}^i(\tau),$$
$$u_i(t) \in U_i, t \in T_h(\tau),$$

which is called a current optimal control problem of the i-th subsystem.

Controls $u^d_i(t|\tau, y^*_{i\tau}(\cdot), u^{d\tau-h}_j(\cdot), j \in I_i)$, $t \in [\tau, t^*]$; $u^d_j(t|\tau - h, y^*_{i\tau-h}(\cdot), u^{d\tau-2h}_k(\cdot), k \in I_j)$, $t \in [\tau, t^*]$, $j \in I_i$, steer the group to the terminal set with guaranty (for all possible parameters) and delivers a maximum value to the guaranteeing cost $\tilde{I}(u_i)$. Controls $u^d_i(t|\tau, y^*_{i\tau}(\cdot), u^{d\tau-h}_j(\cdot), j \in I_i)$, $t \in [\tau, t^*]$, $i \in I$, which are used in the control process do no sutisfy the same property. However, for small h they differ a little from the above guaranteeing controls and can be called suboptimal. The resulted difference in the trajectories generated by optimal and suboptimal current programs can be treated as an additional disturbance in the dynamical systems.

Let $s_i(\tau)$ be time needed to complete calculations of the optimal open-loop control $u^d_i(t|\tau, y^*_{i\tau}(\cdot), u^{d\tau-h}_j(\cdot), j \in I_i)$, $t \in [\tau, t^*]$. On the interval $[\tau, \tau + s_i(\tau)]$, since the current control function is not known yet, the previous optimal program $u^d_i(t|\tau - h, y^*_{i\tau-h}(\cdot), u^{d\tau-2h}_j(\cdot), j \in I_i)$, $t \in [\tau, \tau + s_i(\tau)]$, is fed into the input of the i-th subsystem. Starting with instant $\tau + s_i(\tau)$ a control function is $u^*_i(t) = u^d_i(t|\tau, y^*_{i\tau}(\cdot), u_j(\cdot), j \in I_i)$, $t \ge \tau + s_i(\tau)$. The presence of the delays also results in suboptimality of the constructed current programs. As above they can be considered as disturbances.

At the beginning of this section optimal open-loop control $u^0(t) = (u_i^0(t), i \in I)$, $t \in T$, was introduced. To construct them one has to solve an optimal centralized control problem (time expenses are not essential as this procedure is performed before the real control process starts)

$$\sum_{i\in I} c_i' x_i(t^*) \to \max, \tag{8}$$
$$\dot{x}_i = A_i(t)x_i + \sum_{j\in I_i} A_{ij}(t)x_j + B_i(t)u_i,$$
$$x_i(t_*) = x_{0i}, i \in I,$$
$$g_* - \beta(t_*) \le \sum_{i\in J} H_i x_i(t^*) \le g^* - \alpha(t_*),$$
$$u_i(t) \in U_i, t \in T, i \in I,$$

where $\alpha(t_*) = (\alpha_k(t_*), k \in K)$, $\beta(t_*) = (\beta_k(t_*), k \in K)$;

$$\alpha_k(t_*) = \max h'_{(k)}x,\ x \in X^*(t_*),\ k \in K, \tag{9}$$
$$\beta_k(t_*) = \min h'_{(k)}x,\ x \in X^*(t_*),\ k \in K,$$

and $X^*(t_*)$ is a set of all terminal states of (5), $i \in I$, corresponding to $z_i \in Z_i$, $v_i \in V_i$, $i \in I$.

Thus, the algorithm for calculating the optimal current control $u_i^*(t)$, $t \ge \tau$ at any instant $\tau \in T_h$ is the following: 1) solve $2m$ optimal observation problems (6), obtaining the estimates $\alpha_k^i(\tau), \beta_k^i(\tau), k \in K$; 2) find the coherent estimates $\bar{\alpha}^i(\tau)$, $\bar{\beta}^i(\tau)$; 3) solve optimal control problem (7).

Definition. A device solving an optimal observation problem from (6) and calculating the estimates $\bar{\alpha}^i(\tau), \bar{\beta}^i(\tau)$, is called an optimal estimator; a device solving optimal control problem (7) is called an optimal controller.

Let $s_i^e(\tau)$ be time needed by $2m$ estimators working in parallel to obtain the estimates $\bar{\alpha}^i(\tau), \bar{\beta}^i(\tau)$; and $s_i^r(\tau)$ be time required for the optimal controller to solve the current optimal control problem. If $s_i(\tau) = s_i^e(\tau) + s_i^r(\tau) < h$, $i \in I$, then the optimal estimators and the optimal controller are said to perform real-time optimal decentralized control of the group under uncertainties.

Any common method for solving extremal problems (6–9) may be used. For the inequalities $s_i(\tau) < h$, $i \in I$, to hold, problems (6), (7) at the moment $\tau \in T_h$ are suggested to be solved by dual methods from (Gabasov *et al.*, 2000b; 2002) These methods were elaborated for dynamical problems such as optimal observation and optimal control problems. The peculiarities of these problem originated from their dynamic nature are taken into account to justify algorithms for on-line applications. Their ability to solve problems (6), (7) at the instant $\tau \in T_h$ quickly is due to the fact that switching instants of the optimal open-loop control $u_i^d(t|\tau - h, y^*_{i\tau-h}(\cdot), u_j(\cdot), j \in I_i)$, $t \in [\tau - h, t^*]$, differ just a little from those of $u_i^d(t|\tau, y^*_{i\tau}(\cdot), u_j(\cdot), j \in I_i)$, $t \in [\tau, t^*]$. It is reasonable to take solution of problems (6), (7) for the instant $\tau - h$ as initial approximation for the solution of (6), (7) for the instant τ.

Papers (Gabasov *et al.*, 2000b; 2002) operate with a special structure called a support. Roughly speaking a support is a totality of some switching points of the control ensuring some general constraints of the problem are active. The rest of the switching instants is recovered from the elements accompanying the support. In this terms, the initial supports for problems (6), (7) at the instant $\tau \in T_h$ are the optimal supports of the corresponding previous problems. Note, that optimal support may be communicated between the subsystems instead of the current programs.

5. NUMERICAL EXAMPLE

On the interval $T = [0, 7]$ consider the system

$$\dot{x}_1 = x_3,\quad \dot{x}_2 = x_4 \tag{10}$$
$$\dot{x}_3 = -11x_1 + x_2 + u_1 + w_1,$$
$$\dot{x}_4 = 0.25x_1 - 10.25x_2 + u_2 + w_2,$$

with $x_1(0) = 0.2$, $x_2(0) = -0.2$ and unknown $x_3(0) = z_1$, $x_4(0) = z_2$: $(z_1, z_2) \in Z = \{z \in Z : -0.4 \le z_1 \le 0, 0 \le z_2 \le 0.1\}$, and disturbances of the form $w_1(t) = v_1 \sin(3t)$, $w_2(t) = v_2 \cos(5t)/4$, $t \in T$: $(v_1, v_2) \in V = \{v \in R^2 : |v_i| \le 0.04, i = 1, 2\}$.

Let the measurer at moments $t \in T_h = \{0, h, \dots, 7 - h\}$, $h = 0.07$, returns values

$$y_1 = x_1 + \xi_1,\ y_2 = x_2 + \xi_2,$$

where $\xi_i = \xi_i(t)$, $|\xi_i(t)| \le 0.05$, $t \in T_h$, are bounded errors.

The aim of the control process is to steer system (10) at the moment $t^* = 7$ to the set $X^* = \{x \in R^4 : |x_i| \le 0.1, i = \overline{1,4}\}$; by bounded controls $0 \le u_i(t) \le 0.4$, $i = 1, 2$, $t \in T$; minimizing the functional $J(u) = \int_0^7 (u_1(t) + u_2(t))dt$.

Let in a concrete control process the following values has realized:

$$z_1^* = -0.3;\ z_2^* = 0.07;\ v_1^* = 0.02; v_2^* = -0.02;$$
$$\xi_1^*(t) = 0.03\sin(5t),\ \xi_2^*(t) = 0.04\cos(7t), t \in T_h.$$

Both centralized and decentralized feedbacks has been constructed for the problem in question, where two subsystems has been considered: the first subsystem is

$$\dot{x}_1 = x_3,\ \dot{x}_3 = -11x_1 + x_2 + u_1 + w_1,$$

with a measurer $y_1 = x_1 + \xi_1$; the second is

$$\dot{x}_2 = x_4, \quad \dot{x}_4 = 0.25x_1 - 10.25x_2 + u_2 + w_2,$$

with $y_2 = x_2 + \xi_2$.

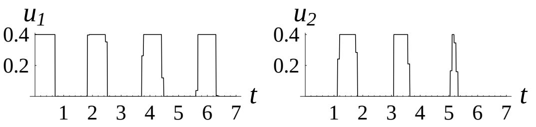

Fig. 1. Optimal centralized feedbacks.

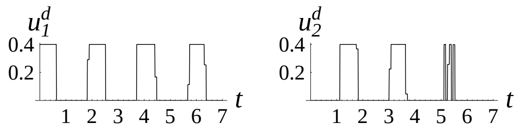

Fig. 2. Optimal decentralized feedbacks.

Centralized feedbacks are presented in Fugure 1; decentralized in Figure 2. Figure 3 shows the projections on the phase planes x_1x_3 and x_2x_4 of the optimal trajectories under optimal centralized (dash curve) and decentralized feedbacks (solid curve). The value of the cost function during centralized control turned out to be equal to 1.62281; in decentralized — 1.66793.

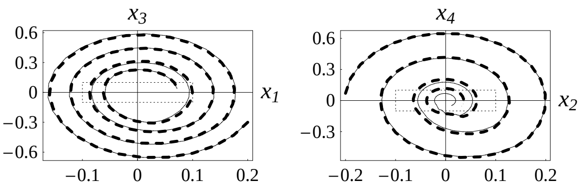

Fig. 3. The projections of the optimal trajectories.

Above the delay $s_i(\tau)$, $i = 1, 2$, $\tau \in T_h$, was not taken into account. The effect of the delays is was studied assuming $s_i(\tau) = h/2$, $i = 1, 2$, $\tau \in T_h$. For this case the value of the cost function increased. For centralized control it turned out to be equal to 1.63768; in decentralized — 1.68985.

CONCLUSION

In this paper, optimal decentralized control of a group of interconnected linear dynamical systems under uncertainties was investigated. A general scheme of optimal decentralized feedback control in real-time was described. This scheme allows hard interconnections between the subsystems in the group which resulted in the fact that only control dimensions were reduced when distributing the control functions among the controllers of subsystems. Authors believe that weak interconnections will also reduce the state dimensions in problems (7). This idea is subject to further research as is a development of the method for nonlinear large-scale problems.

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