A COMPENSATOR FOR ATTENUATION OF WAVE REFLECTIONS IN LONG CABLE ACTUATOR–PLANT INTERCONNECTIONS WITH GUARANTEED TRANSIENT PERFORMANCE IMPROVEMENT [1]

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Abstract: The wave reflection phenomenon that appears when actuator and plant are connected through long cables is studied in this paper. In several applications, the perturbation induced by the presence of these reflected waves is non–negligible and seriously degrades the performance of the control and the operativity of the system. Standard compensation schemes are based on matching impedances at specific frequencies (possibly infinity) and are realized with the addition of linear RLC filters. Impedance matching is clearly ineffective if there is no single dominant frequency in the system and–or the plant is highly uncertain. In a recent paper the authors proposed a novel compensator design framework, based on the scattering representation of the transmission line, that is applicable for the latter scenario. In contrast with the standard schemes the compensators are *active* and require for their implementation regulated sources placed either on actuator or plant side. The use of active compensators raises the issue of *ensuring stability* of the design, a point left open in our previous work, that is fully solved in the present note. We propose a family of compensators that requires only knowledge of cable characteristics and—under some practically reasonable assumptions—guarantees *transient performance improvement* and asymptotic tracking for all (unknown) plants with passive impedance.

Keywords: Infinite dimensional systems, wave equation, transmission lines, motor control, overvoltage, reflection coefficient, PWM inverter, impedance.

Notation We define the differentiation and advance–delay operators, acting on signals $x : \mathbb{R} \to \mathbb{R}$, as $(p^k x)(t) \triangleq \frac{d^k}{dt^k} x(t)$ and $(q^{\pm k} x)(t) = x(t \pm kd)$, respectively, where $d \in \mathbb{R}_+$ and $k \in \mathbb{Z}_+$. Their Laplace transform counterparts, which are used to define transfer functions, are s and $z = e^{ds}$, respectively.

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1. INTRODUCTION

In this paper we complement and extend the material presented in (R. Ortega, 2004) addressing, in particular, the fundamental *stability* issue that arises due to the use of active compensators. This point was left open in our previous work and is fully solved in the present note by ensuring that, for a class of provably stabilizing compensators (that contains as a particular case the scheme proposed in (R. Ortega, 2004)), the operator seen from the plant is *passive*, which warrantees stability and asymptotic tracking for all plants with passive impedance.

The remaining of this paper is organized as follows. In Section 2 we present the model of the system under consideration, including the compensator configuration and the scattering representation. In Section 3 we present the compensator design configuration and the related well–posedness analysis—that requires the additional assumptions of plant linearity and piece–wise approximation of the signals—is carried–out in Section 4. This analysis reveals that any full–decoupling scheme, as well as any voltage–decoupling one, will yield ill–posed interconnections. This motivates the consideration in Section 5 of current–decoupling controllers, for which a complete (transient and asymptotic) stability analysis, given in Section 6, is possible. We wrap–up the paper with some concluding remarks and open problems in Section 7.

2. SYSTEMS MODEL

To model the plant connected to the actuator through long cables we consider the configuration shown in Fig. 1, where we model the *connecting cables* as a two–port system whose dynamics are described via the Telegrapher's equations

$$C\frac{\partial v(t,x)}{\partial t} = -\frac{\partial i(t,x)}{\partial x}, L\frac{\partial i(t,x)}{\partial t} = -\frac{\partial v(t,x)}{\partial x} \quad (1)$$

where $v(t,x)$, $i(t,x)$ represent the line voltage and current, respectively, $x \in [0,\ell]$ is the spatial coordinate, with $\ell > 0$ the cable length and $C, L > 0$, which are assumed constant, are the capacitance and inductance of the line, respectively. As discussed in (R. Ortega, 2004) the use of the scattering representation of the transmission line is instrumental for the compensator design. For, we define the so-called *scattering variables* [2]

$$\begin{bmatrix} s_+(t,x) \\ s_-(t,x) \end{bmatrix} \triangleq T \begin{bmatrix} v(t,x) \\ i(t,x) \end{bmatrix} \quad , \quad T \triangleq \begin{bmatrix} 1 & Z_0 \\ 1 & -Z_0 \end{bmatrix}, \quad (2)$$

with $Z_0 \triangleq \sqrt{\frac{L}{C}}$ the *line characteristic impedance*. Using the well–known relation for the scattering variables (Berg and McGregor, 1966)

$$\begin{bmatrix} s_+(t,\ell) \\ s_-(t,\ell) \end{bmatrix} = \begin{bmatrix} q^{-1} & 0 \\ 0 & q \end{bmatrix} \begin{bmatrix} s_+(t,0) \\ s_-(t,0) \end{bmatrix}, \quad (3)$$

we can establish the following relation between the port variables of the transmission line (1)

$$\begin{bmatrix} v(t,\ell) \\ i(t,\ell) \end{bmatrix} = W(q) \begin{bmatrix} v(t,0) \\ i(t,0) \end{bmatrix} \quad (4)$$

$$W(z) \triangleq T^{-1} \begin{bmatrix} z^{-1} & 0 \\ 0 & z \end{bmatrix} T \in \mathbb{R}^{2\times 2}(z)$$

with $d \triangleq \ell\sqrt{LC}$ the *propagation delay*. The *actuator* is modelled as a one–port whose port variables, $(v(t,0), i(t,0))$, are directly connected to the line. It consists of a voltage source, $v_S(t)$, connected in series with an impedance $Z_a(s) \in \mathbb{R}(s)$, called the surge impedance, that we assume strictly stable. The transmission line is terminated by the *plant*, which is a one–port, with port variables $(v(t,\ell), i(t,\ell))$. If we assume the plant is LTI the dynamics of the overall system is described by (4) together with

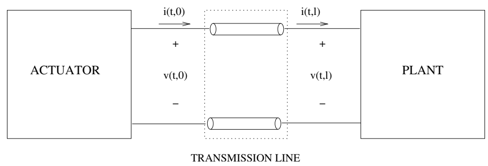

Fig. 1. *Uncompensated systems configuration.*

$$\begin{aligned} v(t,0) &= -Z_a(p)i(t,0) + v_s(t) \\ v(t,\ell) &= Z_p(p)i(t,\ell), \end{aligned} \quad (5)$$

where $Z_p(s) \in \mathbb{R}(s)$ is the plant impedance—that we assume is strictly stable but otherwise *unknown*. We need the following *generic* assumption.

Assumption A.0

$$R_p + Z_0 \neq 0, \quad R_a + Z_0 \neq 0. \quad (6)$$

where $R_p, R_a \in \mathbb{R}$ *are the high–frequency gains of the plant and actuator impedances.*

Under Assumption A.0, it is possible to show that the mapping from the source voltage to the plant voltage is given by the linear *delay–differential* operator

$$\begin{aligned} v(t,\ell) &= K_a(p)K_p(p)v(t-2d,\ell) + \\ &+ \frac{1}{2}[1+K_p(p)][1-K_a(p)]v_s(t-d) + \epsilon_t, \end{aligned} \quad (7)$$

where ϵ_t is an exponentially decaying term, that will be omitted in the sequel, and

$$K_a(s) \triangleq \frac{Z_a(s)-Z_0}{Z_a(s)+Z_0}, \quad K_p(s) \triangleq \frac{Z_p(s)-Z_0}{Z_p(s)+Z_0}, \quad (8)$$

are the so-called actuator and plant *reflection coefficients*, respectively. [3] For further developments it will be assumed that $K_p(s)$ is also strictly stable. We make at this point the following crucial observation: the delayed signal $K_a(p)K_p(p)v(t-2d,\ell)$ is added to the filtered (delayed) pulse $v_s(t-d)$ to generate $v(t,\ell)$. This term captures the physical phenomenon of *wave*

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[2] A normalization factor, that is omitted here for simplicity, is sometimes added in this definition (van der Schaft, 1996).

[3] Assumption A.0 is needed to ensure these transfer functions are well–defined. If $Z_p(s), Z_a(s)$ are LTI RLC filters then, because of Bruni's Theorem, they are positive real transfer functions with $R_p, R_a > 0$, and the assumption may be obviated.

reflection that deforms the transmitted signals and degrades the quality of the control.

To attenuate the wave reflections compensators are added, either at the actuator as shown in Fig. 2 or the plant sides. They may be added at both sides like in teleoperation applications—see (Anderson and Spong, 1989).

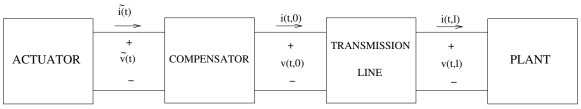

Fig. 2. *Port representation of the system with compensator on the actuator side.*

3. PROPOSED ACTIVE COMPENSATION CONFIGURATION

Standard compensation schemes are based on matching impedances at specific frequencies (possibly infinity) and are realized with the addition of linear RLC filters. Impedance matching is clearly ineffective if there is no single dominant frequency in the system and–or the plant is highly uncertain. Under these conditions the effectiveness of passive LTI RLC filtering, particularly acting only on the actuator side, is severely stymied. Therefore, following (R. Ortega, 2004), we will assume that the compensators may contain *active regulated sources*. Although there are several theoretically admissible configurations to add regulated sources at the line terminations, for technological reasons, we propose the one shown in Fig. 3. [4]

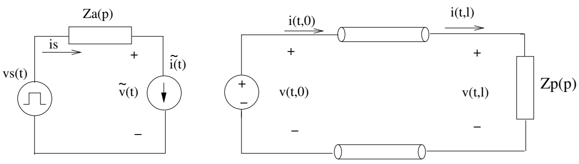

Fig. 3. *Proposed circuit realization of the active compensation scheme.*

To generate the regulated current, $\tilde{i}(t)$, and voltage, $v(t,0)$, we consider discrete–time compensators of the form [5]

$$\begin{bmatrix} \tilde{i}(t) \\ v(t,0) \end{bmatrix} = H(q) \begin{bmatrix} \tilde{v}(t) \\ i(t,0) \end{bmatrix}. \tag{9}$$

Note that if $H(z) \in \mathbb{R}^{2\times 2}(z)$ is proper $\tilde{i}(t)$ and $v(t,0)$ can be causally generated as linear combinations of (delayed and un–delayed) measurable signals $\tilde{v}(t),\ i(t,0)$. Motivated by the representation of the transmission line (4), we rewrite (9) in the equivalent t–parameter representation (see Table 19.1 of (DeCarlo and Lin, 2001))

$$\begin{bmatrix} v(t,0) \\ i(t,0) \end{bmatrix} = C(q) \begin{bmatrix} \tilde{v}(t) \\ \tilde{i}(t) \end{bmatrix}, \tag{10}$$

where $C(z) \in \mathbb{R}^{2\times 2}(z)$ is not necessarily proper. Connecting the compensator with the transmission line yields

$$\begin{bmatrix} v(t,\ell) \\ i(t,\ell) \end{bmatrix} = M(q) \begin{bmatrix} \tilde{v}(t) \\ \tilde{i}(t) \end{bmatrix}, \tag{11}$$

where we have defined the transfer matrix

$$M(z) \stackrel{\triangle}{=} W(z)C(z) \in \mathbb{R}^{2\times 2}(z). \tag{12}$$

4. DISCRETE–TIME REPRESENTATION AND WELL-POSEDNESS ANALYSIS

As explained above the system is described by delay–differential equations. Establishing well–posedness for an interconnected delay–differential system seems to be a formidable task. Therefore, we introduce the following: [6]

Assumption A.1 *The propagation delay $d = \ell\sqrt{LC}$ is sufficiently small so that all signals can be suitably described by their piece–wise approximation. More precisely, for all signals $x : \mathbb{R}_+ \to \mathbb{R}$*

$$x(t) = x(kd), \quad \forall\ t \in [kd, (k+1)d), \quad k \in \mathbb{Z}_+.$$

Before proceeding to explain the significance of Assumption A.1 on the well–posedness analysis notice that its pertinence depends on the order relation between d and the frequency content of the signals—that is, whether d is sufficiently small in comparison to the rate of change of the signals. In this respect, we refer the reader to (R. Ortega, 2004). The "period" of the transient oscillation is $\approx .5$ *ms* while d is of the order of 10 μs—hence the approximation is a little crude in this example. [7] With the "discretization" Assumption A.1 the overall dynamics, at the sampling instants kd, is described by a *purely discrete–time system*, for which the well–posedness analysis follows standard lines. Indeed, under Assumption A.1, the plant voltage $v(t,\ell)$ becomes [8]

$$v(t,\ell) = Z_p^*(q)i(t,\ell), \quad \forall\ t \in [kd,(k+1)d), \quad k \in \mathbb{Z}_+ \tag{13}$$

with $Z_p^*(z) \in \mathbb{R}(z)$ the pulse transfer function representation (with sampling time d) of the plant

[4] Throughout the rest of the paper we will consider only the control configuration of Fig. 2. Totally analogous arguments will apply to the case of plant-side compensators.

[5] Clearly, the realization of this controller assumes knowledge of the line propagation delay d. See also Assumption A.3 below.

[6] Although not explicitly stated, this assumption is required for the proof of Proposition 1 in (R. Ortega, 2004).

[7] Sampling the signals every d units of time is done only for simplicity, and the sampling period can be taken as $\frac{d}{N}$ for any $N \in \mathbb{Z}_+$, making the approximation even better. Unfortunately, taking a smaller sampling period generates repeated poles of $W(z)$ in the unit disk that makes the subsequent stability analysis (which is based on passivity) inapplicable.

[8] To simplify the notation we preserve the continuous–time notation $(\cdot)(t)$ for all signals, in the understanding that they are constant along the sampling periods.

impedance. Similarly, in the presence of a compensator, the actuator equation (5) becomes

$$\tilde{v}(t) = -Z_a^*(q)\tilde{i}(t) + v_S(t), \quad \forall\, t \in [kd, (k+1)d), \quad k \in \mathbb{Z}_+. \tag{14}$$

The dynamics is completed with the equations of the transmission line (4) and the compensator (9) (or (10)). For the well–posedness analysis it is convenient to rewrite the system in the following form. From the transmission line equations (4) and the plant equation (14) we obtain $i(t,0) = -P(q)v(t,0)$, where

$$P(z) \triangleq -\frac{1}{Z_0}\frac{z^2 - K_p^*(z)}{z^2 + K_p^*(z)}, \quad K_p^*(z) = \frac{Z_p^*(z) - Z_0}{Z_p^*(z) + Z_0}. \tag{15}$$

The overall system can then be represented with the block diagram of Fig. 4. We recall Definition 3–9

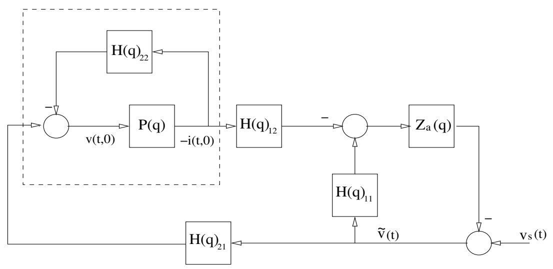

Fig. 4. *Block diagram representation of the overall discrete–time system.*

of (Chen, 1984). As indicated in (Chen, 1984) this definition is needed for a practical design—see also (Calier and C.A., 1982). Indeed, the standard definition that looks only at the overall transfer matrix is *not sufficient* to avoid the presence of "internal" improper loops. On the other hand, the definition of well–posedness used in (R. Ortega, 2004) is more restrictive than the one given below—which is sufficient for our purposes.

Definition 1. Let every subsystem of a composite system be describable by a rational transfer function. Then the composite system is said to be well posed *if the transfer function of every subsystem is proper and the closed-loop transfer function from any point chosen as an input terminal to every other point along the directed path is well defined and proper.*

From properness of $H(z)$ and $P(z)$ the proposition below can be established via direct application of Definition 1 to the block diagram of Fig. 4.

Proposition 1. Consider the system depicted in Fig. 3 with the compensator (9), $H(z) \in \mathbb{R}^{2\times 2}(z)$ proper and the transmission line described by the Telegraphers equation (1). Suppose Assumptions A.0 and A.1 hold. Then, the overall system is well–posed, if and only if

$$\begin{gathered}[Z_a H_{11}](\infty) \neq -1 \\ H_{22}(\infty) \neq Z_0 \\ \left[\frac{P}{1+PH_{22}}H_{12}\frac{Z_aH_{21}}{1+Z_aH_{11}}\right](\infty) \neq 1\end{gathered} \tag{16}$$

where $P(z)$ is defined in (15).

For the ideal compensator proposed in (R. Ortega, 2004) we have that $H_{22}(\infty) = Z_0$ that violates condition (16), confirming that the overall system is not well posed. [9]

5. PROCEDURE FOR THE DESIGN OF ROBUSTLY STABLE COMPENSATORS

The specifications of the wave attenuation problem are given in terms of *transient performance* improvement—namely, reducing the overshoot of the step response. Furthermore, the objective should be achieved *without knowledge* of the plant dynamics. In this paper we aim at a less ambitious objective and identify a family of compensators—parameterized by one tuning coefficient—that does not require prior knowledge of the plant and ensures asymptotic convergence to a loss–less steady–state, that is,

$$\lim_{t\to\infty}[\tilde{v}(t)\tilde{i}(t) - v(t,\ell)i(t,\ell)] = 0.$$

Furthermore, internal stability of the overall system and asymptotic regulation of the terminal voltage is ensured, e.g., all internal signals are bounded and

$$\lim_{t\to\infty}[\tilde{v}(t) - v(t,\ell)] = 0.$$

The transient performance improvement is discussed in Section 6 for a representative example. To establish our results we introduce the following:

Assumption A.2 *The plant impedance $Z_p(s)$ is* strictly positive real.

Assumption A.3 *The voltage source is a step of the form $v_s(t) = V_s u_{-1}(t),\ V_s \in \mathbb{R}$.*

Assumption A.2, which is always verified in practice, allow us to invoke passivity arguments to prove stability without knowledge of the plant. We should underscore that, due to the presence of the voltage source, $v_s(t)$, the actuator subsystem is *not passive*. Therefore, even for passive compensator—for instance, LTI RLC filters—stability cannot be ensured with Assumption A.2 alone. However, under Assumption A.3, the actuator is quasi–passive and the recent interesting results of (Polushin and Marquez, 2004), together with the property of passivity of (power–preserving) interconnected subsystems, can be used to complete the stability analysis. Since the proposed compensator includes active elements we cannot invoke these arguments in our analysis and an alternative route, still relying on passivity, will be taken. Assumption A.3

[9] The proof of Proposition 1 is fully detailed in a complete internal report available from the authors under request.

will be required in our case to be able to prove an asymptotic tracking property.

Motivations for decoupling

To design the compensator we will concentrate on (11) and proceed as follows. Assuming known the line characteristic impedance Z_0, and noting that $W(z)$ is invertible, equation (12) parameterizes the compensator in terms of the matrix $M(z)$. We will show below that the well–posedness restriction of Proposition 1 will translate into some structural constraints for $M(z)$, specifically some non–decoupling and relative degree conditions. The stability objectives can also be expressed in terms of constraints on $M(z)$. Internal stability will be established invoking a passivity argument. Namely, restricting to matrices $M(z)$ such that the operator seen from the plant is passive. As will be shown below, this imposes some degree and parametric restrictions on $M(z)$. Finally, the asymptotic stability condition will be ensured imposing $M(1) = I$. The first natural candidate matrices $M(z)$ are of the form:

$$\begin{bmatrix} m_{11}(z) & 0 \\ 0 & m_{22}(z) \end{bmatrix}, \quad \begin{bmatrix} m_{11}(z) & 0 \\ m_{21}(z) & m_{22}(z) \end{bmatrix},$$

$$\begin{bmatrix} m_{11}(z) & m_{12}(z) \\ 0 & m_{22}(z) \end{bmatrix}$$

where $m_{ij}(z) \in \mathbb{R}(z),\ i,j = 1,2$ are arbitrary and possibly improper, and correspond to full–, voltage– and current–decoupling behaviors, respectively. There are two strong motivations to aim at decoupling. On one hand, it has been shown in (R. Ortega, 2004) that, due to the signal decoupling that permits the definition of a measurable error dynamics, it is possible to design *adaptive* versions of the resulting compensators that estimate the transmission line parameter Z_0. On the other hand, thanks to the diagonal/triangular structure, it is possible to express the conditions for *internal stability* in terms of the transfer functions $m_{ij}(z)$. Indeed, let us consider for illustration the case of current–decoupling for which we have

$$i(t,\ell) = \frac{m_{22}(q)}{m_{12}(q)}[v(t,\ell) - m_{11}(q)\tilde{v}(t)].$$

Terminating with the plant dynamics we obtain the transfer function yields the block diagram representation of Fig. 5, from which we see that stability of $m_{11}(z)$ and *positive realness* of $-\frac{m_{22}(z)}{m_{12}(z)}$ ensure stability for all strictly positive real plants. These conditions, together with the zero steady–state error requirement $M(1) = I$, will be imposed on our design below.

Unfortunately, it can be proven that voltage–decoupling compensators yield to an ill-posed interconnection.

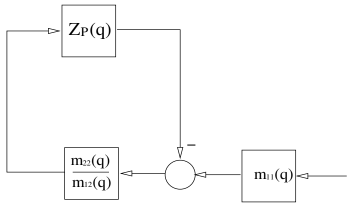

Fig. 5. *Feedback interconnection of interest for the current–decoupled* $M(z)$.

6. MAIN RESULT

We are in position to present the main result of the paper.

Proposition 2. Consider the system depicted in Fig. 3 where the transmission line is described by the Telegraphers equation (1). Suppose Assumptions A.1–A.3 hold. Let the voltage and current of the regulated sources be defined by the proper compensator

$$\begin{bmatrix} \tilde{i}(t) \\ v(t,0) \end{bmatrix} = H(q,\alpha) \begin{bmatrix} \tilde{v}(t) \\ i(t,0) \end{bmatrix}, \qquad (17)$$

where

$$H(q,\alpha) = \begin{bmatrix} \dfrac{q^2-1}{D(q,\alpha)} & \dfrac{-2Z_0 q}{D(q,\alpha)} \\[2ex] \dfrac{2[\alpha(q^2-1) - Z_0]}{D(q,\alpha)} & \dfrac{\gamma(\alpha)(q^2-1)}{D(q,\alpha)} \end{bmatrix},$$

$D(q,\alpha) = 2\alpha(q^2-1) - Z_0(q^2+1)$,
$\gamma(\alpha) = -Z_0(2\alpha + Z_0)$ and $\alpha \leq -\frac{1}{2}Z_0$.

Under these conditions:

1 The overall system is well–posed and internally stable.
2 The following asymptotic behavior is ensured

$$\lim_{t\to\infty}[\tilde{v}(t)\tilde{i}(t) - v(t,\ell)i(t,\ell)] = 0$$
$$\lim_{t\to\infty}[\tilde{v}(t) - v(t,\ell)] = 0.$$

3 The compensator–transmission line subsystem is current–decoupled

$$\begin{bmatrix} v(t,\ell) \\ i(t,\ell) \end{bmatrix} = \begin{bmatrix} \dfrac{1}{q} & \alpha(q - \dfrac{1}{q}) \\[2ex] 0 & -\dfrac{1}{Z_0}\left(\alpha q - \dfrac{Z_0+\alpha}{q}\right) \end{bmatrix} \begin{bmatrix} \tilde{v}(t) \\ \tilde{i}(t) \end{bmatrix}.$$

4 The mapping $v_s(t) \mapsto v(t,\ell)$ is given by

$$v(t,\ell) = \frac{Z_p^*(q)\left(1 + \frac{Z_0}{\alpha}\right)}{Z_p^*(q) + Z_0} v(t-2d,\ell) +$$
$$+ \frac{Z_0 - Z_a^*(q)}{Z_p^*(q) + Z_0} v(t-2d,\ell) + \frac{Z_p^*(q)}{Z_p^*(q) + Z_0} v_s(t-d) +$$
$$+ \frac{(1 + \frac{Z_0}{\alpha})Z_p^*(q)}{Z_p^*(q) + Z_0} v_s(t-3d). \qquad (18)$$

The effective action of the compensator on the transient performance is captured in (18).[10] Before discussing further this equation let us take a particular case of the class given above where expressions are simpler.

Corollary 1. Under the conditions of Proposition 2, and setting $\alpha = -Z_0$, the compensator–transmission line subsystem verifies

$$\begin{bmatrix} v(t,\ell) \\ i(t,\ell) \end{bmatrix} = \begin{bmatrix} \frac{1}{q} & -Z_0(q - \frac{1}{q}) \\ 0 & q \end{bmatrix} \begin{bmatrix} \tilde{v}(t) \\ \tilde{i}(t) \end{bmatrix},$$

and the mapping $v_s(t) \mapsto v(t,\ell)$ is given by

$$v(t,\ell) = \frac{Z_0 - Z_a^*(q)}{Z_p^*(q) + Z_0} v(t-2d,\ell) + \frac{Z_p^*(q)}{Z_p^*(q) + Z_0} v_s(t-d). \tag{19}$$

Comparing (19) with (18) one should remark the effects of the particular choice $\alpha = -Z_0$ on the mapping $v_s(t) \mapsto v(t,\ell)$. Namely, the term $v_s(t-3d)$ that might induce additional oscillations is eliminated, and the term in front of $v(t-2d,\ell)$—the reflection coefficient—has been modified. These two equations should be compared with the uncompensated relation (7)—modulo the discretization Assumption A.1 which is essentially technical. Without further knowledge about $Z_p(s)$ it is difficult to assess the effect of the proposed law on the transient behavior. However, the following analysis is illustrative. If the surge impedance is purely resistive and satisfies $Z_a << Z_0$ the *new reflection coefficient* can be approximated by $\frac{Z_0}{Z_p(s)+Z_0}$. In this case, we observe that the plant reflection coefficient $\frac{Z_p(s)-Z_0}{Z_p(s)+Z_0}$ has been multiplied by a factor $[\frac{Z_p(s)}{Z_0} - 1]^{-1}$. If the relative degree of the plant is zero and $Z_p(\infty) > Z_0$, then we achieve *improved attenuation* at infinite frequency.

7. CONCLUSIONS AND OUTLOOK

We have given in this paper rigorous theoretical foundations for the compensator design framework proposed in (R. Ortega, 2004). In particular, using the adequate definition of well–posedness we have completely characterized the achievable compensator–line behaviors that lead to proper compensators with well–defined interconnections. We have, then, identified a family of current–decoupling (well–posed and proper) schemes that ensure asymptotic stability for all strictly positive real plants. These issues were not properly addressed in (R. Ortega, 2004), where inadequate definitions of well–posedness and positive realness led to overly conservative conditions and no clear explanation—besides simulation evidence—to the interest of current–decoupling was given. Of particular relevance is the new stability analysis presented here—which was mentioned as an open problem in (R. Ortega, 2004). Anyway, we want to underline that, as indicated in Section 4, the discretization Assumption A.1, introduced for the well–posedness analysis, is quite critical. In the case of a purely resistive plant this assumption is not needed. Notice also that in the key well–posedness conditions (16) the model of the plant appears only in the third one, which is generically satisfied. In spite of these arguments it is clear that, to render the result more practical, the relaxation of this assumption is needed. For AC drive applications an approximation of the proposed active compensator with a shunt passive LTI filter would lead to a workable design. It is possible to show that a continuous–time approximation, e.g., with a Pade approximation of the delay, of the compensator proposed in (R. Ortega, 2004) is not positive real—hence not realizable with RLC circuits. However, some preliminary computations for the more general scheme (17) suggests the existence of an interval for the free parameter α for which positive realness is ensured. The outcome of this research will be reported elsewhere. The construction at the University of Illinois of an experimental, low power, rig to test the proposed algorithms is also under investigation.

[10] The proof of Proposition 2 is also fully detailed in a complete internal report available from the authors under request.

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