TIME SERIES ANALYSIS FOR IRREGULARLY SAMPLED DATA

Piet M.T. Broersen

Department of Multi Scale Physics, Delft University of Technology

Abstract: Many spectral estimation methods for irregularly sampled data tend to be heavily biased at higher frequencies or fail to produce a spectrum that is positive for all frequencies. A time series spectral estimator is introduced that applies the principles of a new automatic equidistant missing data algorithm to unevenly spaced data. This time series estimator approximates the irregular data by a number of equidistantly resampled missing data sets, with a special nearest neighbor method. Slotted nearest neighbor resampling replaces a true observation time instant by the nearest equidistant resampling time point, but only if it is within half the slot width. A smaller slot reduces the bias. Therefore, multi shift slotted nearest neighbor uses a slot width that is a fraction of the resampling time, giving equidistant data sets with slightly different starting points, shifted over the slot width. Results can be accurate at frequencies much higher than the mean data rate.

Keywords: autoregressive model, nearest neighbor resampling, slotting, spectral estimation, time series analysis, uneven sampling, order selection, parametric model.

1. INTRODUCTION

Meteorological data, astronomical data or turbulence data obtained by Laser-Doppler anemometry are often irregularly sampled, due to the nature of the observation system. This has the advantage that the highest frequency that can be estimated is higher than half the mean data rate, which is the upper limit for equidistant observations. Many estimation techniques exist for unevenly spaced data. Bos, *et al*. (2002) describe several variations of slotting or resampling methods and the Fourier estimator of Lomb-Scargle. Lahalle, *et al*. (2004) estimate continuous time ARMA models, requiring the explicit use of a model for the irregular sampling instants.

Benedict, *et al*. (2000) give an extensive survey of techniques. Slotting methods estimate an equidistant autocovariance function from the irregularly sampled data. Slotting algorithms have been refined with normalization and fuzzy slotting. Local normalization reduces the variance of the estimated autocovariance function. Fuzzy slotting produces a smoother autocovariance function by distributing products over multiple time slots. Autocovariance functions as estimated by present slotting techniques are not guaranteed to be positive semi-definite. This results in spectra that can become negative at a percentage of the frequencies where the power is weak. Methods to improve the performance of slotting autocovariance methods require very large data sets. Also a priori knowledge about the spectral shape is necessary, or subjective experimental choices have to be made to produce successful spectral estimates.

Resampling techniques reconstruct a signal at equal time intervals. After resampling, the equidistant data can be analyzed using the periodogram or time series models. Spectral estimates at higher frequencies will be severely biased. Adrian and Yao (1987) described Sample and Hold reconstruction as low-pass filtering followed by adding noise. These effects can in theory be eliminated using the refined Sample and Hold estimator (see Benedict, *et al*., 2000). In practice, all spectral details smaller than the bias are lost. Nearest Neighbor resampling has similar characteristics. The spectra are strongly biased for frequencies higher than about 20 % of the mean data rate. The noise and filtering effects of equidistant resampling set limits to the achievable accuracy of resampling methods. This precludes the accurate estimation of spectra at higher frequencies where the resampling noise blurs details smaller than the bias and hides spectral slopes.

Bos, *et al*. (2002) introduced a new idea with time series analysis. Their estimator can be perceived as searching for uninterrupted sequences of data that are almost equidistant. The selected sequences of different lengths can be analyzed with an irregular version of the Burg (1967) algorithm for segments. A slotted nearest neighbor resampling with Burg uses an equidistant signal, with many empty places where no original observation fell within a slot. The bias of slotted nearest neighbor is very much smaller than the bias without slotting. The reason is that a single original irregular observation can never appear at multiple resampled time instants. A disadvantage of this slotted Burg resampling method is that very large data sets are required to obtain some uninterrupted equidistant sequences of sufficient length for the irregular Burg algorithm. It turned out that a non-linear maximum likelihood algorithm for missing data, developed by Jones (1980), could sometimes give a still better solution, also if much less data are available. Whereas the slotted Burg method of Bos, *et al*. (2002) required about 200000 observations, the quasi maximum likelihood solution sometimes converged already to an accurate spectral estimate with 2000 irregular observations.

The behavior of this quasi maximum likelihood method for irregular data has first been investigated for the simpler, related problem of equidistant observations with missing data. In that case, Jones (1980) described an efficient method to calculate the true likelihood. A survey of existing missing data methods and a robust version of the maximum likelihood algorithm for autoregressive models of missing data problems has been given by Broersen, *et al*. (2004a, 2004b). Broersen and Bos (2004) included different types of time series models and order selection in the algorithm. The performance of that robust and automatic maximum likelihood time series algorithm outperforms all other methods for equidistant missing data problems.

The variance of the spectra generally becomes smaller if more data are available. Often, the bias is independent of the sample size, like in Sample and Hold resampling. The variance is generally inversely proportional to the sample size. Therefore, most existing methods will converge to the biased spectral result if enough data are available. Most methods to diminish the bias are only successful if the variance is very small. The purpose of this paper is to develop a spectral estimator that can be used in small data sets and that has a small bias. The modifications required to apply the automatic algorithm for equidistant missing data to irregularly sampled data are given. First, irregular data are approximated by a number of shifted equidistant data sets. Time instants are fixed to an equidistant resampling grid where the original irregular sampling instant is not further away from the grid point than half a slot width. The choices of the grid time, the slot width as well as the automatic selection of the best model order and model type for the time series spectral model are discussed.

2. MULTI SHIFT SLOTTED NN RESAMPLING

The analysis of resampling methods shows that an important problem is the multiple use of a single irregular observation for more resampled data points. This immediately creates a bias term in the estimated covariance function, because the autocovariance $R(0)$ leaks to estimated non-zero autocovariance lags. The analysis of Adrian and Yao (1987) showed that the autocovariance function and the spectrum suffer from bias in Sample and Hold. Nearest Neighbor (NN) resampling has the same problems. The bias is caused by the shift of irregular time intervals to a fixed grid and by the multiple use of the same irregular observation. That gives a colored spectral estimate, even if the true irregular process would be white noise. It will be eliminated in *slotted* NN resampling.

The signal $x(t)$ is measured at N irregular time instants $t_1,\dots,t_N$. The average distance between the samples T_0 is given by $T_0 = (t_N - t_1)/(N-1) = 1/f_0$, with f_0 the mean data rate. The signal is resampled on a grid at kN equidistant time instants at grid distance $T_r = T_0 / k$ (for simplicity in notation, k or $1/k$ is limited to integer numbers). The resampled signal exists only for $t = nT_r$ with n integer. The spectrum can be calculated up to frequency $kf_0 / 2$. The usual Nearest Neighbor resampling substitutes at all grid points nT_r the closest irregular observation $x(t_i)$, with

$$|t_{i-1}-nT_r| > |t_i-nT_r| \quad ; \quad |t_{i+1}-nT_r| > |t_i-nT_r|. \qquad (1)$$

The uninterrupted resampled signal contains kN equidistant observations. For $k > 1$, that means that many of the original N irregular observations have to be used for more resampled observations.

Slotted Nearest Neighbor resampling accepts only a resampled observation at $t=nT_r$ if there is an irregular observation $x(t_i)$ with t_i within the time slot w

$$nT_r - 0.5w \ < \ t_i \ \leq \ nT_r + 0.5w. \qquad (2)$$

If there is more than one irregular observation within a slot, the one closest to nT_r is used for resampling; if there is no observation within the slot, the resampled signal at nT_r is left empty. For small T_r and w equal to T_r, the number N_0 of non-empty points nT_r becomes close to N because almost every irregular time point falls into another time slot. For larger T_r with $k < 1$, more irregular observations may fall within one slot and only the one closest to the grid point survives in the slotted NN resampled signal. Taking $w = T_r / M$, with integer M, gives disjunct intervals in (2) where some irregular times t_i are not within any slot of (2). Therefore, multi shift slotted NN resampling is introduced, where M different equidistant missing data signals are extracted from one irregular data set

$nT_r+mw–0.5w< t_i \leq nT_r+mw+0.5w, m=0,1,.,M-1.$ (3)

Now, all slots of width w are connected in time. The number of possible grid points is $N*M*T_0/ T_r$. Hence, the remaining fraction γ is approximately given by $1/Mk$. Experience with missing data problems of Broersen, *et al*. (2004b) shows that time series models can be easily estimated for $\gamma > 0.1$. It may become difficult if γ is less than 0.01, unless the number of observations is very large. This limits the useful range of the resampling frequency $1/T_r$ and the slot width w for a given number of observations.

3. TIME SERIES MODELS

Three different linear types of time series models can be distinguished: autoregressive or AR, moving average or MA and combined ARMA models. An ARMA(p,q) model can be written as (Priestley, 1981)

$$x_n + a_1 x_{n-1} + \cdots + a_p x_{n-p} = \varepsilon_n + b_1 \varepsilon_{n-1} + \cdots + b_q \varepsilon_{n-q} \ , \quad (4)$$

where ε_n is a purely random process of independent identically distributed stochastic variables with zero mean and variance σ_ε^2. It is purely AR for $q = 0$ and purely MA for $p = 0$. Assume that data represent a stationary stochastic process. The power spectral density $h(\omega)$ of an ARMA(p,q) model is completely determined by the parameters in (4) together with the variance σ_ε^2 and is given by:

$$h(\omega) = \frac{\sigma_\varepsilon^2}{2\pi} \frac{\left| B_q\left(e^{j\omega i}\right) \right|^2}{\left| A_p\left(e^{j\omega i}\right) \right|^2} = \frac{\sigma_\varepsilon^2}{2\pi} \frac{\left| 1 + \sum_{i=1}^{q} b_i \, e^{-j\omega i} \right|^2}{\left| 1 + \sum_{i=1}^{p} a_i \, e^{-j\omega i} \right|^2} . \quad (5)$$

The autocovariance $r_{AR}(k)$ of the AR part of (4) is found with the standard AR theory (Priestley, 1981). It simply follows that

$$\begin{aligned} r(k) &= E\{ x_n x_{n-k} \} \\ &= \sum_{m=-q}^{q} \left[r_{AR}\left(k+m\right) \sum_{i=0}^{q} b_i b_{i+|m|} \right] , \ \forall k . \end{aligned} \quad (6)$$

This result shows that the parameters of a time series model completely describe the power spectral density and the autocorrelation function of the data x_n in (4).

Furthermore, results of the theory for equidistant data show that transformations like (5) and (6) of efficiently estimated time series parameters represent efficient estimates of the power spectrum and the autocorrelation function; see Priestley (1981). The freely available program ARMAsel of Broersen (2002) computes hundreds of candidate time series models, AR, MA and ARMA, for uninterrupted equidistant data and automatically selects one single model type and model order. That selected model is used to compute the spectrum and the autocorrelation function of the data with (5) and (6), respectively. In simulations, the quality of the selected model is often close to the best achievable accuracy for the given number of observations.

A similar automatic, approximate maximum likelihood, ARMAsel-mis program has been developed for the equidistant missing data problem. The accuracy of the ARMAsel-mis spectra is better than the spectra obtained with many other methods from the literature, in simulations with missing data. Broersen, *et al*. (2004b) have given examples where the estimation of time series models in missing data problems was efficient, meaning that the accuracy of the resulting model approached the limit of the achievable accuracy. The experimental accuracy of spectra for irregular data will be studied here.

4. ARMASEL FOR IRREGULAR DATA

Input for the estimation are the M equidistant missing data sequences or segments obtained with the multi shift slotted nearest neighbor algorithm of (3). The segments are all derived from original observations in the same irregularly sampled time interval. In principle, the data in the different segments are correlated and not independent. However, the most influential parts of each segment are found at places where only few data are missing. Generally, those places are at different times for the various segments and the assumption that the segments are more or less independent is justified. However, the method will not be maximum likelihood, even not approximately because the true process is always continuous if the observations are irregular. Jones (1980) computes the likelihood of an equidistant missing data problem by relating observations to all previous observations that are present in one single segment. In the irregular case, the data are distributed over different segments if M is greater than 1. The 'likelihood' is computed separately for each segment and added afterwards in the minimization procedure. Therefore, not all contributions to the true likelihood are taken into account. Using all of the almost independent M segments, each with about N/M observations, gives a much better accuracy than using only one segment.

All elements for an automatic ARMAsel algorithm for irregular data can be copied from the algorithm that has been developed for missing data by Broersen, *et al.* (2004a, 2004b). Only the creation of M equidistant segments with (3) has to be added.

- The 'likelihood' for AR models is computed with the method of Jones (1980) or with ARfil (Broersen, *et al.*, 2004b), depending on γ.
- The tangent of $\pi/2$ times the AR reflection coefficients is used in the minimization to guarantee estimated reflection coefficients with absolute values less than 1.
- The starting values for the AR(p+1) model are the estimated reflection coefficients of the AR(p) model with an additional zero for order p+1.
- AR(p) order selection uses as criterion:
  GIC(p) = the 'likelihood' + αp,
  with $\alpha = 3$ for less than 25 % missing, $\alpha = 5$ for less than 25 % remaining and $\alpha = 4$ otherwise.

- The maximization of the likelihood of MA and ARMA models gives problems with MA starting values and with order selection. Those models are much better estimated from the parameters of an intermediate AR model, with a reduced statistics method; see Broersen and Bos (2004).
- The order of that intermediate AR model is chosen as the highest AR order with a spectrum close to the spectrum of the selected AR model.
- Order selection for MA and ARMA is based on the 'likelihood' plus three times the number of estimated parameters. The same criterion is used to determine the preferred model type for the irregular data; see Broersen and Bos (2004).
- The quantity γN can roughly be considered as the effective number of observations. The remaining fraction γ depends on the choice of the slot width and on the resampling period.

5. BIAS OF SLOTTED NN RESAMPLING

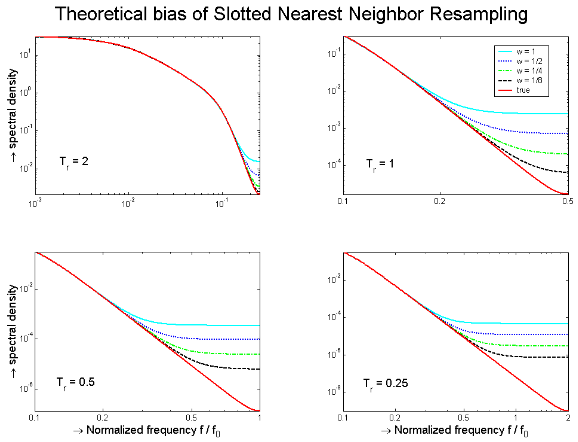

Fig.1. Theoretical expectation of the bias of slotted nearest neighbor resampling for a turbulence spectrum, for 4 resampling times T_r and four slot widths w. The slot width is always given as a fraction of T_r . In every plot, the sequence from above to below is the same sequence of the legend.

Whereas sample and hold or nearest neighbor resampling always cause a filtering operation and additive noise in the frequency domain, this effect may disappear by using the slotting variant. As an example, the expectation of a white noise spectrum remains white and unchanged after slotted nearest neighbor resampling. The major part of the bias in resampling is caused by the multiple use of the same irregular observation for more resampled values. This causes a cross over of the autocorrelation $R(0)$ to the autocorrelation at $R(T_r)$, $R(2T_r)$,… . Slotting prevents this because an irregular observation can only be in one time slot. The bias depends on the distribution of the sampling instants. The variation of the autocorrelation function over the width of the slot is the remaining cause of the bias with slotting. It can be described easily if the sampling moments have a Poisson distribution, by calculating the probability density function of those continuous time correlation lags τ that contribute to the resampled autocorrelation $R_{res}(nT_r)$. This bias depends strongly on the shape of the autocorrelation function and hence, of the spectrum. Rather flat spectra have no visible bias; spectra with a large dynamic range can have a strong bias. Two types of spectra are treated here. The first has a constant slope in the double logarithmic presentation, that descends at a rate of $\sim f^{-5/3}$ from $0.01f_0$. The second example starts similar but it has an extra declining slope at a rate of $\sim f^{-7}$ for frequencies f above $0.1f_0$. These types of spectra are representative for turbulence data. A narrow peak can be added to study the possibility of retrieving spectral details.

The bias in the first example with one slope is hardly visible in spectral plots. Like many examples with a limited dynamic range, the bias is negligible and accurate spectra can be estimated until frequencies far beyond the mean data rate. Results of the second example with two slopes are presented in Fig. 1. The first figure shows whole the frequency range. The resampling time $T_r = 2/f_0$ permits to compute spectra up to $f_0/4$. The other figures give only the higher part of the frequency range, to increase the visibility of the bias. The bias becomes important in weak parts of the spectrum and becomes less if the slot width is reduced. If the slot width is taken small enough, the bias will disappear eventually. That requires a very small slot if the dynamic range of the true spectrum is large, like in Fig. 1 for $T_r = 0.25$. However, the small slot also reduces the remaining fraction γ, because more resampling instants are used for the same amount of data. A smaller slot reduces the bias of the spectral estimate, but it gives an increased variance because the remaining fraction γ becomes smaller and the estimation of parameters is more difficult.

To illustrate what happens in the areas with a large spectral bias, a small and narrow peak at $0.6f_0$ has been added in Fig. 2, on the steep slope of the true

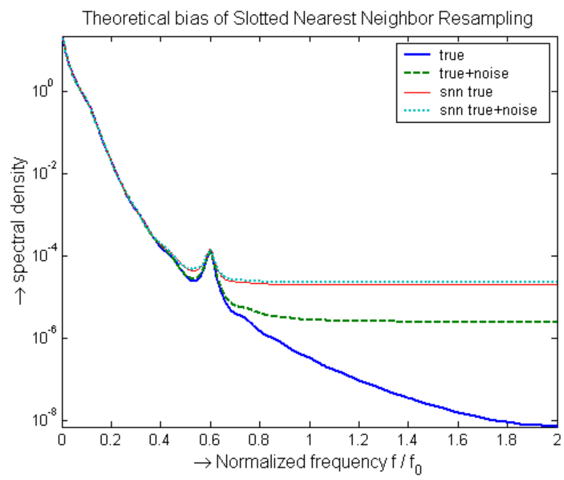

Fig.2. Theoretical expectation of the bias of slotted nearest neighbor resampling for 2 true double slope turbulence spectra, with or without additive white noise, with signal to noise power ratio 10^5. Linear frequency scale, $T_r = 0.25/f_0$, slot width $w = T_r/2$.

turbulence spectrum of Fig.1. Moreover, a second spectrum with the same peak has been generated, with an extra additive white noise at a level of 0.00001 times the power of the turbulence signal. Although the true spectra in Fig. 2, with or without noise, are completely different for $f / f_0 > 0.6$, the expectations of the biased slotted nearest neighbor resampled spectra are almost identical. This demonstrates that all true spectral details which are under the bias level of the slotting will not appear in the spectrum after the slotted resampling operation. Different true spectra, with or without noise, give the same biased estimates. This shows that reconstruction methods trying to remove the resampling bias will not be reliable. If different true spectra produce almost the same biased spectrum, it is not possible to undo the bias and to reconstruct the true spectrum without using additional information about the data. That additional or a priori information is not an outcome of the irregular data which are analyzed.

The peak at $f / f_0 = 0.6$ is still visible in both slotted spectra in Fig. 2. This indicates that it will be still be possible to detect spectral details above half the mean data rate, despite the bias. That would be impossible for equidistantly sampled observations. If the power of the noise is increased to more than 0.001 times the signal power, the true noisy and slotted noisy spectra virtually coincide and the peak vanishes. Both spectra are flat in the higher frequency range and only the very low frequency part of the spectrum raises above that noise level. The bias of slotted nearest neighbor resampling depends on the spectral shape. It is only important in a frequency range with low power.

6. PERFORMANCE OF THE NEW ESTIMATOR

Simulations with a known (aliased) spectrum are a first step in testing new algorithms. Test data was generated using the following procedure. First 128N equidistant data points were generated using a high order AR process. Then, randomly 127N data points were discarded. Each data point had a probability of 127/128 to be discarded. The resulting irregular data was non-equidistant and time intervals between arrivals were roughly Poisson distributed. In simulations, the true properties of the data are known. Hence, the quality of estimated results can be established. A quality measure for the fit is the aliased model error ME_T. That supposes that no anti-aliasing filter was used prior to resampling. It has been defined by de Bos *et al.* (2002) as

$$ME_T(p) = N\left[\frac{PE_T(p)}{\sigma^2_{\varepsilon,T}} - 1\right]. \qquad (7)$$

$PE_T(p)$ is the squared error of prediction of a model with p parameters. The true autocorrelation at time scale $T_r = kT_0$ has been used to compute $\sigma^2_{\varepsilon,T}$. This normalizing variance measures how the process $x(t)$ is predicted optimally by using $x(t-T_r)$, $x(t-2T_r)$, $\cdots$.

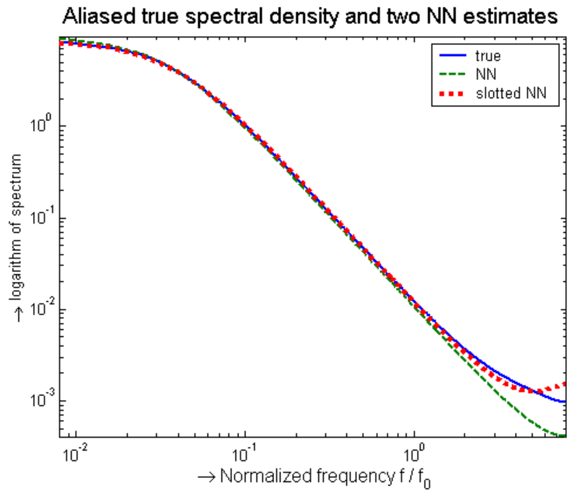

Fig.3. True spectrum and the ordinary and slotted NN estimates for a single slope turbulence spectrum. $T_r = 1/16f_0$ with slot width $w = T_r$, $N = 1000$. NN selected AR(1) with $ME_T = 40.6$, with slotted NN the AR(2) model was selected with $ME_T = 16.1$.

The estimation with the new algorithm may take a long time, because a non-linear search program is used to estimate the time series parameters. If the sample size is large enough, a very small slot width can be chosen to reduce the bias and accurate results can be obtained. Sometimes, low order models with a few parameters are sufficient for accurate estimation of the spectrum. They can be estimated with small data sizes. Higher order models require more data.

Fig. 3 gives a simulation result with the single slope example. The usual NN interpolation with uninterrupted equidistant resampled data selected the AR(1) model with ARMAsel. The new ARMAsel algorithm for irregular data could estimate 10 or 15 AR parameters and selected the AR(2) model. The spectral estimate is accurate up to high frequencies, much higher than the mean data rate f_0. Even for N = 200, often good results are obtained for $T_r = 1/16$, with AR(1) or AR(2) selected. The result of the slotted Burg irregular algorithm of Bos, *et al*. (2002) is not shown in Fig.3. It could estimate only one parameter for N =1000, with $ME_T = 38.9$. It requires very large data sets and fails for N =200, because no single parameter could be estimated with sufficient accuracy. ARMAsel for irregular data can estimate one AR parameter as long as γN, the effective number of observations, is greater than about 10. Hence, for N = 50, the resampling time T_r should be greater than about 0.2 to estimate an AR(1) model if w is taken as T_r. The AR(1) model still gives a rather accurate spectral density in this one-slope example without sharp true spectral details. The reasonable accuracy of the AR(1) model until the frequency 8f_0 demonstrates that low order AR models are good representatives for turbulence with a $\sim f^{-5/3}$ slope.

Accurate estimation is somewhat more difficult for the second example with two spectral slopes. Fig. 4 gives the spectra if enough observations are available

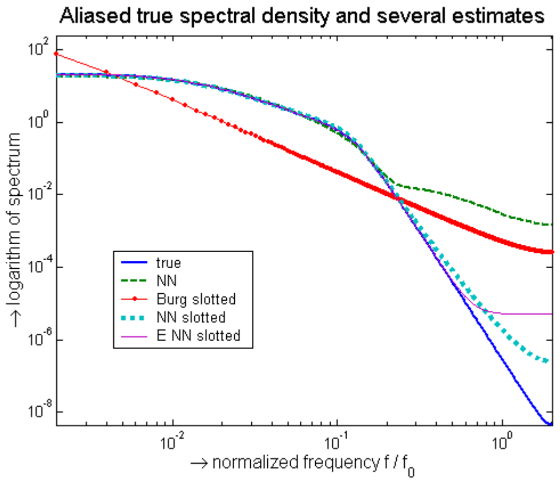

Fig. 4. True spectrum, 3 estimates and the NN expectation for a double-slope turbulence spectrum. T_r = 1/4f_0 with slot width w = T_r /4. N = 5000. The AR(3) model was selected with ARMAsel irregular applied to the multi shift slotted nearest neighbor resampled signal, with ME_T = 5200.

to let the ARMAsel algorithm for irregular data select a good fitting model. As a comparison, also the results of nearest neighbor resampling without slotting and of the slotted Burg irregular algorithms are presented. ARMAsel selected the ARMA(7,6) model for an uninterrupted nearest neighbor resampled signal with ME_T = 1.8*10^6. It is almost coinciding with the biased expectation of nearest neighbor resampling. The enormous improvement in nearest neighbor with the slotting procedure is obvious in Fig. 4: the bias becomes about 1000 times smaller for frequencies greater than f_0/2. The irregular slotted Burg algorithm could only estimate one AR parameter here and the ME_T was 2.7*10^6. As a comparison, the ME_T of the white noise AR(0) model is 7*10^8. That number is calculated by using the variance of the turbulence signal for PE_T (0) in (7). It has been verified that the selected spectral estimates for NN and slotted NN are always rather close to their ***biased*** expectations, if sufficient data are available. It is also possible to compute models for the combinations T_r = 0.5, w= T_r /8 or T_r = 1, w= T_r /16, which have about the same remaining fraction $\gamma \approx 1/16$. In most cases, the spectrum of the selected ARMAsel-irreg model was close to the expectation of the biased spectrum, as given in Fig. 1, but the automatic selection is not always reliable. If more observations are available, the multi shift slotted nearest neighbor spectra converge to their biased expectations. The bias can still be diminished by using smaller slots then. It turns out that 100 observations are sufficient for the two slope example to have a rather good AR(2) estimate of the spectrum for T_r = 0.5, w= T_r /2 or T_r =1, w= T_r /4, with the ME_T of the selected model less than 0.01*ME_T (0) of the white noise model. The irregular Burg algorithm will become comparable with ARMAsel-irreg if the sample size is great enough to estimate 3 or 4 AR parameters. However, the variance of the estimated Burg parameters will always be greater.

6. CONCLUSIONS

A new robust estimator is introduced that fits a time series model to multi shift slotted nearest neighbor resampled segments from irregularly sampled data. The ARMAsel-irreg algorithm gives rather accurate results at frequencies higher than the mean data rate f_0. In simulations with few data, the results are much better than those that can be obtained from the same data with other known existing techniques. In many examples, order and type of the best time series model for the data can be selected automatically, without user interaction, with a selection criterion. However, selection is not yet always reliable.

Multi shift slotted nearest neighbor resampling will give very accurate spectra if the dynamic spectral range is limited. For a large dynamic range, a small slot width will reduce the expectation of the bias. That still requires very large data sets to obtain accurate estimates at frequencies higher than f_0.

REFERENCES

Adrian R. J. and C. S. Yao (1987). Power spectra of fluid velocities measured by laser Doppler velocimetry. *Exper. in Fluids*, **5**, pp. 17-28.

Benedict, L. H., H. Nobach and C. Tropea (2000). Estimation of turbulent velocity spectra from Laser Doppler data. *Measurement Sci. Technol.*, **11**, pp. 1089-1104.

Bos, R., S. de Waele and P. M. T. Broersen (2002). Autoregressive spectral estimation by application of the Burg algorithm to irregularly sampled data. *IEEE Trans. on Instrumentation and Measurement*, **51**, pp. 1289-1294.

Broersen, P.M.T. (2002). Automatic spectral analysis with time series models. *IEEE Trans. on Instrument. and Measurement*, **51**, pp. 211-216. *ARMASA Matlab toolbox*, freely available at www.mathworks.com/matlabcentral/fileexchange.

Broersen, P.M.T. and R. Bos (2004). Estimation of time series spectra with randomly missing data. *Proc. IEEE/IMTC Conf., Como*, pp. 1718-1723.

Broersen, P.M.T., S. de Waele and R. Bos (2004a). Application of autoregressive spectral analysis to missing data problems. *IEEE Trans. on Instrument. and Measurement*, **53**, pp. 981-986.

Broersen, P.M.T., S. de Waele and R. Bos (2004b). Autoregressive spectral analysis when observations are missing. *Automatica*, **40**, pp. 1495-1504.

Burg, J. P. (1967). Maximum entropy spectral analysis. *Proc 37th Meeting Soc. Of Exploration Geophysicists*, 6 pp.

Jones, R.H. (1980). Maximum likelihood fitting of ARMA models to time series with missing observations. *Technometrics*, **22**, pp. 389-395.

Lahalle, E., G. Fleury and A. Rivoira (2004). Continuous ARMA spectral estimation from irregularly sampled observations. *Proc. IEEE/IMTC Conf., Como*, pp. 923-927.

Priestley, M. B. (1981). *Spectral Analysis and Time Series*. London, U.K.: Academic Press.