A MOVING SWITCHING PLANE FOR THE SLIDING MODE CONTROL OF THE THIRD ORDER SYSTEM

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Abstract: In this paper a new sliding mode control algorithm for the third order uncertain, nonlinear and time-varying dynamic system subject to unknown disturbance and input constraint is proposed. The algorithm employs a time-varying switching plane. At the initial time $t = t_0$, the plane passes through the point determined by the system initial conditions in the error state space. Afterwards, the plane moves with a constant velocity to the origin of the space. The plane is selected in such a way that the integral of the absolute value of the system error over the whole period of the control action is minimised and the input constraint is satisfied. By this means, the reaching phase is eliminated, insensitivity of the system to the external disturbance is guaranteed from the very beginning of the proposed control action and fast, monotonic error convergence to zero is achieved.

Key Words: Sliding mode control, variable structure systems, switching surface.

1. INTRODUCTION

In recent years much of the research in the area of control systems theory focused on the design of a discontinuous feedback which switches the structure of the system according to the evolution of its state vector. This technique, usually called sliding mode control, provides an effective and robust means of controlling nonlinear plants (DeCarlo, *et al*., 1988; Hung, *et al*., 1993; Slotine and Li, 1991; Utkin, 1997). The main advantage of this technique is that once the system state reaches a sliding surface, the system dynamics remain insensitive to a class of parameter variations and disturbances.

However, robust tracking is assured only after the system state hits the sliding surface, i.e. the robustness is not guaranteed during the reaching phase. Provided a conventional time-invariant sliding plane is considered, the advantage of the sliding mode control, namely the desired dynamic behaviour of the system, is not obtained for some time from the beginning of its motion. Furthermore, usually for the given initial conditions there is a trade-off between the short reaching phase and the fast system response in the sliding phase. In order to overcome these problems the idea of the time-varying switching lines applied for the sliding mode control of the second order systems was introduced by Choi, *et al*. (1993); Choi and Park (1994); Choi, *et al*. (1994) and further discussed by Bartoszewicz (1995, 1996). The control algorithms proposed in these papers eliminate the reaching phase and guarantee fast error convergence rate for the second order uncertain systems with arbitrary initial conditions. Further results on the application of the time-varying switching lines for the sliding mode control of the second order systems have recently been reported by Tokat, *et al.* (2002).

In this paper we consider the third order, nonlinear, time-varying system subject to the input constraint. We introduce a continuously time-varying switching plane adaptable to the initial conditions of the system and we prove the existence of a sliding mode on the plane. At the time $t = t_0$ the plane passes through the representative point, specified by the initial conditions of the system, in the error state space. Afterwards, the plane moves smoothly, with a constant velocity, to the origin of the space. Thus the proposed control algorithm eliminates the reaching phase and forces the representative point of the system to stay always on the switching plane. As a consequence, our control is robust with respect to the external disturbance and parameter uncertainties from the very beginning of the system motion. Furthermore, the plane is designed in such a way that the integral of the absolute value of the system error over the whole period of the control action is minimised and the input constraint is satisfied. Therefore, good dynamic performance of the considered system is ensured.

The remainder of the paper is organised as follows. The problem considered in this paper is formulated in Section 2. Section 3 presents the proposed control law and gives details of the switching plane design procedure. Finally, Section 4 presents conclusions of the paper.

2. PROBLEM STATEMENT

Let us consider the time-varying and nonlinear, third order system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= f(\boldsymbol{x},t) + \Delta f(\boldsymbol{x},t) + b(\boldsymbol{x},t)u + d(t) \end{aligned} \quad (1)$$

where x_1, x_2, x_3 are the state variables of the system and $\boldsymbol{x}(t) = [x_1(t)\ \ x_2(t)\ \ x_3(t)]^T$ is the state vector, t denotes time, u is the input signal, b, f – are a priori known, bounded functions of time and the system state, Δf and d are functions representing the system uncertainty and external disturbances respectively. Further in the paper, it is assumed that there exists a strictly positive constant δ which is the lower bound of $b(\boldsymbol{x}, t)$, i.e. $0 < \delta = \inf\{|b(x, t)|\}$. Furthermore functions Δf and d are unknown and bounded. Therefore, there exists a constant μ which for every pair $(\boldsymbol{x},t)$ satisfies the following condition $|\Delta f(\boldsymbol{x},t) + d(t)| \le \mu$. Initial conditions of the system are denoted as x_{10}, x_{20}, x_{30}, where $x_{10} = x_1(t_0)$, $x_{20} = x_2(t_0)$, $x_{30} = x_3(t_0)$. The system (1) is supposed to track the demand trajectory given as a function of time $\boldsymbol{x}_d(t) = [x_{1d}(t)\ \ x_{2d}(t)\ \ x_{3d}(t)]^T$, where $x_{2d}(t) = \dot{x}_{1d}(t)$, $x_{3d}(t) = \dot{x}_{2d}(t)$ and $x_{3d}(t)$ is a differentiable function of time. The trajectory tracking error is defined by the following vector

$$\boldsymbol{e}(t) = [e_1(t)\ \ e_2(t)\ \ e_3(t)]^T = \boldsymbol{x}(t) - \boldsymbol{x}_d(t) \quad (2)$$

Hence, we have $e_1(t) = x_1(t) - x_{1d}(t)$, $e_2(t) = x_2(t) - x_{2d}(t)$, $e_3(t) = x_3(t) - x_{3d}(t)$. In this paper it is assumed that at the initial time $t = t_0$, the tracking error and the error derivatives

$$e_1(t_0) = e_0\,,\ e_2(t_0) = 0\,,\ e_3(t_0) = 0 \quad (3)$$

where e_0 is an arbitrary real number. The purpose of this paper is to design a sliding mode control strategy for the system (1) which:

- Makes the system insensitive to the disturbance $d(t)$ and the model uncertainty $\Delta f(\boldsymbol{x}, t)$ for any time $t \ge t_0$, i.e. from the very beginning of the system motion;
- Drives the system error to zero monotonically. In other words, we require error convergence without oscillations, overshoots or undershoots;
- Satisfies the following input constraint

$$|u(t)| \le u_{\max} \quad (4)$$

- Ensures minimisation of the following control quality criterion

$$J = \int_{t_0}^{\infty} |e_1(t)|\,dt \quad (5)$$

3. SWITCHING PLANE DESIGN

Let us consider a time-varying switching plane with the constant angle of inclination. Originally the plane moves uniformly (i.e. with a constant velocity) in the state space and then it stops at the time instant t_f. Consequently, for any $t \le t_f$ the switching plane is described by the following equation

$$\begin{aligned} &s(\boldsymbol{e},t) = 0 \ \text{ where} \\ &s(\boldsymbol{e},t) = e_3(t) + c_2 e_2(t) + c_1 e_1(t) + A + Bt \end{aligned} \quad (6)$$

where c_1, c_2, A and B are some constants. The selection of these constants will be considered further in the paper. Since the plane stops at the time t_f, for any $t \ge t_f$ it is described as follows

$$\begin{aligned} &s(\boldsymbol{e},t) = 0 \ \text{ where} \\ &s(\boldsymbol{e},t) = e_3(t) + c_2 e_2(t) + c_1 e_1(t) \end{aligned} \quad (7)$$

First, the constants c_1, c_2, A and B should be chosen in such a way that the representative point of the system at the initial time $t = t_0$ belongs to the switching plane. For that purpose, the following condition must be satisfied

$$s(\boldsymbol{e}(t_0),t_0)=e_3(t_0)+c_2e_2(t_0)+c_1e_1(t_0)+ \\ +A+Bt_0=0 \tag{8}$$

Notice that the input signal

$$u=\frac{-f(\boldsymbol{x},t)-c_2e_3(t)-c_1e_2(t)+\dot{x}_{3d}(t)-B-\gamma\operatorname{sgn}[s(\boldsymbol{e},t)]}{b(\boldsymbol{x},t)} \tag{9}$$

where $\gamma = \eta + \mu$ and η is a strictly positive constant, ensures the stability of the sliding motion on the switching plane (6). In order to verify this property we consider the product

$$s(\mathbf{e},t)\dot{s}(\mathbf{e},t) = s(\mathbf{e},t)[\dot{e}_3(t) + c_2e_3(t) + c_1e_2(t) + B] \tag{10}$$

Substituting relations (1), (2) and (9) into (10) we get

$$s(\mathbf{e},t)\dot{s}(\mathbf{e},t) = s(\mathbf{e},t)\{\Delta f(\mathbf{x},t) + d(t) - \gamma\operatorname{sgn}[s(\mathbf{e},t)]\} \le \\ \le -\eta|s(\mathbf{e},t)| \tag{11}$$

which proves the existence and stability of the sliding motion on the plane described by equations (6) and (7). Consequently, for any time $t \in \langle 0, t_f \rangle$ the system dynamics is described by equation (6) with the initial conditions (3). Therefore, we consider the following equation

$$e_3(t)+c_2e_2(t)+c_1e_1(t)+A+Bt=0 \tag{12}$$

In order to solve it, we consider

$$e_3(t)+c_2e_2(t)+c_1e_1(t)=0 \tag{13}$$

Since the tracking error convergence to zero without oscillations is required, the characteristic polynomial of equation (13) should have one, double real root. Hence, we get another condition

$$c_2=2\sqrt{c_1} \tag{14}$$

Furthermore, the parameters c_1 and c_2 must be strictly positive to make the system (1) stable in the sliding mode. Solving equation (12) with condition (14) and assuming for the sake of clarity that $t_0 = 0$ we get the tracking error and its derivatives for the time $t \in \langle 0, t_f \rangle$

$$e_1(t)=\left[e_0+\frac{A}{c_1}-\frac{2B\sqrt{c_1}}{c_1^{\,2}}+\left(\frac{A}{\sqrt{c_1}}+\sqrt{c_1}e_0+ \\ -\frac{B}{c_1}\right)t\right]e^{-\sqrt{c_1}t}-\frac{A}{c_1}+\frac{2B\sqrt{c_1}}{c_1^{\,2}}-\frac{B}{c_1}t \tag{15}$$

$$e_2(t)=\left[\frac{B}{c_1}-\left(A+c_1e_0-\frac{B\sqrt{c_1}}{c_1}\right)t\right]e^{-\sqrt{c_1}t}-\frac{B}{c_1} \tag{16}$$

$$e_3(t) = \left[-A - c_1e_0 + c_1\left(\frac{A}{\sqrt{c_1}} + e_0\sqrt{c_1} - \frac{B}{c_1}\right)t\right]e^{-\sqrt{c_1}t} \tag{17}$$

Taking into account condition (8) and the assumption that $t_0 = 0$ one gets

$$A = -c_1e_0 \tag{18}$$

Then formulae (15), (16) and (17) can be written as

$$e_1(t)=\left(-\frac{2B\sqrt{c_1}}{c_1^2}-\frac{B}{c_1}t\right)e^{-\sqrt{c_1}t}+ \\ +\frac{2B\sqrt{c_1}}{c_1^{\,2}}+e_0-\frac{B}{c_1}t \tag{19}$$

$$e_2(t) = \frac{B}{c_1}\left(1 + \sqrt{c_1}t\right)e^{-\sqrt{c_1}t} - \frac{B}{c_1} \tag{20}$$

$$e_3(t) = -Bte^{-\sqrt{c_1}t} \tag{21}$$

Next, we will analyse the behaviour of the system in the second phase of its motion, that is when the switching plane does not move. Notice that for the time $t \ge t_f$ the switching plane is fixed and passes through the origin of the error state space. This leads to the condition

$$A + Bt_f = 0 \tag{22}$$

From equations (22) and (18) we have

$$t_f=\frac{e_0c_1}{B} \tag{23}$$

The time invariant switching plane is described by relation (7), which is equivalent to equation (13). The initial conditions which are necessary to solve equation (13) can be determined from equations (19), (20) and (21) whose values are evaluated at the time instant t_f. With the following notation

$$k = \frac{c_1\sqrt{c_1}e_0}{B} \tag{24}$$

the initial conditions for the second phase of the system motion can be written as

$$e_1(t_f)=\left(-\frac{2B\sqrt{c_1}}{c_1^{\,2}}-e_0\right)e^{-k}+\frac{2B\sqrt{c_1}}{c_1^{\,2}} \tag{25}$$

$$e_2(t_f) = -\frac{B}{c_1} + \left(\frac{B}{c_1} + e_0\sqrt{c_1}\right)e^{-k} \tag{26}$$

$$e_3(t_f) = -e^{-k} c_1 e_0 \tag{27}$$

The parameter k defined by equation (24) is strictly positive. Solving equation (13) with initial conditions (25), (26), (27) and using relation (18) we get

$$e_1(t) = e^{-\sqrt{c_1}t}\left[\left(-\frac{2B\sqrt{c_1}}{c_1^2} + \frac{2B\sqrt{c_1}}{c_1^2}e^k - e_0 e^k\right.\right.$$
$$\left.\left. + \left(-\frac{B}{c_1} + \frac{B}{c_1}e^k\right)t\right)\right] \tag{28}$$

$$e_2(t) = e^{-\sqrt{c_1}t}\left[\frac{B}{c_1} - \frac{B}{c_1}e^k + e_0 e^k\sqrt{c_1} + \right.$$
$$\left. - \left(-\frac{B}{\sqrt{c_1}} + \frac{B}{\sqrt{c_1}}e^k\right)t\right] \tag{29}$$

$$e_3(t) = e^{-\sqrt{c_1}t}\left[-e_0 e^k c_1 + B\left(e^k - 1\right)t\right] \tag{30}$$

These three equations describe the tracking error for any time $t \geq t_f$. Notice that the tracking error described by equations (19) and (28) does not exhibit any overshoots. This can be demonstrated as follows. For the time $t \in \langle 0, t_f\rangle$ the tracking error is described by equation (19). During this interval the error either monotonically decreases for $e_0 > 0$ to a value $e_1(t_f) > 0$ or monotonically increases for $e_0 < 0$ to a value $e_1(t_f) < 0$. This can be seen from equations (20) and (25) reformulated as

$$e_2(t) = \frac{B}{c_1}e^{-\sqrt{c_1}t}\left[\left(1 + \sqrt{c_1}t\right) - e^{\sqrt{c_1}t}\right] \tag{31}$$

$$e_1(t_f) = \frac{2B\sqrt{c_1}}{c_1^2}e^{-k}\left[-\left(1 + \frac{k}{2}\right) + e^k\right] \tag{32}$$

The signs of B and e_0 are the same, so it can be seen from equation (31) that the derivative of the tracking error has the opposite sign to e_0 for any $t \in \langle 0, t_f\rangle$. Equation (32) shows that the signs of the tracking error at the time instants t_0 and t_f are the same. Hence, the error does not exhibit any overshoot during this time interval. On the other hand, for the time $t \geq t_f$ the tracking error is given by equation (28). If $e_0 > 0$, then the tracking error decreases from the positive value $e_1(t_f)$ to zero. If $e_0 < 0$, then the error increases from the negative value $e_1(t_f)$ to zero. The tracking error converges to zero monotonically because from equation (29)

$$e_2(t) = \frac{B}{c_1}e^{-\sqrt{c_1}t}\left[1 - e^k + ke^k - \sqrt{c_1}\left(e^k - 1\right)t\right] \tag{33}$$

and for any $t \geq t_f$, the expression in the square bracket $1 - e^k + ke^k - \sqrt{c_1}\left(e^k - 1\right)t \leq 1 - e^k + ke^k + -\sqrt{c_1}\left(e^k - 1\right)t_f = 1 - e^k + k$ is negative. Thus it follows from equation (33) that the derivative of the tracking error has the opposite sign to e_0.

In the sequel a method of choosing the switching plane parameters will be proposed. We will consider the control quality criterion given by equation (5). Since we have demonstrated that the tracking error converges monotonically in the considered system, this criterion is equivalent to

$$J = \left|\int_0^\infty e_1(t)dt\right| \tag{34}$$

Substituting equations (19) and (28) into (34) and calculating appropriate integrals we get

$$J = \frac{2|e_0|}{\sqrt{c_1}} + \frac{e_0^2 c_1}{2|B|} \tag{35}$$

In order to calculate the parameters B and c_1 of the switching plane, criterion (35) should be minimised with input constraint (4), where u_{max} is a constant, which satisfies the following condition

$$u_{max} > \frac{\left[\left|\dot{x}_{3d} - f(x,t)\right| + \gamma\right]}{|b(x,t)|} \tag{36}$$

Conditions (4) and (36) imply that there exists such a strictly positive constant

$$U = u_{max} - \max\left[\frac{\left|\dot{x}_{3d} - f(x,t)\right| + \gamma}{|b(x,t)|}\right] \tag{37}$$

that condition (4) is satisfied if the following relation holds

$$|\dot{e}_3(t)| \leq |b(x,t)|U \tag{38}$$

Furthermore, clearly inequality (38) always holds if

$$|\dot{e}_3(t)| \leq \delta U \tag{39}$$

Let us calculate the greatest value of $\dot{e}_3(t)$. For $t \leq t_f$ the tracking error is given by equation (19). Differentiating this equation three times we get

$$\dot{e}_3(t) = Be^{-\sqrt{c_1}t}\left(t\sqrt{c_1} - 1\right) \tag{40}$$

Notice that at the initial time $\dot{e}_3(0) = -B$. Then we calculate the extreme value of this signal. The value is achieved at the time

$$\tilde{t} = 2/\sqrt{c_1} \quad (41)$$

and it is equal to

$$\dot{e}_3(\tilde{t}) = Be^{-2} \quad (42)$$

Note that $|Be^{-2}| < |B|$. Furthermore at the time $t = t_f$, $\dot{e}_3(t_f) = Be^{-k}(k-1)$. Notice that for $k > 0$ we have $|Be^{-k}(k-1)| \leq |B|$. This can be proved considering the right hand side of the above relation as a function of $k > 0$. Let $f(k) = Be^{-k}(k-1)$. This function for $k = 0$ equals $f(0) = -B$ and for $k \to \infty$, $f(k) \to 0$. The extreme of this function is achieved when $k = 2$ and it is equal $f(2) = Be^{-2}$. Then $|Be^{-2}| < |B|$.

On the other hand, for $t \geq t_f$ the tracking error is described by equation (28). Differentiating this equation three times we get

$$\begin{aligned}\dot{e}_3(t) = &-\sqrt{c_1}\, e^{-\sqrt{c_1}t}\left[-e_0 e^k c_1 + B\left(e^k - 1\right)t\right] + \\ &+ B\left(e^k - 1\right)e^{-\sqrt{c_1}t}\end{aligned} \quad (43)$$

Consequently, at the time t_f

$$\dot{e}_3(t_f) = B\left[e^{-k}(k-1) + 1\right] \quad (44)$$

The absolute value of the right hand side of the above equation may be greater than $|B|$ and could possibly present the greatest value of $|\dot{e}_3(t)|$. Furthermore, for $t \in \langle t_f, \infty)$ the extreme value of (43) is reached at the time

$$\tilde{\tilde{t}} = 2/\sqrt{c_1} + t_f e^k / \left(e^k - 1\right) \quad (45)$$

This extreme value is

$$\dot{e}_3(\tilde{\tilde{t}}) = B\left(e^k - 1\right)\exp\left[-\left(2 + \frac{ke^k}{e^k - 1}\right)\right] \quad (46)$$

Let us consider the following expression $|(e^k - 1)\exp\{-[2 + ke^k/(e^k - 1)]\}| = |\exp[-2 - k/(e^k - 1)] - \exp[-2 - ke^k/(e^k - 1)]| < 1$. Therefore, $|B(e^k - 1)\exp\{-[2 + ke^k/(e^k - 1)]\}| < |B|$. Finally, let us notice that the third derivative of the tracking error described by relation (45) converges to zero for $t \to \infty$. Consequently, we conclude that the greatest value of $|\dot{e}_3(t)|$ is represented by the maximum of the two expressions: $|B|$ and the absolute value of expression (44). Therefore, we get the following constraints

$$|B| \leq \delta U \quad (47)$$

and

$$|B| \leq \delta U / \left[e^{-k}(k-1) + 1\right] \quad (48)$$

Since constraint (48) is expressed in terms of k rather than c_1, it will be convenient to consider quality criterion $J(k, B)$ instead of $J(c_1, B)$. For that purpose we calculate c_1 from equation (24)

$$c_1 = \left(\frac{Bk}{e_0}\right)^{2/3} \quad (49)$$

Substituting (49) into (35), the control quality criterion can be presented as

$$J = \frac{|e_0|^{4/3}}{|B|^{1/3}}\left(2k^{-1/3} + \frac{1}{2}k^{2/3}\right) \quad (50)$$

It can be easily noticed that for any given value of k, the minimum of criterion (50) is obtained for the greatest value of $|B|$ satisfying constraints (47) and (48). These constraints are illustrated in Figure 1. The optimal solution of the minimisation problem considered here is represented by a point which belongs to the curve bounding the admissible set shown in the figure. Therefore, this solution can be found as minimum of a single variable function of k. Consequently, in order to minimise criterion (50) with constraints (47) and (48) the following two cases $k < 1$ and $k \geq 1$, should be considered.

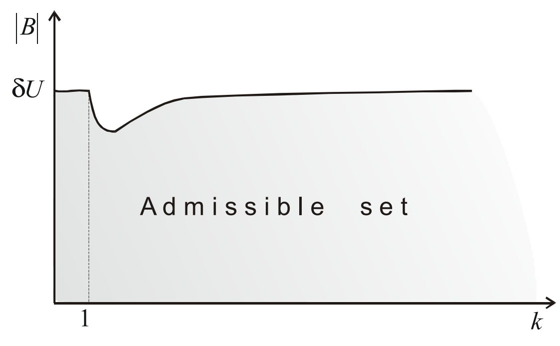

Fig. 1. Optimisation constraints.

In the first case we minimise criterion (50) with constraint (47), and in the latter one constraint (48) is taken into account.

Now we will precisely analyse the minimisation task. First we consider the case when $k < 1$. In this situation, from relations (50) and (47) we get

$$J = \frac{|e_0|^{4/3}}{(\delta U)^{1/3}}\left(2k^{-1/3} + \frac{1}{2}k^{2/3}\right) \quad (51)$$

This criterion value decreases when the argument k increases form zero up to one. This leads to the conclusion that the minimum value of (51) is equal to

$$J = \frac{5|e_0|^{4/3}}{2(\delta U)^{1/3}} \tag{52}$$

In the second case, i.e. when $k \geq 1$ we minimise

$$J = \frac{|e_0|^{4/3}\left[e^{-k}(k-1)+1\right]^{1/3}}{(\delta U)^{1/3}}\left(2k^{-1/3} + \frac{1}{2}k^{2/3}\right) \tag{53}$$

The smallest value of criterion (53) is achieved for $k = 2$. It equals

$$J = \frac{3|e_0|^{4/3}\left(e^{-2}+1\right)^{1/3}}{(\delta U)^{1/3}\,2^{1/3}} \tag{54}$$

Notice that the right hand side of equation (52) is greater than that of (54). Hence (54) is the minimum value of criterion (50), and finally we have the following optimal solution

$$k_{opt} = 2 \tag{55}$$

$$B_{opt} = \frac{\delta U \operatorname{sgn}(e_0)}{\left(e^{-2}+1\right)} \tag{56}$$

Consequently, from expression (49) we get the following value of the parameter c_1

$$c_{1opt} = \left(\frac{2\delta U}{|e_0|\left(e^{-2}+1\right)}\right)^{2/3} \tag{57}$$

The other parameters of the switching plane can be calculated from equations (14) and (18)

$$c_{2opt} = 2\left(\frac{2\delta U}{|e_0|\left(e^{-2}+1\right)}\right)^{1/3} \tag{58}$$

$$A_{opt} = -e_0\left(\frac{2\delta U}{|e_0|\left(e^{-2}+1\right)}\right)^{2/3} \tag{59}$$

The plane described by these parameters stops moving at the time instant $t_f = 2^{2/3}\left[|e_0|\left(e^{-2}+1\right)/(\delta U)\right]^{1/3}$.

4. CONCLUSIONS

In this paper a new sliding mode control method, for the third order dynamic system has been proposed. The method employs a time-varying switching plane which moves with a constant velocity and the constant angle of inclination to the origin of the error state space. The switching plane parameters are chosen in such a way that the control quality criterion is minimised and the input constraint is satisfied. The moving switching plane ensures that the tracking error converges to zero monotonically and the system is insensitive with respect to its model uncertainty and external disturbance since the very beginning of the control action.

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